1. (2 pts) Assume that \( M \subset \mathbb{R}^N \) is a hypersurface and \((\Psi, U)\) is a coordinate chart for \( M \). Assume also that \( U' \subset \mathbb{R}^{N-1} \) is an open set and \( \Phi : U' \to U \) is a diffeomorphism, and put \( \Psi' := \Psi \circ \Phi \); then \((\Psi', U')\) is another coordinate chart for \( M \). Without using the surface measure \( \lambda_M \), show that for each Borel measurable nonnegative \( f : M \to \mathbb{R} \),
\[
\int_U (f \circ \Psi) J_{\Psi} \, d\lambda_{\mathbb{R}^N-1} = \int_{U'} (f \circ \Psi') J_{\Psi'} \, d\lambda_{\mathbb{R}^N-1}
\]
where \( J_{\Psi}, J_{\Psi'} \) are defined by (5.2.12). (Hint: use Jacobi's formula and the fact that \( J_{\Psi}(y) = \sqrt{\det(d\Psi(y)^T d\Psi(y))} \).)

2. (2 pts) Do Exercise 6.1.6.

3. (2 pts) Do Exercise 6.1.7.


5. (1 pt) Let \((E, \mathcal{B}, \mu)\) be a measure space and \( f : E \to \mathbb{R} \) a measurable function. Show that for all \( p \in [1, \infty) \),
\[
\left( \|f\|_{L^p} \right)^p = \int_0^\infty p t^{p-1} \mu(\{x \in E : |f(x)| \geq t\}) \, dt.
\]

6. (1 pt) Assume that \((E, \mathcal{B}, \mu)\) is a finite measure space and \( f : E \to \mathbb{R} \) is a bounded measurable function. It is easy to see that \( f \in L^p(E, \mu) \) for all \( p \). Show that for any sequence \( p_j \in [1, \infty] \) converging to some \( p \in [1, \infty] \), we have \( \|f\|_{L^{p_j}} \to \|f\|_{L^p} \).