Errata

p. 2, 15 up: replace \( \frac{\ell!}{k!(\ell-k)!} \) by \( \frac{\ell!}{k!(\ell-k)!} \).

p. 3, 5 dn: replace \((C^n)_{\ell-1} + (C^n)_{\ell-1}\) by \((C^n)_{\ell-1} + (C^n)_{\ell}\).

p. 3, 6 up: replace “numbers” by “number”

p. 3, 7 up: replace \(S_\ell - S_{m-1}\) by \(S_m - S_{m-1}\)

p. 3, footnote: replace “infemum” by “infimum”

p. 4, 9 dn: remove “is”

p. 4, 7 up: remove “of.”

p. 7, eq. (1.1.15): replace “2p \lor q” by “2(p \lor q)”

p. 8, 3 dn: remove “and using (1.1.16) with \(a = 1\)”

p. 8, 6 dn: should be “\(\mathbb{E}[s_{\Sigma^01}]\)”

p. 9, 5 up: replace “if” by “if and only if.”

p. 10, 4 up: remove “by (1.3.13)”

p. 11, bottom half: replace “\(\ell\)” by “1”

p. 13, 2 dn: remove “only”

p. 13, 17 up: “\(\exp\left(-\ell \log \frac{\ell}{n+1}\right)\)” should be “\(\exp\left(\ell \log \frac{\ell}{n+1}\right)\)”

p. 13, 4 up: replace “which” by “with”

p. 14, 12 dn: replace “\(n\)” by “\(\frac{n}{\ell}\)”

p. 15, 11 up: replace “see” by “sees”

p. 17, 15 dn: replace “assume” by “assumed.”
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p. 17, 17 dn: replace $\sum_{m=1}^{n} = \sum_{m=1}^{n} Y_m$. 

p. 18, 7 dn: replace “Starting again from (*)” by “Starting from (bf c)”

p. 20, 1 up: replace $E[|X_n|/n]$ by $E[|X_m|/n]$

p. 21, 4 dn: replace “(1.3.10)” by “(1.2.3)”

p. 26, 11 up: change $\mu_{|\{1,\ldots,N\}}$ to $\mu_{|S}$

p. 33, eq. (2.3.1): Replace $P_f$ by $P_n f$

p. 36, 3 up: replace $= \lim_{N \to \infty}$ by $\geq \lim_{N \to \infty}$.

p. 40, 15 up: replace “$2(N-1)/ne$” by “$N-1/ne$”

p. 47, 5 dn: should be $\sum_{n=1}^{\infty}$.

p. 48, 5 dn: replace $= \mathbb{P}(\rho_j < \infty | X_0 = i)$ by $= \mathbb{P}(\rho_j < \rho_i | X_0 = i)$.

p. 49, 11 dn: change to

$$E[u(X_{n,\Gamma}) | X_0 = i] \leq u(i) \text{ for } n \geq 0 \text{ and } i \in S \setminus \Gamma.$$ 

p. 49, 13 up: change “any $i$” to “any $i \notin \Gamma$”

p. 49, 12–11 up: replace $\sum_{k \notin S}$ by $\sum_{k \notin \Gamma}$.

p. 50, 14 up to p. 51, 14 dn: change to:

3.1.8 Lemma. If $P$ is irreducible on $S$, then, for any finite subset $F \neq S$, $\mathbb{P}(\rho_{S \setminus F} < \infty | X_0 = i) = 1$ for all $i \in F$.

Proof: Set $\tau = \rho_{S \setminus F}$. By irreducibility, $\mathbb{P}(\tau < \infty | X_0 = i) > 0$ for each $i \in F$. Hence, because $F$ is finite, there exists a $\theta \in (0,1)$ and an $N \geq 1$ such that $\mathbb{P}(\tau > N | X_0 = i) \leq \theta$ for all $i \in F$. But this means that, for each
i ∈ F, \( P(\tau > (\ell + 1)N \mid X_0 = i) \) equals
\[
\sum_{k ∈ F} P(\tau > (\ell + 1)N & X_{\ell N} = k \mid X_0 = i)
\]
\[
= \sum_{k ∈ F} P(X_n ∈ F \text{ for } \ell N + 1 ≤ n ≤ (\ell + 1)N, \tau > \ell N, \& X_{\ell N} = k \mid X_0 = i)
\]
\[
= \sum_{k ∈ F} P(\tau > N \mid X_0 = k)P(\tau > \ell N & X_{\ell N} = k \mid X_0 = i)
\]
\[
≤ \theta P(\tau > \ell N \mid X_0 = i).
\]
Thus, \( P(\tau > \ell N \mid X_0 = i) ≤ \theta^\ell \), and so \( P(\tau_m = \infty \mid X_0 = j) = 0 \) for all \( i ∈ F \). □

3.1.9 Theorem. Assume that \( P \) is irreducible on \( S \), and let \( u : S → [0, \infty) \) be a function with the property that \( \{ k : u(k) ≤ L \} \) is finite for each \( L ∈ (0, \infty) \). If, for some \( j ∈ S \), \( (Pu)_i ≤ u(i) \) for all \( i ≠ j \), the chain determined by \( P \) is recurrent on \( S \).

Proof: If \( S \) is finite, then (cf., for example, Exercise 2.4.2) at least one state is recurrent, and therefore, by irreducibility, all are. Hence, we will assume that \( S \) is infinite.

Given \( i ≠ j \), set \( F_L = \{ k : u(k) ≤ u(i) + u(j) + L \} \) for \( L ∈ \mathbb{N} \), and denote by \( \rho_L \) the first return time \( \rho_{(S \setminus F_L)∪\{j\}} \) to \( (S \setminus F_L) ∪ \{j\} \). By Lemma 3.1.6,
\[
u(i) ≥ E[u(X_{n∧\rho_L}) \mid X_0 = i] ≥ (u(i) + u(j) + L)P(\rho_{S \setminus F_L} < n ∧ \rho_j \mid X_0 = i)
\]
for all \( n ≥ 1 \). Hence, after letting \( n → \infty \), we conclude that, for all \( L ∈ \mathbb{N} \),
\[
u(i) ≥ (u(i) + u(j) + L)P(\rho_{S \setminus F_L} < \rho_j \mid X_0 = i)
\]
\[
≥ (u(i) + u(j) + L)P(\rho_j = \infty \mid X_0 = i),
\]
since, by Lemma 3.1.8, we know that \( P(\rho_{S \setminus F_L} < \infty \mid X_0 = i) = 1 \). Thus, we have now shown that \( P(\rho_j < \infty \mid X_0 = i) = 1 \) for all \( i ≠ j \). Since
\[
P(\rho_j < \infty \mid X_0 = j) = (P)_{jj} + \sum_{i≠j} P(\rho_j < \infty \mid X_0 = i)(P)_{ji},
\]
it follows that \( P(\rho_j < \infty \mid X_0 = j) = 1 \), which means that \( j \) is recurrent. □

p. 57, 12–11 up: change to: If \( j \) is transient, then \( \pi_{ij} = 0 \) for all \( i \) and therefore \( \mu_j = 0 \). If \( j \) is recurrent, then either \( i \) is transient, and so \( \mu_i = 0 \), or \( i \) is recurrent, in which case, by Theorem 3.1.2, either \( \pi_{ij} = 0 \) or \( i → j \) and \( \pi_{ij} = \pi_{jj} \).

p. 57, 3 up: change to “and, for each \( i ∈ C \) and \( s ∈ (0,1) \), \( (R(s))_{ik} > 0 \iff k ∈ C \).”

p. 57, 2 up: change “In particular,” to “In addition,”
p. 58, 16 up: change to $b = a = a'$

p. 65: Here is a more conceptual way to prove that

$$\sum_{r=1}^{n} \mathbb{P}(\rho_j \geq r \mid X_0 = j)(\mathbf{P}^{n-r})_{jj} = 1.$$ 

Take $\rho_j^{(0)} = 0$ and, for $m \geq 1$, $\rho_j^{(m)}$ to be the time of the $m$th return to $j$. In addition, set $T_j^{(n-1)} = \sum_{\ell=0}^{n-1} \mathbf{1}_{\{j\}}(X_\ell)$. Then

$$X_0 = j \implies \{T_j^{(n-1)} = m + 1\} = \{\rho_j^{(m)} < n \leq \rho_j^{(m+1)}\}.$$ 

Hence,

$$\sum_{r=1}^{n} \mathbb{P}(\rho_j \geq r \mid X_0 = j)(\mathbf{P}^{n-r})_{jj} = \sum_{r=0}^{n-1} \mathbb{P}(\rho_j \geq n - r \mid X_0 = j)(\mathbf{P}^r)_{jj}$$

$$= \sum_{r=0}^{n-1} \sum_{m=0}^{r} \mathbb{P}(\rho_j \geq n - r \mid X_0 = j)\mathbb{P}(\rho_j^{(m)} = r \mid X_0 = j)$$

$$= \sum_{r=0}^{n-1} \sum_{m=0}^{r} \mathbb{P}(\rho_j^{(m+1)} \geq n \& \rho_j^{(m)} = r \mid X_0 = j)$$

$$= \sum_{m=0}^{n-1} \sum_{r=m}^{n-1} \mathbb{P}(\rho_j^{(m+1)} \geq n \& \rho_j^{(m)} = r \mid X_0 = j)$$

$$= \sum_{m=0}^{n-1} \mathbb{P}(\rho_j^{(m)} < n \leq \rho_j^{(m+1)} \mid X_0 = j)$$

$$= \sum_{m=0}^{n-1} \mathbb{P}(T_j^{(n-1)} = m + 1) = \mathbb{P}(T_j^{(n-1)} \leq n \mid X_0 = j) = 1.$$ 

p. 67, 7 dn: change to “$i \in S_{r+n} \implies 1 = \sum_{r'=0}^{d-1} (\mathbf{P}^n\mathbf{1}_{S_{r'}})_{ij} = (\mathbf{P}^n\mathbf{1}_{S_r})_{ij}.$”

p. 67, 17 dn: replace $\pi_j^{(r)}$ by $\pi_j^{(r)}.$

p. 68, Exercise 3.3.3: One needs to add that assumption that $(\mathbf{P}u)_j < \infty.$

p. 70, 4 dn: replace $\alpha > 0$ by $\alpha \geq 1.$

p. 72, 3 dn: change “Exercise (3.3.8)” to “Exercise (3.3.7)”
p. 72, 12 up: replace “to see . . . as $n \to \infty$.” by “to see that $(\nu)_j \geq (\mu)_j$ for all $j \in S$. Now consider $\omega \equiv \nu - \mu$, and conclude that $\omega = 0.$”

p 73, 15 dn: replace $= \pi_{ii} \pi_{jj}$ by $\pi_{jj} \pi_{ii}$.

p. 74, 9 up: replace $\epsilon > 0$ by $0 < \epsilon < 1$.

p. 74, 7 up: replace $\sup_m \geq N\epsilon$ by $\sup_m \geq n\epsilon$.

p. 76, 9 dn: replace $N(t) - N(s)$ by $N(s + t) - N(s)$

p. 76, 1 up: replace “of positive” by “are positive”

p. 77, 10 up: insert “$e^{-t}$” into the integrand of both integrals

p. 81, 9 dn: replace $E_n = \frac{J_n - J_{n-1}}{R_{n-1}}$ by $E_n = R_{X_{n-1}}(J_n - J_{n-1})$

p. 81, replace $R_{J_{m-1}}$ in the second line of (4.2.3) by $R_{X_{m-1}}$

p. 81, 16 & 13 up: replace $\Phi^{(R,P)}$ by $\Phi^R$

p. 81, 1 up: should be $\{\bar{X}(t \wedge \zeta) : t \geq 0\}$

p. 82, 15 dn: replace $\Phi^{(R,P)}$ by $\Phi^R$

p. 83, 9 & 8 up: replace $\Phi^{(R,P)}$ by $\Phi^R$

p. 83, 4 up: replace $\{N(s) = m\}$ by $\{J_m \leq s < J_{m+1}\}$

p. 84, 3 dn: replace $\Phi^{R,P}$ by $\Phi^R$

p. 92, 10 up: Starting here and running through 8 dn on p. 93, the proof of Theorem 4.3.2 can be replaced by: In particular, for any $T > 0$ and $m \geq 1$,

$$
P(J_n \leq T \mid X(0) = i) \leq P(J_n \leq T \& \rho_i^{(m)} \leq n \mid X(0) = i) + P(\rho_i^{(m)} > n \mid X(0) = i),
$$

where $\rho_i^{(m)}$ is the time of the $m$th return of $\{X_n : n \geq 0\}$ returns to $i$. Because, $i$ is recurrent for $\{X_n : n \geq 0\}$, the second term tends to 0 as
At the same time, 
\[
\rho_i^{(m)}(n) \leq n \implies J_n \geq \frac{1}{R_i} \sum_{\ell=1}^{m} E_{\rho_i^{(\ell)}},
\]
and so the first term in dominated by the probability that the sum of \(m\) mutually independent, unit exponential random variable is less than or equal to \(R_i T\), and this probability tends to 0 as \(m \to \infty\). Hence, we have shown that, for all \(T > 0\), 
\[
P(J_{\infty} \leq T \mid X(0) = i) = \lim_{n \to \infty} P(J_n \leq T \mid X(0) = i) = 0,
\]
and so 
\[
P(\epsilon < \infty \mid X(0) = i) = P(J_{\infty} < \infty \mid X(0) = i) = \lim_{T \to \infty} P(J_{\infty} \leq T \mid X(0) = i) = 0.
\]
p. 134, 15dn: Change to
\[ \int_{s}^{s+T(s,i,\xi)} S(\tau, i, L) \, d\tau = \xi. \]

p. 139, 5 up: replace \( 2\psi(s) \) by \( 2\psi(t) \)

p. 140, 20 up: replace \((P^T P)_{ii}\) by \(PP^T)_{ii}\)

p. 140, 8 up: replace \( \sum_{m=1}^{\infty} \) by \( \sum_{k=1}^{\infty} \) twice

p. 141, 3 dn: replace \( \theta_{d}^{rm} \) by \( \theta_{d}^{-rm} \)

p. 141, 6 dn: replace “Let \( H \) be the subspace of . . . .” by “Let \( H \) be the linear subspace spanned by . . . .”

p. 142, 18 dn: replace by “by a reversible probability vector for \( P \).”

p. 142: change (5.6.11) to \( \beta_+ \geq \frac{2#E}{\text{diam}(P) \text{diam}(P)} \) and (5.6.12) to \( \beta_- \geq \frac{2}{\text{diam}(P) \text{diam}(P)} \)

p. 143, 15 up: replace by \( \rho^k(\omega) \) if \( \eta = \hat{\omega}^k \)

p. 143, 10 up: change left hand side to \( -(g, Q^\mu f)_\mu \)