18.125 Homework 7

due Wed Mar 30 in class

1. (1 pt) Let $f \in L^1(\mathbb{R})$ and Mf be the Hardy–Littlewood maximal function of f. Show that for each x,

$$Mf(x) = \sup_{h \neq 0} \frac{1}{|h|} \int_{I_h(x)} |f(y)|\, dy,$$

where $I_h(x) \subset \mathbb{R}$ is the interval of points lying between x and $x + h$.

2. (1 pt) Consider the following function $f : (0,1)^2 \to \mathbb{R}$, defined Lebesgue almost everywhere:

$$f(x,y) = \begin{cases} x^{-2}, & 0 < y \leq x < 1; \\ -y^{-2}, & 0 < x < y < 1. \end{cases}$$

Show that both repeated integrals below exist and

$$\int_0^1 \left(\int_0^1 f(x,y)dy \right) dx \neq \int_0^1 \left(\int_0^1 f(x,y)dx \right) dy.$$

3. (1 pt) Do Exercise 4.1.7.

4. (2 pts) Do Exercise 4.1.8.

5. (1 pt) Do Exercise 4.1.9.

6. (1 pt) Do Exercise 4.1.10.

7. (1 pt) Do Exercise 4.2.8.

8. (2 pts) Do Exericse 4.2.9.