18.315

1 Problem Set 1

Due Monday October 3

For a graph H and a positive integer n, let $c(H, n)$ denote the fraction of copies of H in K_n which must be monochromatic in any 2-edge-coloring of K_n.

1. Prove that $c(H) = \lim_{n\to\infty} c(H, n)$ exists and is positive.
2. Prove that $c(K_3) = 1/4$.
3. Prove that $c(K_{s,t}) = 2^{1-st}$, where $K_{s,t}$ is the complete bipartite graph with parts of order s and t.
4. Prove that for each r there is $N = N(r)$ such that the following holds:

For every r-coloring of the complete graph on the grid $[N] \times [N]$, there is a rectangle whose opposite edges have the same color. That is, there are $x \neq x'$ and $y \neq y'$ such that the edges $((x, y), (x, y'))$ and $((x', y), (x', y'))$ have the same color, and the edges $((x, y), (x', y))$ and $((x, y'), (x', y'))$ have the same color.