Syllabus for 18.997: Probabilistic Methods in Combinatorics, Spring 2011

Instructor: Jacob Fox, fox@math.mit.edu, Room 2-363c

Time: Tuesdays and Thursdays, 11am-12:30pm

Location: Room 2-105

Office hours: Tuesdays 12:30-1:30 p.m. or by appointment

Prerequisites: Permission of instructor. Some probability and combinatorics would be helpful but can be learned in the course.

Textbook: The Probabilistic Method, by N. Alon and J. Spencer, 3rd edition, Wiley, 2008

Grades: Grades will be based on 6 problem sets. No exams.

Objective: To develop an appreciation for the strength and beauty of the probabilistic method

Suggestions: --Class participation and discussion are highly encouraged.
--Please feel free to ask me questions before, during, or after class.

Rough Outline

Chapter 1: The Basic Method

- 2/1 Ramsey numbers, winning sets in tournaments, sum-free sets
- 2/3 Dominating sets in graphs, 2-coloring hypergraphs, Erdos-Ko-Rado, disjoint pairs in set-systems

Chapter 2: Linearity of Expectation

- 2/8 Max-cut, Hamiltonian paths, Ramsey multiplicity
- 2/10 Balancing vectors, unbalancing lights

Chapter 3: Alterations

- 2/15 Improved bounds for Ramsey numbers, Turan's theorem, graphs of large girth and chromatic number
- 2/17 Recoloring and 2-coloring hypergraphs

Chapter 4: The Second Moment Method

- 2/24 Turan's proof of the Hardy Ramanujan theorem
- 3/1 Random graphs: threshold functions and clique number
- 3/3 Distinct sums, Rodl nibble.

Chapter 5: Lovasz Local Lemma

- 3/8 2-coloring hypergraphs, lower bounds for Ramsey numbers, decomposing sphere coverings
- 3/10 Coloring reals with all shifts multicolored, linear arboricity of graphs

Appendix A: Bounding of Large Deviations

- 3/15 Chernoff bounds and applications

Chapter 7: Martingales and Tight Concentration

- 3/17 Azuma's inequality
- 3/29 Applications of Azuma's inequality: chromatic number of random graphs, isoperimetric inequality for the Hamming cube
- 3/31 Applications of Talagrand's inequality: longest increasing subsequence in a random permutation, clique number of random graphs

Chapter 6: Correlation Inequalities

- 4/5 Four Functions Theorem of Ahlswede and Daykin, Kleitman's Lemma, FKG Inequality, correlated events in random graphs

Chapter 9: Pseudorandomness

- 4/7 Explicit constructions, the Quadratic Residue Tournament
- 4/12 Eigenvalues and expanders
- 4/14 Quasirandom graphs

Dependent Random Choice

- 4/21 Turan numbers of bipartite graphs, Ramsey number of the cube, embedding 1-subdivided graphs
- 4/26 Variation, Balog-Szemeredi-Gowers
- 4/28 Ramsey numbers of sparse graphs

Probabilistic Gems

- 5/3 Crossing numbers, incidences, sums and products
- 5/5 Independence number of triangle-free graphs, local coloring
- 5/10 Weierstrass Approximation Theorem, maximal antichains

Chapter 13: Discrepancy

- 5/12: Chernoff estimate, six standard deviations suffice