Math 54-1, HW 6

4.1 2

(a) Yes: \( u \in W \implies u_1 \cdot u_2 \geq 0 \), but
\[
cu = [c_{u_1}], \quad cu \cdot cu_2 = c^2 u_1 \cdot u_2 \geq 0 \implies cu \in W
\]

(b) \( \vec{u} = [1, 0], \quad \vec{v} = [0, -1] \)

(c) Not a subspace, as it does not contain the zero polynomial: \( a + t^2 \) is not the zero polynomial for any \( a \).

(12) We have \( W = \text{Span} \{ \vec{u}, \vec{v} \} \), where \( \vec{u} = [1, 0], \vec{v} = [0, -1] \).

By Theorem 1, \( W \) is a subspace of \( \mathbb{R}^2 \).

24 (a) True, by definition (p. 216)
(b) True, by (3) on p. 217
(c) True: for example, a vector space is a subspace of itself.
(d) False, see Example 8.

(1) \( H \) \( OEH \)
(2) \( u, v \in H \implies u + v \in H \)
(3) \( u \in H, c \in \mathbb{R} \implies cu \in H \)

We need to prove:

(1) \( \text{HOK} \implies OEH \) and \( OEK \).
(2) \( \text{HOK} \implies u + v \in \text{HOK} \).
(3) \( u \in \text{HOK} \), \( c \in \mathbb{R} \implies cu \in \text{HOK} \).

We also need to prove:

(1) \( \text{HOK} \implies \text{HOK} \).
(2) \( u, v \in \text{HOK} \implies u + v \in \text{HOK} \).
(3) \( u \in \text{HOK} \), \( c \in \mathbb{R} \implies cu \in \text{HOK} \).

4.2 8 (b) Not a vector space of \( \mathbb{R}^2 \), as does not contain the zero vector. Indeed, if \( r = s = 0 \), then \( 5r - 1 \neq 5 + 2t \).

(12) Not a subspace of \( \mathbb{R}^4 \), as does not contain the zero vector. Indeed, otherwise there is a \( b, d \) such that \( b - 5d = 2b = 2d + 1 = d = 0 \). But \( d \neq 0 \implies 2d + 1 \neq 0 \), a contradiction.
30. Let $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Basis for $\text{Null } A$ is

$\text{Null } A \rightarrow B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Row vectors $A \rightarrow B = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

$A^T = 0$.