MAT 307 - Spring 2009

Assignment 5

Due: April 29

Problem 1. [21 points]
Let G be a connected graph on n vertices with minimum degree d and maximum degree D, A be the adjacency matrix of G with eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$, and $W(m)$ denote the number of closed walks of length m in G. Prove the following:

(a) For each positive integer m, $W(m) = \mathrm{Tr}(A^m) = \sum_{i=1}^n \lambda_i^m$.

(b) $\lambda_1 \geq -\lambda_n$.

(c) G is bipartite if and only if $\lambda_n = -\lambda_1$.

(d) $\lim_{m\to\infty} W(2m)^{\frac{1}{2m}} = \lambda_1$, where m ranges over positive integers.

(e) G is not bipartite if and only if the $\lim_{m\to\infty} W(m)^{\frac{1}{m}}$ exists.

(f) $\max(d, \sqrt{D}) \leq \lambda_1 \leq D$.

(g) The chromatic number of G is at most $\lambda_1 + 1$.