1. (5 pt) Consider the series
\[ \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}. \]

(a) Find the radius of convergence and the interval of convergence.
(b) Differentiate the series to obtain a new series.
(c) Find the interval of convergence of the differentiated series. (You may use that this series has the same radius of convergence as the original series.)

**Ratio Test:**
\[ \lim_{n \to \infty} \left| \frac{\frac{(-1)^{n+1} x^{n+1}}{\sqrt{n+1}}}{\frac{(-1)^n x^n}{\sqrt{n}}} \right| = \lim_{n \to \infty} \frac{|x|}{\sqrt{1+\frac{1}{n}}} = |x|. \]

- \( |x| < 1 \) \implies series converges
- \( |x| > 1 \) \implies series diverges

**Endpoints:**
- \( x = 1 \) \implies \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \]
  - Converges by Alternating Series Test,
  - \( 0 \leq \frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}} \),
  - \( \lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0 \).

\[ \sum_{n=1}^{\infty} \sqrt[n]{(-1)^n} = 1 \] \hspace{1cm} \( R = 1 \), same endpoints

- \( x = -1 \) \implies \[ \sum_{n=1}^{\infty} \frac{(-1)}{\sqrt{n}} \]
  - Diverges by p-series test
  - \( p = \frac{1}{2} < 1 \)

**Interval of Convergence:**
\( (-1, 1] \)

**Determine endpoints:**
- \( x = 1 \) \implies \[ \sum_{n=1}^{\infty} (-1)^n \sqrt{n} \]
- \( x = -1 \) \implies \[ \sum_{n=1}^{\infty} (-1)^n \sqrt{n} \]
  - Both diverge by Test for Divergence

**Interval of Convergence:**
\( (-1, 1) \)
2. (5 pt) Find a power series representation (in terms of powers of $x$) of the function 

$$f(x) = \ln(1-x^2).$$

Determine the radius of convergence of the series.

$$f'(x) = -\frac{2x}{1-x^2} = -2x \cdot \frac{1}{1-x^2} =$$

$$= -2x \cdot \sum_{n=0}^{\infty} (x^2)^n = -2x \cdot \sum_{n=0}^{\infty} x^{2n},$$

Converges for $|x| < 1 \Rightarrow |x| < 1 \quad [R=1]$.

$$f(x) = \mathcal{C} + \int f'(x) \, dx = -2 \int \sum_{n=0}^{\infty} x^{2n+1} \, dx =$$

$$= -2 \sum_{n=0}^{\infty} \frac{x^{2n+2}}{2n+2} = -\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n+1} =$$

$$= -x^2 - \frac{x^4}{2} - \frac{x^6}{3} - \cdots$$

Same $R$ of convergence.

$$f(x) = -\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n+1}$$

$R=1$.