

## **Syllabus for 18.315: Combinatorial Theory**

Fall 2011

- Instructor: Jacob Fox, fox@math.mit.edu, Room 2-363c
- Time: Mondays and Wednesdays 12:30-2:00
- Location: Room 2-139
- Office hours: Fridays 3:00-4:00 or by appointment
- Prerequisites: Permission by instructor. Basic knowledge of combinatorics, graph theory, and linear algebra.
- Textbooks:  
(recommended) Ramsey theory by R. Graham, B. Rothschild, J. Spencer,  
Extremal combinatorics by S. Jukna  
Modern Graph Theory by B. Bollobas  
Linear algebra methods by L. Babai and P. Frankl
- Grades: No exams; grades will be based on 6 problem sets.
- Objective: To learn and appreciate the problems and methods in extremal combinatorics
- Suggestions: Class participation and discussion are highly encouraged. Please feel free to ask me questions before, during, or after class.
- Class website: <http://math.mit.edu/~fox/18315/>

## **Rough Outline**

- 9/7 Ramsey's theorem and its applications
- 9/12 Hypergraph Ramsey numbers I
- 9/14 Hypergraph Ramsey numbers II
- 9/19 Theorems of van der Waerden, Hales-Jewett, and Szemerédi
- 9/21 Student holiday – no class
- 9/26 Turán's theorem and its applications
- 9/28 Erdős-Stone-Simonovits theorem
- 10/3 Bipartite Turán problems, algebraic constructions
- 10/5 Szemerédi's regularity lemma, statement and proof
- 10/10 Columbus day – no class
- 10/12 Applications I - graph removal lemma, (6,3)-theorem, Roth's theorem, Ajtai-Szemerédi corner theorem
- 10/17 Roth's theorem – proof using Fourier analysis
- 10/19 Applications II - Ramsey numbers of bounded degree graphs, induced Ramsey theorem
- 10/24 Hypergraph regularity, removal, and the multidimensional Szemerédi theorem
- 10/26 Weak regularity lemmas and algorithmic aspects
- 10/31 Property testing I
- 11/2 Property testing II
- 11/7 Sunflower lemma
- 11/9 LYM inequality, Sperner's theorem, and Littlewood-Offord theorem
- 11/14 Sauer-Shelah and Kruskal-Katona
- 11/16 Linear algebra in extremal set theory: Odd and even towns, Fisher's inequality, Ray-Chaudhuri-Wilson theorem
- 11/21 Two-distance sets, Graham-Pollack
- 11/23 modular intersection theorems and the chromatic number of  $n$ -space
- 11/28 Frankl-Wilson theorem, constructive lower bounds for Ramsey numbers, and Borsuk's conjecture.
- 11/30 Dvir's proof of the finite field Kakeya problem, the joints problem
- 12/5 Combinatorial Nullstellansatz – statement and proof, Cauchy-Davenport, Chevalley-Warning
- 12/7 Restricted sums
- 12/12 Regular subgraphs, covering cubes
- 12/14 Graph coloring