Syllabus for 18.318: Probabilistic Methods in Combinatorics

Spring 2014

Instructor:	Jacob Fox, fox@math.mit.edu, Room E17-316
Time:	MWF 1:00 - 2:00
Location:	Room E17-133
Office hours:	Fridays 3:00-4:00 or by appointment
Prerequisites:	Permission by instructor
Textbooks:	The Probabilistic Method, 3rd Edition, by Noga Alon and Joel Spencer, Wiley, 2008
Grades:	No exams; grades are based on 6 problem sets
Objective:	To learn and appreciate the probabilistic method
Suggestions:	Class participation and discussion are highly encouraged. Please feel free to ask questions before, during, or after class.

Rough Outline

Chapter 1: The Basic Method

2/5 Ramsey numbers, winning sets in tournaments.

2/7 sum-free sets, dominating sets, Property B,

2/10 Erdős-Ko-Rado, disjoint pairs in set-systems

Chapter 2: Linearity of Expectation

2/12 Max-cut, Hamiltonian paths,

2/14 Ramsey multiplicity, Balancing vectors, unbalancing lights

2/18 Stanley-Wilf limits

Chapter 3: Alterations

2/19 Ramsey numbers revisited, Turán's theorem,

2/21 Graphs of large girth and chromatic number

2/24 Recoloring and 2-coloring hypergraphs

Chapter 4: The Second Moment Method

2/26 Turán's proof of the Hardy-Ramanujan theorem

2/28 Random graphs: threshold functions and clique numbers

3/3 Distinct sums

3/5 Rödl's nibble method

Chapter 5: Lovász Local Lemma

3/7 Local lemma, 2-coloring hypergraphs

3/10 Ramsey numbers revisited, Decomposing sphere covers

3/12 Real colorings with multicolored shifts, linear arboricity of graphs

3/14 Algorithmic local lemma

Appendix A: Bounds of Large Deviations

3/17 Chernoff bounds and applications

Chapter 7: Martingales and Tight Concentration

3/19 Azuma's inequality

3/21 Applications of Azuma's inequality: chromatic number of random graphs, isoperimetric inequality for the Hamming cube

3/31 Applications of Talagrand's inequality: longest increasing subsequence of a random permutation, clique number of random graphs

Chapter 6: Correlation Inequalities

4/2 Four Functions Theorem, Kleitman's Lemma

4/4 FKG inequality, correlated events in random graphs

Chapter 9: Pseudorandomness

4/7 Explicit constructions, the quadratic residue tournament

4/9 Eigenvalues and expanders

4/11 Quasirandom graphs

4/14 Quasirandom graphs II

Dependent Random Choice

4/16 Basic lemma

4/18 Turán numbers of bipartite graphs, Ramsey number of the cube, embedding 1-subdivided graphs

4/23 Balog-Szemeredi-Gowers lemma

4/25 Ramsey numbers of sparse graphs

4/28 Sidorenko's conjecture

4/30 Ramsey-type conjecture of Erdős

5/2 Erdős-Fajtlowicz conjecture

Probabilistic Gems

5/5 Crossing numbers, incidence geometry, sums and products

5/7 Independence number of triangle-free graphs, local coloring

5/9 Weierstrass Approximation theorem, maximal antichains

Discrepancy

5/12 Chernoff estimate, six standard deviations suffice

5/14 Algorithmic proof of six standard deviations suffice