18.950 Homework 4

Due in lecture Wednesday October 3. Book references are from do Carmo.


6. Let $I$ be an interval, $\alpha : I \to \mathbb{R}^3$ be a regular parameterized curve, and $\beta : I \to \mathbb{R}^3$ a smooth function with $\beta \neq 0$. We define a parameterized surface by

$$x(u, v) = \alpha(u) + v\beta(u), \quad (u, v) \in I \times \mathbb{R}.$$ 

This is called a ruled surface, with rulings $\beta$ and directrix $\alpha$ (e.g. a cylinder, having $\alpha$ a circle and $\beta$ constant). Show that a regular ruled surface has Gaussian curvature $K \leq 0$. 
