18.315: Problem Set 2

Due: Wednesday October 19

Prove 5 of 6 of the following.

1. Every n-vertex graph containing no path with 3 edges has at most n edges.

2. Every n-vertex graph with minimum degree at least $n/2$ contains a cycle of length n.

3. Every 3-uniform hypergraph G on n vertices such that no 6 vertices contains 3 edges has $o(n^2)$ edges.

4. Every graph on $n \geq 2$ vertices contains an ϵ-regular pair with each part of order at least δn with $\delta = 2^{-\epsilon^{-O(1)}}$.

5*. For every graph H and $\epsilon > 0$, there is $\delta = \delta(\epsilon, H) > 0$ such that every graph on n vertices without an induced copy of H contains an induced subgraph on at least δn vertices which has edge density at most ϵ or at least $1 - \epsilon$.

6*. For each $c > 0$ there is $c' > 0$ such that every graph G with n vertices and cn^2 edges contains a d-regular subgraph with $d \geq c'n$.