MAT 307 - Spring 2009

Assignment 3

Due: April 1

The solution for each problem should be no longer than one page.

Problem 1. [4 points]
Prove that every graph with m edges has a k-colorable subgraph with at least $(1 - \frac{1}{k})m$ edges.

Problem 2. [6 points]
Prove that every tournament has a hamiltonian path, i.e., a directed path which visits every vertex once.

Problem 3. [6 points]
Prove that there exists a tournament T on n vertices with at least $n!2^{1-n}$ hamiltonian paths.

Problem 4. [8 points]
Let $v_1, \ldots, v_n$ be n vectors in $\mathbb{R}^n$ of unit length $|v_i| = 1$. Prove that there are signs $\epsilon_i = \pm 1$ such that

$$|\epsilon_1 v_1 + \ldots + \epsilon_n v_n| \leq \sqrt{n}.$$

Show that this estimate is tight, i.e., the $\sqrt{n}$ estimate cannot be improved.