2.2 \[ \begin{pmatrix} 3 & 2 \\ 7 & 4 \end{pmatrix}^{-1} = \frac{1}{12-14} \begin{pmatrix} 4 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 7/2 & -3/2 \end{pmatrix} \]

8. Given: \( A \) is invertible, \( A D = I \)

Need to prove: \( D = A^{-1} \)

Since \( A \) is invertible, there exists \( A^{-1} \). Multiply both sides of \( AD = I \) to the left by \( A^{-1} \):

\( A^{-1}AD = A^{-1} \); but \( A^{-1}AD = (A^{-1}A)D = I \cdot D = D \).

10. a. False: \( (AB)^{-1} = B^{-1}A^{-1} \)
   
   b. True, Theorem 6(a)
   
   c. True, Theorem 4
   
   d. True, Theorem 7
   
   e. False, see Theorem 7

17. \( AB = BC \), \( A, B, C \) square, there exists \( B^{-1} \).

Multiply by \( B^{-1} \) to the left:

\[ ABB^{-1} = BC \cdot B^{-1} \]. But \( AB = BC \).

\( A = AI = A(BB)^{-1} \). So, \( A = BCB^{-1} \).

2.3 \( A \) is already in REF and has 4 pivot positions; therefore, it is invertible.

15. No, it cannot. If, for simplicity, \( A = [\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n] \) with \( \vec{a}_1 = \vec{a}_2 \), then the vectors \( \vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n \) are linearly dependent.

with a linear dependence relation \( \vec{a}_1 - \vec{a}_2 = 0 \).

16. No, by IMT (h)

17. If \( A \) is invertible, then \( A^{-1} \) is invertible, so the columns of \( A \) are lin. ind. By IMT (e).

22. \( Hx = \vec{c} \) is inconsistent for some \( \vec{c} \) \( \Rightarrow \) by IMT (g), \( H \) is not invertible \( \Rightarrow \) by IMT (d), the \( \vec{c} \) is a nonexistent solution. \( \Rightarrow \) by IMT (e), \( A \) is invertible \( \Rightarrow \) \( A^2 = A \cdot A \) is invertible \( \Rightarrow \) by IMT (h), the columns of \( A \) span \( \mathbb{R}^n \).