# 18.125 Homework 8
## due Wed Apr 6 in class

**1.** (1 pt) Define the Borel measure $\mu$ on $\mathbb{R}^2$ by $\mu(A) = \lambda_{\mathbb{R}^2}(A \cap B_1)$ where $B_1$ denotes the unit ball. Let $\Phi : \mathbb{R}^2 \to \mathbb{R}$ be defined by $\Phi(x, y) = x$. Compute the measure $\Phi_* \mu$.

**2.** (2 pt) Using Tonnelli's Theorem, show that for a measurable nonnegative function $f$ on a $\sigma$-finite measure space $(E, \mathcal{B}, \mu)$,
$$\int_E f \, d\mu = \int_0^\infty \mu(\{x \in E \mid f(x) \geq t\}) \, dt.$$
Use this to determine for which $s \in \mathbb{R}$ the following integrals are finite:
$$\int_{B_1} |x|^s \, dx, \quad \int_{\mathbb{R}^N \setminus B_1} |x|^s \, dx$$
where $B_1$ is the unit ball in $\mathbb{R}^N$.

**3.** (2 pts) Do Exercise 5.1.13 (i)–(iii).

**4.** (3 pts) Do Exercise 5.2.20.

**5.** (2 pts) Do Exercise 5.2.25.