Recall the notation
\[ \langle x \rangle := \sqrt{1 + |x|^2}. \]

**Exercise 5.1.** (a) Let \( s \in \mathbb{R} \). Show that
\[ m(x, \xi) = \langle x \rangle^s, \quad m(x, \xi) = \langle \xi \rangle^s, \quad m(z) = \langle z \rangle^s \]
are order functions, where we denote \( z = (x, \xi) \).

(b) Show that the order functions in part (a) satisfy \( m \in S(m) \).

**Exercise 5.2.** Show that
\[ a(x, \xi) = \sum_{|\alpha| \leq k} a_\alpha(x) \xi^\alpha \]
where each \( a_\alpha(x) \) has all derivatives bounded, lies in \( S(\langle \xi \rangle^k) \).

**Exercise 5.3.** (a) Let \( m_1, m_2 \) be order functions. Show that \( m_1 m_2 \) is an order function as well.

(b) Show that if \( a_1 \in S(m_1), a_2 \in S(m_2), \) then \( a_1 a_2 \in S(m_1 m_2) \).

**Exercise 5.4.** (a) Arguing similarly to the proof in the lecture, show that if \( m \) is an order function and \( a \in S(m) \), then \( \text{Op}_h(a)^* \) is a continuous operator on \( \mathcal{S}(\mathbb{R}^n) \).

(b) Using part (a), show that \( \text{Op}_h(a) \) is a continuous operator on \( \mathcal{S}'(\mathbb{R}^n) \).

**Exercise 5.5.** Using Exercise 3.5(b) and following the proof for standard quantization given in the lecture, show that if \( m \) is an order function and \( a \in S(m) \), then the Weyl quantization \( \text{Op}_h^w(a) \) is a continuous operator on \( \mathcal{S}(\mathbb{R}^n) \) and on \( \mathcal{S}'(\mathbb{R}^n) \).

**Exercise 5.6.** (a) Show that \( a(x, \xi) = e^{-i(x, \xi)} \) does not lie in \( S(m) \) for any order function \( m \).

(b) With \( a \) defined in part (a) and \( h := 1 \), show that \( \text{Op}_h(a) \) does not map \( \mathcal{S}(\mathbb{R}^n) \) to itself, and it does not map \( \mathcal{S}'(\mathbb{R}^n) \) to itself either.

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