BRAIDING OF SMEARED PRIMARY FIELDS

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Start by discussing braiding:

When we see braiding, we have a braid group $B_n = \pi_1(Conf_n(\mathbb{C}))$. This contains $P_n$ the set of loops in the configuration space of $n$ distinguished points in the plane.
In order to get configurations of points, we fix $\infty$ and a tangent vector. The compliment is $\mathbb{C}$ with a preferred framing.

4 point functions: Take as input 4 points on $\mathbb{C}P^1$ together with $IPER$ (irreducible positive energy representations) assigned to those points. Elements of these $IPER$’s and tangent vectors at the points and produce elements of a vector space. Call this the space of 4 point functions.

Geometric picture of fusion:
When they collide, the function that gets assigned is the function that gets attached to a single IPER at some point. This is the fusion object.

Why do we look at 4 points? There is an action of $PGL_2(\mathbb{C})$ on $\mathbb{C}P^1$ that is sharply triply structure i.e. given $a, b, c \in \mathbb{C}$ there is a unique $\gamma \in PGL_2(\mathbb{C})$ such that $\gamma(a) = 0$, $\gamma(b) = 1$ and $\gamma(c) = \infty$. If we have 4 points, we can send the first three to 0, 1 and $\infty$ and the fourth goes to a unique element called the cross ratio. In the second picture, if $\alpha = 1$, $z$ describes the set of these points up to isomorphism.

**Definition.** A configuration of 4 points in the sphere is a function on $\mathbb{C}P^1 \setminus \{0, 1, \infty\}$.

In fact, we will have a multi-valued function: 4 point functions will satisfy the KZ equation and the solutions don’t necessarily exist globally.

Toly mentioned a correspondence between differential equations of regular singular type and local systems (or locally constant sheaves). The latter corresponds to representations of the fundamental group of the configuration space by assigning to each point a vector space $p \to P_p$. To each path $\gamma$ connecting $p$ and $q$ we get an isomorphism $V_p \to V_q$ dependent on the homotopy group.

Pushing $z$ around endows the space of 4 point functions at some fixed configuration with an action of the fundamental group of $\mathbb{C}P^1 \setminus \{0, 1, \infty\}$.

Now onto primary fields: We are given $V$ and $U$ representations of $SU(N)$ and $H_i, H_j, H_k$, positive energy reps of $LSU(N)$ primary field. $LSU(N)$ is
the equivalent map $V[z, z^{-1}] \otimes H_j \to H_i$. Primary fields can be “smeared” by integrating against a smooth function.

Suppose $f$ is a smooth function $f = \sum_n f_n z^n$ then $\varphi'_{ji}(v, f) = \sum \varphi_{ji}(v, n)f_n$. This eats $z^n$ and produces a map $H_j \to H_i$.

To describe the braiding, we compare $\varphi^u_{kj}(u, g)\varphi^v_{ji}(v, f)$ with $\varphi^u_{kh}(v, f)\varphi^u_{ki}(u, g)$.

The formula: $\varphi^u_{kj}(u, f)\varphi^v_{ji}(v, g) = \sum_n c_{jk}\varphi^u_{kh}(v, e_{\mu_{jh}} \cdot g)\varphi^u_{hi}(u, e_{-\mu_{jh}} \cdot f)$ where the $c_{ij}$ are the transport coefficients.
$e_{\mu jh}$ is a multiplicity phase $e_\mu(f(e^{i\theta}) = e^{i\mu \theta} f(e^{i\theta})$. What is $\mu jh$? It is determined by the representations $V$, $U$, and a subset of $H_i$, $H_k$, $H_l$ and $H_h$. From Nick’s talk, the Sugawarn construction produced a construction of the Virasaro algebra (which is the span of $\{L_n\}_{n \in \mathbb{Z}} \oplus \mathbb{C} k$) on $H_i$, $H_j$ etc. There is a distinguished operator $L_0$. PER’s are graded by $L_0$ eigenvalue $\in \mathbb{C}$. Each IPER has $L_0$ eigenvalues in some coset of $\mathbb{Z}$ and there is a well defined lowest energy. If $L_0$ acts by integers then the action integrates to an action of $\mathbb{T}_{\text{rot}}$. Otherwise we get an action of some finite cover of $\mathbb{T}_{\text{rot}}$. 