Errata

p. 2, line 4 down: Change \( \bigcup_{i \in I} \) to \( \bigcup_{i \in \Lambda} \)

p. 2, line 11 down: \( A_{i_1} \cap \cdots \cap A_{i_n} \)

p. 4, line 4 down: \( \mathbb{P}(N < \infty) > 0 \implies \mathbb{E}[N] < \infty \)

p. 6, line 13 up: \( \omega \in [0, 1) \)

p. 8, line 13 down: Change \([0, \infty)^2\) to \([0, 1)^2\).

p. 27, line 11 up: Change \( \Xi^{-1} \) to \( (\Lambda')^{-1} \)

Exercise 1.3.19 on pp. 32–33: There are a number of annoying errors here. In line 4 up on p. 32, \( e^{tz} \) should be \( e^{-tz} \). In line 6 down on p. 33, the last expression should be \( 2t^2 e^{-t \delta} \). The most serious error is in the estimation of \( E(t, \delta) \) starting on line 8 on p. 33. One should write \( E(t, \delta) \) as

\[
\int_0^\delta e^{-\frac{z^2}{2}} \left( e^{tR(z)} + e^{tR(-z)} - 2 \right) dz.
\]

One should then use a second order Taylor’s expansion to write \( e^{tR(z)} + e^{tR(-z)} - 2 \) as \( t(R(z) + R(-z)) + F(t, z) \), where \( |F(t, z)| \leq \frac{t^2}{2} (R(z)^2 + R(-z)^2) e^{tR(|z|)} \).

Now observe that

\[
|R(z) + R(-z)| = \sum_{m=2}^{\infty} \frac{z^{2m}}{m} \leq \frac{z^4}{2(1 - z^2)}.
\]

Thus, for \( \delta \leq \frac{1}{2} \), one can dominate \( |E(t, \delta)| \) by a constant times the sum of

\[
t \int_0^\delta |z|^4 e^{-\frac{z^2}{2}} dz \quad \text{and} \quad t^2 \int_0^\delta |z|^6 e^{-\frac{z^2}{2}} dz,
\]

and so there is a \( C < \infty \) such that \( |E(t, \delta)| \leq Ct^{-\frac{3}{2}} \) for all \( t \geq 1 \) and \( 0 < \delta \leq \frac{1}{2} \).

Finally, take \( \delta = \sqrt{3t^{-1} \log t} \) to arrive at

\[
\frac{\Gamma(t + 1)}{t^{t+1} e^{-t} - \sqrt{\frac{2\pi}{t}}} \leq Ct^{-\frac{3}{2}}
\]

for \( t \geq 9 \), and from this get (1.3.21).
Errata

p. 43, line 7 up: sup_{n \geq 1} \mathbb{E}^p[X_n^2] < \infty

p. 43, line 3 up: Change to
\[ \mathbb{P}\left( \max_{1 \leq m \leq n} |S_m| \geq 2t \right) \leq \frac{\mathbb{P}(|S_n| \geq t)}{1 - \max_{1 \leq m \leq n} \mathbb{P}(|S_n - S_m| \geq t)}. \]

p. 45, line 6 down: 2^{-\frac{1}{p}} \mathbb{E}^p[|X|^p]^{\frac{1}{p}}

p. 50, line 6 down: Theorem 1.3.15

p. 55, line 6 down: Change to
\[ \mathbb{P}\left( |\tilde{S}_n| \geq \beta \right) \leq 3 \exp\left[ -\beta \log(2) |\beta| \right]. \]

p. 55, line 11–12 down: delete “for every \( k \geq 2 \)

p. 55, lines 13–14 down: Change to
\[ \lim_{n \to \infty} \left| \tilde{S}_n - a \right| \leq \inf_{k \geq 2} \lim_{m \to \infty} \left| \tilde{S}_{km} - a \right| = \lim_{k \to \infty} \lim_{m \to \infty} \left| \frac{S_{km} - S_{km-1}}{\Lambda_{km}} - a \right| \quad \text{P-almost surely.} \]

p. 55, line 2 up: \[ \mathbb{P}\left( |\tilde{S}_n - a| \right) \]

p. 62, line 15 up: Change “it introduced” to “it was introduced”

p. 64, line 10 up: Change \( C^\infty_c(\mathbb{R}^N; \mathbb{R}) \) to \( C^\infty_c(\mathbb{R}; \mathbb{R}) \)

p. 66, line 10 down: Change to “Show that (2.1.1) for \( \varphi \in C^\infty_c(\mathbb{R}; \mathbb{R}) \)”

p. 82, eq. (2.3.1): Change \( x \in \mathbb{R}^N \) to \( \xi \in \mathbb{R}^N \)

p. 83, line 1 down: Change \( \mathbb{C} \)-valued to \( \mathbb{R} \)-valued.

p. 83, line 18 up: Change 2.3.23 to 2.4.37

p. 83, line 12 up: Change “in \( \mathbb{C} \)” to “in \( \mathbb{R}^N \)”

p. 85, line 1 down: Insert period at end of line

p. 88, line 1 up: Delete \( \pm \)

p. 89, line 6 down: \( (e, X)_{\mathbb{R}^N} \)

p. 89, line 1 up: Change \( e^{\beta_n |\xi|^2} \) to \( e^{\beta_2 |\xi|^2} \)

p. 91, line 6 down: \( \leq 2\epsilon^{-2} \)

p. 90, lines 10 and 4 up: Change to \( \leq \frac{\xi^2 a^2}{25\epsilon^2} \) and \( \leq \frac{R^2 r^2}{4} \)
p. 91, line 5 down: \( \leq 2\epsilon^{-2} \)

p. 95, lines 9–8 up: The hint should read \((\Pi\xi)(2) = C_{(22)}^{-1} C_{(21)}\xi_{(1)} \) if \( \xi_{(2)} = 0_{(2)} \)
and \( \Pi\xi = \xi \) if \( \xi_{(1)} = 0 \).

p. 98, footnote: Change to “footnote in Exercise 2.1.13”

p. 101, line 15 down: Delete “is trivial”

p. 103, line 9 up: Delete “In”

p. 108, line 15–14 up: Change \( F_{h0} = \left(\sqrt{-1}\right)^n h_0 \) to \( F_{hn} = \left(\sqrt{-1}\right)^n h_n \).

p. 110, line 14 up: Change \( p, 2 \) to \( p, q \)

p. 111, line 5 up: Change \( p \in (0, 1) \) to \( p \in (1, \infty) \)

p. 116, line 2 down: Change \( \mu_\ell \) to \( \nu_\ell \).

p. 122, line 19 down: Change \( +A((1 - \eta R)\varphi) \) to \( -A((1 - \eta R)\varphi) \) in final expression.

p. 133, line 4 up: Change \( (*)\) to \( (**) \)

p. 136, line 8 down: Change \( \sqrt{-1} \int_{\mathbb{R}^N} \) to \( \int_{\mathbb{R}^N} \)

p. 134, line 5 up: Change the second expression to

\[
|\xi| \int_{B(0,r)} \frac{1 - \eta(y)}{|y|^2} |y|^2 M(dy)
\]

p. 136, line 3 down: Change \( \ell_{\mu}(R^{-1}\xi' - x) \) to \( \ell_{\mu}(R^{-1}\xi' - \xi) \) in final expression.
Errata

p. 136, line 6 down: Insert \( \lim_{R \to \infty} \) before \( \int_{\mathbb{R}^N} \eta_R(x') \, dx' \).

p. 136, line 9 up: Insert \( y \) before \( \mu_{\frac{1}{n}}(dy) \).

p. 138, line 2 down: \( \delta_{m} \ast \pi_{M^r}((-\infty,0)) = 0 \)

p. 138, line 8 down: \( m^{\infty} \geq \int_{\mathbb{R}} \eta_0(y) y M(dy) \)

p. 139, line 6 down: Change \( 2^{\frac{2}{\alpha}} \) to \( 2^{-\frac{2}{\alpha}} \)

p. 141, line 9 down: Change \( t = 2 \) to \( t = 2^{\frac{1}{\alpha}} \).

p. 142, line 5 down: Change “Exercise 1.2.12” to “Exercise 1.2.13”

p. 142, line 7 down: Insert \( \varphi_2(|y|) \) before \( M(dy) \).

p. 143, line 12 down: Change righthand side to \( -\Gamma(1-\alpha) \frac{\alpha}{\zeta \sqrt{-1}} \)

p. 146, line 5 down: Change to “rotationally invariant, symmetric, \( \alpha \)-stable”

p. 146, line 1 up: Change to:

1-stable, one can use \( \ell_{\mu}(\xi) = \ell_{\mu}(-\xi) \) to see that \( m = 0 \) and that the \( \nu \) in the expression for \( \ell_{\mu}(\xi) \) can be taken to be symmetric.

p. 147, lines 3 & 7 down: Insert “symmetric” after “rotationally invariant” in line 3, and delete \( \int_{\mathbb{S}^{N-1}} \) in line 7.

p. 148, line 9 up: Change “Exercise 3.3.11” to “Exercise 3.3.12”

p. 157, line 5 down: Change to \( \nu_\tau = \delta_{y_\tau} \)

p. 160, lines 5–7 down: Although it is true that \( \mathcal{F}_{D(\mathbb{R}^N)} \subseteq B_{D(\mathbb{R}^N)} \), the given open set does not provide a proof. Instead, let \( \Delta \) be a subset of \([0,1]\), and set \( A = \{1_{[t,\infty)} : t \in \Delta \} \). Show that \( A \) is a closed subset of \( D(\mathbb{R}) \) and that the map \( t \mapsto 1_{[t,\infty)} \in D(\mathbb{R}) \) is measurable as a map from \((\mathbb{R}, B_{[0,1]})\) into \((D(\mathbb{R}), F_{D(\mathbb{R})})\). Conclude that if \( A \) were an element of \( \mathcal{F}_{D(\mathbb{R})} \), then \( \Delta \) would have to be Borel measurable. Hence, \( A \in B_{D(\mathbb{R})} \setminus F_{D(\mathbb{R})} \) if \( \Delta \notin B_{[0,1]} \).

p. 160, line 15 up: Change \( C(\mathbb{R}^N) \) to \( D(\mathbb{R}^N) \)
p. 162, line 6 up: Change the description of the set $B$ to
\[
\bigcap_{k=1}^{K} \left\{ (\tau_1, \ldots, \tau_{nK}) \in (0, \infty)^{nK} : \sum_{m=1}^{n_k-1} \tau_m > t_{k-1} \& \sum_{m=1}^{n_k} \tau_m \leq t_k \right\}
\]

The reduction to the case when $1 \leq n_1 < \cdots < n_K$ is not so simple as indicated. Perhaps a better approach is to work by induction on $K \geq 1$. The case $K = 1$ is already covered. Thus, assume that $K \geq 2$ and that the result holds for $K - 1$. If $n_K = 0$, there is nothing to do. If $n_K \geq 1$ and $n_1 = 0$, set $K_0 = \min\{k : n_k \geq 1\}$. Then $2 \leq K_0 \leq K$ and, by the induction hypothesis,

\[
P(N(t_k) = n_k, 1 \leq k \leq K) = \mathbb{P}\left( \tau_1 > t_{K_0-1} & T_{n_k} \leq t_k < T_{n_k+1}, K_0 \leq k \leq K \right)
\]

\[
= \int_{t_{K_0-1}}^{t_{K_0}} e^{-t} \mathbb{P}\left( T_{n_k-1} \leq t_k - t < T_{n_k}, K_0 \leq k \leq K \right) dt
\]

\[
= e^{-t_k} \prod_{K_0 < k \leq K} \frac{(t_k - t_{k-1})^{n_k-n_k-1}}{(n_k-n_k-1)!} \int_{t_{K_0-1}}^{t_{K_0}} \frac{(t_{K_0} - t)^{n_{K_0}-1}}{(n_{K_0}-1)!} dt
\]

If $n_1 \geq 1$, then, by the preceding case when $n_1 = 0$,

\[
P(N(t_k) = n_k, 1 \leq k \leq K)
\]

\[
= \mathbb{P}(T_{n_1} \leq t_1 & T_{n_1+1} - T_{n_1} > t_1 - T_{n_1} \\
& T_{n_k} - T_{n_{k-1}} \leq t_k - T_{n_k} \leq T_{n_k+1} - T_{n_{k-1}}, 2 \leq k \leq K)
\]

\[
= \frac{1}{(n_1-1)!} \int_{0}^{t_1} t_1^{n_1-1} e^{-t} \mathbb{P}(N(t_1 - t) = 0 \& N(t_k - t) = n_k - n_1, 2 \leq k \leq K) dt
\]

\[
= e^{-t_k} \prod_{K_0 < k \leq K} \frac{(t_k - t_{k-1})^{n_k-n_k-1}}{(n_k-n_k-1)!} \int_{0}^{t_k} \frac{(t_{K_0} - t_k)^{n_{K_0}-1}}{(n_{K_0}-1)!} dt = e^{-t_k} \prod_{k=1}^{K} \frac{(t_k - t_{k-1})^{n_k-n_k-1}}{(n_k-n_k-1)!}.
\]

p. 163, line 5 up: Change $1_{[T_{n}(\omega), \infty)}(t)$ to $1_{[T_{n}(\omega), \infty)}(at)$

p. 167, line 10–9 up: Change $M(dy)$ to $M(y)$ and

\[
\mathbf{Z}(t) = \mathbf{Z}(t) - \int_{\mathbb{R}^N} |y| M(dy) \text{ to } \mathbf{Z}(t) = \mathbf{Z}(t) - t \int_{\mathbb{R}^N} |y| M(dy)
\]

p. 168, line 13 up: Change “subsets sets” to “subsets”

p. 170, line 17 down: Change 4.3.11 to 4.3.21
Errata

p. 182, line 5 up: Change \( \frac{X((m+1)2^{-n})-X(m2^{-n})}{2} \) to \( \frac{X((m+1)2^{-n})+X(m2^{-n})}{2} \)

p. 182, line 4 up: \( \mathbb{E}^p \left[ \sup_{t \in [0,1]} \|X_{n+1}(t)-X_n(t)\|_B^p \right]^{\frac{1}{p}} \)

p. 188, line 6 down: Change to \( P(\sup_{t \in [0,T]} |(e, B(t))_\mathbb{R}^N| \geq R) \leq 2e^{-\frac{R^2}{2\pi}} \)

In order to get the second inequality, one has to show that \( \gamma_{0,1}(|R, \infty)) \leq \frac{1}{2}e^{-\frac{R^2}{2\pi}} \).

This follows trivially from \( \int_R^\infty e^{-\frac{x^2}{2\pi}} \, dx \leq R^{-1}e^{-\frac{R^2}{2\pi}} \) when \( R \geq \sqrt{\frac{2\pi}{\pi}} \). When \( R \leq \sqrt{\frac{2}{\pi}} \), one has to show that \( \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^R e^{-\frac{x^2}{2\pi}} \, dx \leq \frac{1}{2}e^{-\frac{R^2}{2\pi}} \), or, equivalently, that \( 1 \leq e^{-\frac{R^2}{2\pi}} + \sqrt{\frac{2}{\pi}} \int_0^R e^{-\frac{x^2}{2\pi}} \, dx \). Finally, observe that the right hand side is non-decreasing of \( 0 \leq R \leq \sqrt{\frac{2}{\pi}} \).

p. 190, lines 16 & 8 up: Change 16 up to \( \mathbb{E}^p \left[ \left( \sup_{x,y \in [0,T]^v} \frac{\|\tilde{X}(y)-\tilde{X}(x)\|_B}{|y-x|^{\alpha}} \right) \right]^{\frac{1}{p}} \leq K(\nu, r, \alpha)CT^{\frac{p}{p}+r-\alpha} \)

and \( \prod_{i=1}^N \) to \( \prod_{i=1}^\nu \) in line 8 up

p. 191, line 11 down: On \( B(1)^2 \) righthand side should be replaced by \( \frac{B(1)}{2} \).

p. 192, line 7 up: \( \sum_{n=m}^\infty L(2^{-n-1}) \leq CL(2^{-m-1}) \)

p. 192, line 4 up:

\( \mathbb{P}(\|B-B_n\|_{[0,1]} \geq RL(2^{-n-1})) \leq \sum_{m=n}^\infty \mathbb{P}(M_{m+1} \geq C^{-1}RL(2^{-m-1})) \)

p. 192, line 2 up:

\( \mathbb{P}(M_n \geq RL(2^{-n})) \leq 2^{n(1-2^{-1}R^2)+1} \)

p. 199, line 14 up: Change \( \langle \mathbb{E}^p[X], x^* \rangle = \mathbb{E}^p[\langle X, x^* \rangle] \) to \( \langle \mathbb{E}^p[X], x^* \rangle = \mathbb{E}^p[\langle X, x^* \rangle] \).

p. 205, 4th line of Exercise 5.1.20: Insert “relies” between “Lemma 5.1.10” and “fails”
Errata

p. 222, line 3 up: Change $\int\int_B$ to $\int\int$

p. 230, line 12 up: Replace $P \upharpoonright \mathcal{F}_n$ by $\mathbb{P} \upharpoonright \mathcal{F}_n$.

p. 237, line 4 down: Change the right hand side to

\[ \frac{1}{\alpha} \int \{ M^{(\eta)} f \geq \alpha \} \cap B(0, R) f(y) \, dy \]

p. 238, line 10 down: Delete $f$ at end of sentence

p. 259, line 11 up: Change $u : E \times \mathcal{F} \to \mathbb{R}$ to $u : E^2 \to \mathbb{R}$

p. 259, lines 4 up: Change $(y, \|y\|_F, h)$ to $(y, \|y\|_E, h)$

p. 263, line 13 down:

\[ \sup_{n \in \mathbb{Z}^+} \left\| \mathbb{E} \left[ \| X_n - X_{n-1} \|_F^2 \mid \mathcal{F}_{n-1} \right] \right\|_{L^{2p}(\mu; \mathbb{R})} \]

p. 268, lines 2 & 6 down: Insert $\frac{1}{2}$ before $(\xi, C\xi)_{\mathbb{R}^N}$ in line 2, and change $x$ to $\xi$ in line 6

p. 269, line 6 down: Change $\omega \in \Theta$ to $\omega \in \Omega$

p. 273, line 10 down: Change $A \in \mathcal{B}_t \times \mathcal{F}_t$ to $A \in \mathcal{B}_{[0,t]} \times \mathcal{F}_t$.

pp. 283 & 284, lines 1 up & 3, 6, 8 down: Change $e^{\frac{\xi^2}{2}}$ to $e^{\frac{\|\xi\|^2}{2}}$

p. 285, line 7 up: One doesn’t need Doob’s inequality to prove $\zeta_n(\omega) \nearrow \infty$ almost surely, it follows from almost sure continuity.

p. 287, line 6 up: Change $\mathcal{F}_{k-1,m}$ to $\mathcal{F}_{\zeta_{k-1,m}}$

p. 293, line 4 down: Change $Z^\gamma$ to $\tilde{Z}$

p. 296, line 2 down: Change $\{ \zeta' > t \}$ to $\{ \zeta'(x + B(\cdot)) \}$

p. 296, lines 11 down, 8 up, and 5 up: Replace $p^{(-1,1)}(r^{-2}, r^{-1}(x - c), r^{-1}(y - c))$ by $p^{(-1,1)}(r^{-2}t, r^{-1}(x - c), r^{-1}(y - c))$ in 11 down, change $\{ x + \psi(s) + \delta_s \psi(\tau), \tau \in [0, t - s] \}$ to $\{ x + \psi(s) + \delta_s \psi(\tau) \in I, \tau \in [0, t - s] \}$ in 8 up, and change $(x - x)$ to $(e - x)$ in 5 up

p. 298, line 10 down:

\[ -\mathbb{E} \left[ \gamma_{a,(1-\zeta')_I}(\Gamma - x - \psi(c^G_x), c^G_x \leq t) \right]. \]

p. 301, Lemma 8.1.2: The statement of this lemma should be amended to say that $\hat{\mu}$ is a continuous function of weak* convergence on bounded subsets of $E^*$. The point is that Lebesgue’s Dominated Convergence Theorem applies to sequences but not to nets. By Exercise 5.1.9, weak* convergence of bounded sets admits a metric and therefore continuity can be checked with sequences.
304, lines 13 & 19 down: Change $H(R^N)$ to $H^1(R^N)$

p. 306, line 3 up: Replace $\mathbb{P}(\|X\|_E \leq r) \geq \frac{3}{4}$ by $\mathbb{P}(\|X\|_E \leq r) \geq \frac{9}{10}$.

p. 307, lines 14 down & 7 up: In 14 down, replace $P(\|X\|_E \leq R) \geq \frac{3}{4}$ by $P(\|X\|_E \geq 3\frac{3}{4}R)$ and in line 7 up, replace $P(\|X\|_E \geq 3\frac{3}{4}R)$ by $P(\|X\|_E \geq 3\frac{3}{4}R)$.

p. 308, line 9 down: Replace $\|x^*\|_{E^*}$ by $\|h_{x^*}\|_H$.

p. 308, line 15 up: Change $x^*$ to $x^*$

p. 309, line 14 down: Replace $g = \sum_{m=0}^{\infty}$ by $g = \sum_{m=0}^{\infty}$.

p. 310, line 2 down: Replace $\|h_{x^*}\|^2_H$ by $(h_{x^*}, h_{y^*})_H$.

p. 311, Theorem 8.2.6: Like the analogous one in Lemma 3.1.2, the initial statement of continuity in this result should be changed to $x^* \in E^* \mapsto h_{x^*} \in H$ continuous from the weak* topology to the strong topology on bounded subsets of $E^*$. In fact, only when $H$ is finite dimensional does this continuity hold for the whole space.

p. 311, line 17 down: Replace $h \in H \mapsto h \in E$ by $h \in B_H(0,R) \mapsto h \in E$.

p. 311, line 11-9 up: Change these lines to

$$|\langle h_k - h, x^* \rangle| \leq \epsilon + \min_{1 \leq m \leq n} |(h_k - h, h_{x^*} - h_{x^*_m})_H| + 2R\epsilon \leq (2R + 1)\epsilon,$$

which proves that $\|h_k - h\|_H \leq (2R + 1)\epsilon$ for $k \geq \ell$.

p. 314, lines 11 & 10 up: Change $n + 1^3$ to $(n + 1)^3$ and remove $\{X^n : n \geq 0\}$

p. 315, line 11 & 9 up: Replace 72 by 180.

p. 319, line 1 up: Change first sum to

$$\sum_{k=0}^{\infty} (t - k)^+ \land 1X_{k,0}$$

p. 323, line 4 up: Replace $\sum_{i=1}^{\ell} \tilde{y}_{m,i} - g_{m,i} \|_H$ by $\sum_{i=1}^{\ell} \|\tilde{y}_{m,i} - \Pi L_n g_{m,i}\|_H$.

p. 325, line 5 down: Change $\|Q_n x_k\|$ to $\|Q_n x_k\|_E$

p. 328, lines 2 & 15 down: Change “rotation” to “orthogonal”
Errata

p. 331, line 4–9 down:
**Hint:** Begin by showing that the inequality is trivial when either $n = 1$ or $\|AA^\top\|_{\text{op}} \geq 2$. To prove it when $n \geq 2$ and $\|AA^\top\|_{\text{op}} \leq 2$, set $\Delta = \mathbf{I}_{\mathbb{R}^n} - AA^\top$,
show that

$$|a_{\ell\ell}a_{n\ell}| \leq |\Delta_{n\ell}| + (AA^\top)^{\frac{1}{2}}_{nn} \left( \sum_{j=1}^{n-1} a_{j\ell}^2 \right)^{\frac{1}{2}}$$
for $1 \leq \ell < n$

and proceed by induction on $n$.

p. 332, line 6 up:
$$\sum_{m=1}^{\infty} \frac{X_n(\theta)^2 + (-1)^{m+1}\sqrt{\pi}X_n(\theta)X_n(\theta)}{m^2}$$

p. 332, line 1 up:
$$\sinh z = z \prod_{m=1}^{\infty} \left( 1 + \frac{z^2}{m^2\pi^2} \right).$$

p. 334, line 1 up:
$$\theta_T \{ [0, T] \} \in \Theta_T(\mathbb{R}^N).$$

p. 335, lines 18, 11, & 7 up: Change $H(\mathbb{R}^N)$ to $H^1(\mathbb{R}^N)$

p. 337, line 3 up: Change $W_e(B_E(h_{x^*}, r))$ to $W_e(B_E(h, r))$.

p. 340, lines 3 & 9 down: Change $\|x\|_{E^*} = 1$ to $\|x^*\|_{E^*} = 1$ in line 3 and $x^* \in E^*$ to $x^* \in B_{E^*}(0, 1)$ in line 9

p. 343, line 4 up: Change “they played” to “that played”

p. 346, line 6 up: Replace $h \in H(\mathbb{R}^N)$ by $h \in H^1(\mathbb{R}^N)$

p. 347, lines 3 & 13 down: Change $\|\theta\|$ to $\|\theta\|_{E^1(\mathbb{R}^N)}$ in line 3 and $\|\eta^U\|_u$ to $\|\eta^U\|_{E^1(\mathbb{R}^N)}$ in line 13

p. 348, line 8 down: Change to

$$\int_{(0, \infty)} \left( \partial_x u_0(s, \tau) \partial_x u_0(t, \tau) + \frac{1}{2} u_0(s, \tau) u_0(t, \tau) \right) d\tau = u_0(s, t)$$

p. 350, line 13 up: Change $\mathcal{S}(\mathbb{R}^N; \mathbb{R})$ to $\mathcal{S}^\prime(\mathbb{R}^N; \mathbb{R})$

p. 357, line 11 down: Change “translation invariant” to “Euclidean invariant”

p. 359, line 12 up: Change $\epsilon_n^2$ for $\epsilon_n$

p. 364, lines 4 & 5 down: Change $\delta \frac{\lambda_{[\beta \Lambda_{\delta^m-1}]}}{\beta \Lambda_{\delta^m-1}}$ to $\delta \frac{\lambda_{[\beta \Lambda_{\delta^m-1}]}}{\beta \Lambda_{\delta^m-1}}$

p. 381, line 5 down: Change $L(\mu_k, (Y_{k,n})_*)$ to $L(\mu_k, (Y_{k,n})_*^\mathbb{F})$
p. 383, line 11 down: change the statements of parts (ii) and (iii) of Exercise 9.1.17 as follows.

(ii) For $\ell \in \mathbb{Z}^+$, let $\pi_\ell$ be the natural projection map from $E$ onto $E_\ell$, and show that $K \subseteq E$ if

$$K = \bigcap_{\ell \in \mathbb{Z}^+} \pi_\ell^{-1}(K_\ell), \quad \text{where } K_\ell \subseteq E_\ell \text{ for each } \ell \in \mathbb{Z}^+.$$ 

Conclude from this that $A \subseteq M_1(E)$ is tight if and only if $\{(\pi_\ell)_* \mu : \mu \in A\} \subseteq M_1(E_\ell)$ is tight for every $\ell \in \mathbb{Z}^+$.

Next, set $E_\ell = \prod_{k=1}^\ell E_k$, and let $\pi_\ell$ denote the natural projection map from $E$ onto $E_\ell$. Show that for each $\varphi \in \mathcal{U}_{\text{R}}$ $(E; \mathbb{R})$ and $\epsilon > 0$ there is an $\ell \geq 1$ such that $|\varphi(y) - \varphi(x)| < \epsilon$ for all $x, y \in E$ with $\pi_\ell(x) = \pi_\ell(y)$. Use this to show that $\mu_n \Rightarrow \mu$ in $M_1(E)$ if and only if $\langle \varphi \circ \pi_\ell, \mu_n \rangle \rightarrow \langle \varphi \circ \pi_\ell, \mu \rangle$ for every $\ell \geq 1$ and $\varphi \in C_0(E_\ell; \mathbb{R})$.

(iii) Let $\mu_{[1,\ell]}$ be an element of $M_1(E_\ell)$, and assume that the $\mu_{[1,\ell]}$'s are consistent in the sense that, for every $\ell \in \mathbb{Z}^+$,

$$\mu_{[1,\ell+1]}(\Gamma \times E_{\ell+1}) = \mu_{[1,\ell]}(\Gamma) \quad \text{for all } \Gamma \in \mathcal{B}_{E_\ell}.$$

Show that there is a unique $\mu \in M_1(E)$ such that $\mu_{[1,\ell]} = (\pi_\ell)_* \mu$ for every $\ell \in \mathbb{Z}^+$.

p. 396, line 2 down: Change to “In order to remove the assumption on the fourth moment”

p. 399, line 6 up: Change § 4.2.2 to § 4.2.1

p. 401, line 21 down: Change § 8.1.3 to § 8.1.1

p. 403, line 9 down: Change $C^{1,2}((0, T) \times \mathbb{R}^N; \mathbb{R})$ to $C^{1,2}((0, T) \times \mathbb{R}^N; \mathbb{R}) \cap C([0, T] \times \mathbb{R}^N; \mathbb{R})$

p. 403, lines 10–9 up & 5–4 up & 1–2 up: Change $V(\tau, \psi(\tau))$ to $V(T - \tau, \psi(\tau))$

p. 406, lines 1 down and 1 up: Change “Exercise 8.2.16” to “Exercise 8.3.19”

p. 407, line 6 down: Change $\partial_t u = \frac{1}{2} \Delta u + Vu$ to $\partial_t u = \frac{1}{2} \Delta u + Vu + f$

p. 413, lines 13 & 17 down: Change $f_N$ to $f_N \circ \psi$

p. 415, line 10 down: Change the RHS to $\frac{b(T, x, y)}{g^{(T, \psi(T) - x)}}$

p. 419, line 3 down: Change “one solution” to “one bounded solution”

p. 419, line 5 up: Delete $u$ at beginning of sentence
p. 421, line 13 down: Change $f(\psi(t))$ to $f(\psi(\zeta^G))$

p. 421, line 2 up: Change $\mathcal{G}$ to $G$

p. 431, line 10 down: Change $g^{(N)}(t, \eta - \xi)$ to $g^{(N)}(t, y - \xi)$

p. 432, line 2 down: Change $g^{(N)}(t, y - \psi(\zeta^G))$ to $g^{(N)}(t - \zeta^G, y - \psi(\zeta^G))$

p. 434, line 3 up: Change $\phi \in C^2_b(G; \mathbb{R})$ to "bounded $\phi \in C^2(G; \mathbb{R})$

p. 438, line 3 up: Change $E_{\mathcal{W}(N)}^\iota$ to $E_{\mathcal{W}(N)}^\psi$

p. 440, line 10 down: Change $= 0$ to $= 1$

p. 441, line 9 up: Change $R(t, \psi)$ to $R(t, \psi)$

p. 442, line 13 down: Change $\int_{\mathbb{R}^N} p^\iota(t, \psi(s), dy) dy$ to $\int_{\mathbb{R}^N} p^\iota(t, \psi(s), dy) \varphi(y) dy$

p. 443, 2 & 6 down: Change $\varphi(t \wedge \zeta^{B(x, \lambda)})$ to $\varphi(t, \psi(t \wedge \zeta^{B(x, \lambda)})$

p. 443, 3 & 6 down: Change $f(\psi(\tau))$ to $f(\tau, \psi(\tau))$

p. 445, line 11 down: Change "Corollary 10.3.27" to "Lemma 10.3.28"

p. 450, line 11 down: Change $\rho$ to $\rho$

p. 451, line 8 down: Change $\leq \frac{e^{-\lambda + y}}{(\pi t)^{\frac{3}{2}}}$ to $\frac{e^{-\lambda + y}}{(\pi t)^{\frac{3}{2}}}$ and (ii) to (iii)

p. 464, line 8 up: Change $C_c(G; \mathbb{R})$ to $C^1_c(G; \mathbb{R})$

p. 472, line 5 up: The assertion in Exercise 11.1.36 is false and so this exercise should be ignored.

p. 473, lines 1 & 10 down: Change (ii) to (i) and (iii) to (ii).

p. 473, line 2 up: $W_{\mathcal{Y}}^{(N)}(\forall t \in [0, \infty) \psi(t) = \varphi(t))$

p. 474, line 7 down: Change RHS to $\sup_{y \in \mathbb{R}} |u(r, y)| \vee |u(-r, y)|$

p. 477, line 5 down: Delete $\lim_{s \to 0} \int_0^s (\int_G \varphi(y) p^G(t, x, y) dy) dt$ on RHS

p. 479, line 2 up: Change $C_c(G; \mathbb{R})$ to $C^1_c(G; \mathbb{R})$

p. 481, line 10 up: Replace $B$ by $B^2$

p. 491, line 5 up: Change $\{B_t : t \in [0, \infty]\}$ to $\{F_t : t \geq 0\}$

p. 502, line 1 down: Theorem 11.4.6

p. 508, line 11 down: Change (ii) to (iii)

p. 515, line 1 down: Let $G$ be a connected open subset