Errata

Exercise 1.3.19 on pp. 32–33: There are a number of annoying errors here. The last line of p. 32 should be

$$\le 2\exp\left[1-\frac{t\delta^2}{2}+\frac{t\delta^3}{3(1-\delta)}\right].$$

The most serious error is in the estimation of $E(t,\delta)$ starting on line 8 on p. 33. One should write $E(t,\delta)$ as

$$\int_0^\delta e^{-\frac{tz^2}{2}}\big(e^{tR(z)}+e^{tR(-z)}-2\big)\,dz.$$

One should then use a second order Taylor's expansion to write $e^{tR(z)}+e^{tR(-z)}-2$ as $t\big(R(z)+R(-z)\big)+F(t,z)$, where $|F(t,z)|=\frac{t^2}{2}\big(R(z)^2+R(-z)^2\big)e^{t|R(z)|}$. Now observe that

$$|R(z)+R(-z)|=\sum_{m=2}^\infty\frac{z^{2m}}{m}\le\frac{|z|^4}{2(1-z^2)}.$$

Thus, for $\delta\le\frac12$, one can dominate $|E(t,\delta)|$ by a constant times the sum of

$$t\int_0^\delta |z|^4 e^{-\frac{tz^2}{2}}\,dz \quad\text{and}\quad t^2\int_0^\delta |z|^6 e^{-\frac{tz^2}{6}}\,dz,$$

which shows that $|E(t,\delta)|$ is dominated by a constant times $t^{-\frac32}$. Finally, take $\delta=\sqrt{3t^{-1}\log t}$ to arrive at

$$\left|\frac{\Gamma(t+1)}{t^{t+1}e^{-t}}-\sqrt{\frac{2\pi}{t}}\right|\le Ct^{-\frac32}$$

for $t\ge 9$, and from this get (1.3.21).

p. 43, line 7 up: $\sup_{n\ge1}\mathbb{E}^{\mathbb{P}}[X_n^2]<\infty$

p. 45, line 6 down: $2^{-\frac1p\vee 1}\mathbb{E}\mathbb{P}\big[|X|^p\big]^{\frac1p}$

p. 58, lines 1–10: Change $\tilde{\mathbf{s}}$ to $\tilde{\mathbf{S}}$

p. 90, lines 10 and 4 up: $\le\frac{\xi^2\sigma_m^2}{2\Sigma_n^2}$ and $\le\frac{R^4 r_n^2}{4}$

p. 91, line 5 down: $\leq 2\epsilon^{-2}$

p. 95, lines 9–8 up: The hint should read $(\Pi\boldsymbol{\xi})_{(2)} = C_{(22)}^{-1}C_{(21)}\boldsymbol{\xi}_{(1)}$ if $\boldsymbol{\xi}_{(2)} = \mathbf{0}_{(2)}$ and $4\Pi\boldsymbol{\xi} = \boldsymbol{\xi}$ if $\boldsymbol{\xi}_{(1)} = \mathbf{0}$.

p. 111, line 5 down: convergence in $L^2(\gamma_{0,1}^2;\mathbb{R})$

p. 111, line 6 down: Using the generating function in (2.4.5)

p. 113, line 12 up: Replace $-\dot{q}(t)$ by $\dot{q}(t)$

p. 114, line 6 up: Change 8.4.8 to 8.5.12

p. 122, line 2 down: Reduce to the case when μ is symmetric, $\mu(\{0\}) > 0$, and therefore

p. 138, line 2 down: $\delta_{m_\mu} \star \pi_{M^r}\big((-\infty,0)\big) = 0$

p. 138, line 8 down: $m^{\eta_0} \geq \int_R \eta_0(y)y\, M(dy)$

p. 143, line 12 down: $\frac{\Gamma(1-\alpha)}{\alpha}$

p. 182, line 4 up: $\mathbb{E}^{\mathbb{P}}\Big[\sup_{t\in[0,1]} \|X_{n+1}(t) - X_n(t)\|_B^p\Big]^{\frac{1}{p}}$

p. 188, line 6 down:

$$\mathbb{P}\left(\sup_{t\in[0.T]} \big|\big(\mathbf{e},\mathbf{B}(t)\big)_{\mathbb{R}^N}\big| \geq R\right) \leq 2\mathbb{P}\Big(\big|\big(\mathbf{e},\mathbf{B}(T)\big)_{\mathbb{R}^N}\big| \geq R\Big)$$

p. 192, line 7 up: $\sum_{n=m}^{\infty} L(2^{-n-1}) \leq CL(2^{-m-1})$

p. 192, line 4 up:

$$\mathbb{P}\big(\|B - B_n\|_{[0,1]} \geq RL(2^{-n-1})\big) \leq \sum_{m=n}^{\infty} \mathbb{P}\big(M_{m+1} \geq C^{-1}RL(2^{-m-1})\big)$$

p. 192, line 2 up:

$$\mathbb{P}\big(M_n \geq RL(2^{-n})\big) \leq 2^{n(1-2^{-1}R^2)+1}$$

p. 262, line 14 down:

$$\sup_{n\in\mathbb{Z}^+} \left\| \mathbb{E}^{\mathbb{P}}\Big[\|X_n - X_{n-1}\|_E^{2p} \,\Big|\, \mathcal{F}_{n-1}\Big]^{\frac{1}{2p}} \right\|_{L^{2p}(\mu;\mathbb{R})}$$

p. 298, line 10 down:

$$-\mathbb{E}\mathbb{P}\Big[\gamma_{\mathbf{0},(1-\zeta_{\mathbf{x}}^G)\mathbf{I}}\big(\Gamma - \mathbf{x} - \boldsymbol{\psi}(\zeta_{\mathbf{x}}^G), \zeta_{\mathbf{x}}^G \leq t\Big].$$

p. 331, line 4–9 down:
Hint: Begin by showing that the inequality is trivial when either $n = 1$ or $\|AA^\top\|_{\mathrm{op}} \geq 2$. To prove when $n \geq 2$ and $\|AA^\top\|_{\mathrm{op}} \leq 2$, set $\Delta = \mathbf{I}_{\mathbb{R}^n} - AA^\top$, show that

$$|a_{\ell\ell}a_{n\ell}| \leq |\Delta_{n\ell}| + (AA^\top)^{\frac{1}{2}}_{nn}\left(\sum_{j=1}^{\ell-1} a_{jj}^2\right)^{\frac{1}{2}} \qquad \text{for } \mathbf{1} \leq \ell < n$$

$$|1 - a_{nn}| \leq |1 - a_{nn}^2| \leq \Delta_{nn} + \sum_{\ell=1}^{n-1} a_{n\ell}^2,$$

and proceed by induction on n.

p. 332, line 6 up: $\sum_{m=1}^{\infty} \frac{X_m(\theta)^2 + (-1)^{m+1}\sqrt{8}X_0(\boldsymbol{\theta})X_m(\boldsymbol{\theta})}{m^2}$

p. 332, line 1 up: $\sinh z = z\prod_{m=1}^{\infty}\left(1 + \frac{z^2}{m^2\pi^2}\right)$.

p. 334, line 1 up: $\boldsymbol{\theta}_T \restriction [0,T] \in \Theta_T(\mathbb{R}^N)$.

p. 356, line 9 up: Change $\langle \varphi \log \varphi \rangle_{\gamma_{0,1}}$ to

$$\left\langle \varphi \log \frac{\varphi}{\langle \varphi \rangle_{\gamma_{0,1}}} \right\rangle_{\gamma_{0,1}}$$

p. 382, line 11 down: change the statements of parts (**ii**) and (**ii**) of Exercise 9.1.17 as follows.

(**ii**) For $\ell \in \mathbb{Z}^+$, let π_ℓ be the natural projection map from $\mathbf{E}$ onto E_ℓ, and show that $\mathbf{K} \subset\subset \mathbf{E}$ if

$$\mathbf{K} = \bigcap_{\ell \in \mathbb{Z}^+} \pi_\ell^{-1}(K_\ell), \quad \text{where} \quad K_\ell \subset\subset E_\ell \text{ for each } \ell \in \mathbb{Z}^+.$$

Conclude from this that $\mathbf{A} \subseteq \mathbf{M}_1(\mathbf{E})$ is tight if and only if $\big\{(\pi_\ell)_*\mu : \mu \in \mathbf{A}\big\} \subseteq \mathbf{M}_1(E_\ell)$ is tight for every $\ell \in \mathbb{Z}^+$.

Next, set $\mathbf{E}_\ell = \prod_{k=1}^{\ell} E_k$, and let $\boldsymbol{\pi}_\ell$ denote the natural projection map from $\mathbf{E}$ onto $\mathbf{E}_\ell$. Show that for each $\varphi \in U_{\mathrm{b}}^{\mathbf{R}}(\mathbf{E};\mathbb{R})$ and $\epsilon > 0$ there is an $\ell \geq 1$ such that $|\varphi(\mathbf{y}) - \varphi(\mathbf{x})| < \epsilon$ for all $\mathbf{x}, \mathbf{y} \in \mathbf{E}$ with $\boldsymbol{\pi}_\ell(\mathbf{x}) = \boldsymbol{\pi}_\ell(\mathbf{y})$. Use this to show that $\mu_n \Longrightarrow \mu$ in $\mathbf{M}_1(\mathbf{E})$ if and only if $\langle \varphi \circ \boldsymbol{\pi}_\ell, \mu_n \rangle \longrightarrow \langle \varphi \circ \boldsymbol{\pi}_\ell, \mu \rangle$ for every $\ell \geq 1$ and $\varphi \in C_{\mathrm{b}}(\mathbf{E}_\ell;\mathbb{R})$.

(**iii**) Let $\mu_{[1,\ell]}$ be an element of $\mathbf{M}_1\big(\mathbf{E}_\ell\big)$, and assume that the $\mu_{[1,\ell]}$'s are **consistent** in the sense that, for every $\ell \in \mathbb{Z}^+$,

$$\mu_{[1,\ell+1]}\big(\Gamma \times E_{\ell+1}\big) = \mu_{[1,\ell]}(\Gamma) \quad \text{for all } \Gamma \in \mathcal{B}_{\mathbf{E}_\ell}.$$

Show that there is a unique $\mu \in \mathbf{M}_1(\mathbf{E})$ such that $\mu_{[1,\ell]} = (\boldsymbol{\pi}_\ell)_*\mu$ for every $\ell \in \mathbb{Z}^+$.

p. 415, line 10 down: $\frac{h(T,\mathbf{x},\mathbf{y})}{g^{(N)}(T,\boldsymbol{\psi}(T)-\mathbf{x})}$

p. 450, line 2 down & 14 up: Change $e^{\frac{\pi^2}{4}t}$ to $e^{\frac{\pi^2}{8}t}$ and (**ii**) to (**iii**)

p. 452, line 8 down: $\leq \frac{e^{-\frac{t\lambda_M}{2}}}{(\pi t)^{\frac{N}{2}}}$

p. 472, line 5 up: The assertion in Exercise 11.1.36 is false and so this exercise should be ignored.

p. 472, lines 1 & 10 down: Change (**ii**) to (**i**) and (**iii**) to (**ii**).

p. 473, line 2 up: $\mathcal{W}_{\mathbf{y}}^{(N)}\big(\exists t \in [0,\infty)\ \boldsymbol{\psi}(t) = \boldsymbol{\varphi}(t)\big)$

p. 502, line 1 down: Theorem 11.4.6

p. 515, line 1 down: Let G be a connected open subset