Math 1B worksheet

Nov 2-4, 2009

In this worksheet:
DE means ‘differential equation’,
IVP means ‘initial value problem’.

1-4. Find general formulas for the solutions to the following DE and solve the following IVP:

\[
\begin{align*}
\begin{cases}
y' = 2xy^2, \\
y(0) = 1/2,
\end{cases} 
\quad (1)
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
y' = 2y, \\
y(1) = 7;
\end{cases} 
\quad (2)
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
(1 + \cos x)y' = (1 + e^{-y})\sin x, \\
y(0) = 0;
\end{cases} 
\quad (3)
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
y' = 3x^2 e^y, \\
y(0) = 1.
\end{cases} 
\quad (4)
\end{align*}
\]

5-6. Find the curve passing through the given point and orthogonal to all curves in the given family:

\[
y = e^{kx}, (1, 2); \quad (5)
\]

\[
y = ke^x, (1, 1). \quad (6)
\]

7-8. Prove that the following power series solves the given DE:

\[
y(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{1 \cdot 3 \cdot 5 \cdots (2n+1)}, \quad y' = 1 + xy; \quad (7)
\]

\[
y(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^n n!}, \quad y' = -xy; \quad (8)
\]

9. Sketch the direction fields for the following ODE and sketch the graph \(y = y(x)\), where \(y\) solves the given IVP:

\[
y' = 1 - xy, \quad y(0) = 0. \quad (9)
\]
10. If $y$ solves the IVP from problem 1, find $y(1)$ approximately using Euler’s method with step $1/2$.

11. A tank contains 10 L of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. The solution is kept thoroughly mixed and drains from the tank at the same time. How much salt is in the tank after one hour?