

### 18.125 Homework 3

due Wed Feb 24 in class

1. (2 pts) Consider the map  $\Phi : (0, 1) \rightarrow \{1, 2, \dots, 9\}$  from Problem 1 in the previous homework, and let  $\lambda$  be the Lebesgue measure on  $\mathbb{R}$ . Show that  $\lambda(\Phi^{-1}(9)) = 1$ , that is  $\Phi = 9$  almost everywhere. Show that  $\lambda(\Phi^{-1}(j))$  is uncountable for  $j = 1, \dots, 8$ , giving examples of uncountable sets of zero measure. (You may use the fact that the power set of  $\mathbb{N}$  is uncountable.)
2. (2.5 pts) Do Exercise 2.2.30.
3. (1 pt) Do Exercise 2.2.31. (For the latter part, only show that the existence of  $\lambda_{\mathbb{R}^n}$  implies that  $\mathbb{R}^n$  is uncountable.)
4. (1 pt) Let  $(E, \mathcal{B}, \mu)$  be a measure space and assume  $\mu(E) < \infty$ . We call  $x \in E$  an *atom* of  $\mu$ , if  $\mu(\{x\}) > 0$ . Show that the set of atoms of  $\mu$  is at most countable.
5. (1 pt) Do Exercise 2.2.33.
6. (2.5 pts) Do Exercise 2.2.37.