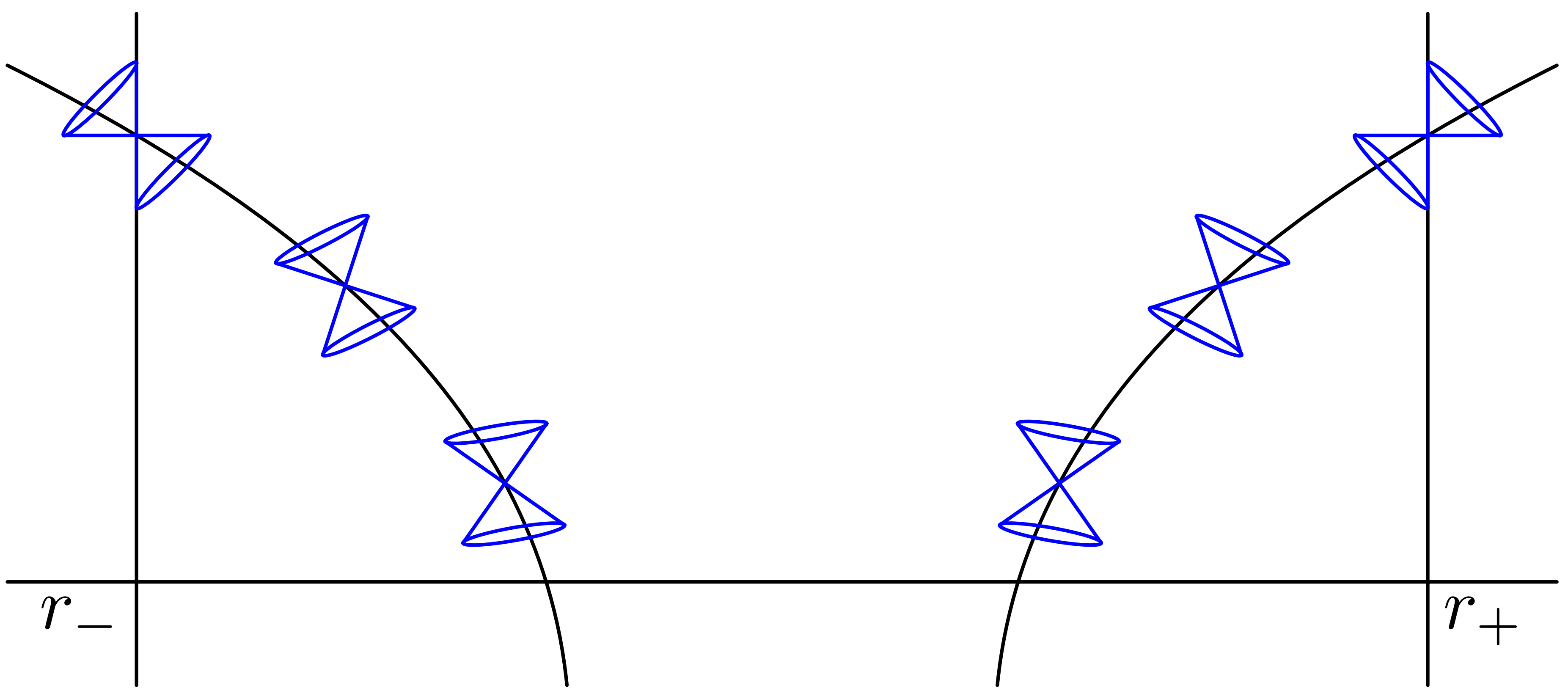
Kerr–de Sitter black hole

The Kerr–de Sitter metric models a rotating black hole in a universe with positive cosmological constant. We are working with the spacetime

$$N = \mathbb{R}_{\text{time}} \times (r_-, r_+) \times \mathbb{S}^2.$$

The two surfaces $\{r = r_\pm\}$ are called event horizons; any positively time oriented curve can only cross them in the direction that leaves N.



Two radial timelike curves and light cones.

The metric has three parameters: M, the mass of the black hole; $\Lambda > 0$, the cosmological constant, and a, the angular momentum. We assume that the black hole is rotating slowly; that is, $|a|$ is small.

Methods of the proofs

Theorem 1 does not follow from the general meromorphic continuation results of scattering theory, such as Mazzeo–Melrose theorem; one reason is that the operator $P_g(\omega)$ is not elliptic inside the two ergospheres close to the event horizons. Instead, we use separation of variables and reduce our problem to finding inverses for the operators in (3) and (4). (We have to take some care here as the operator in (4) is not self-adjoint for $a \neq 0$ and $\omega \notin \mathbb{R}$; thus, it is not guaranteed to have a complete set of eigenfunctions.)

The problem (4) is equivalent to the eigenvalue problem for an $O(h)$ nonselfadjoint perturbation of a semiclassical self-adjoint operator; its eigenvalues satisfy a certain complex quantization condition. The problem (3) is equivalent to finding barrier-top resonances in one-dimensional potential scattering. Detailed analysis of these two problems gives Theorems 1 and 2.

References

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