(Some)
Numbers and Maths
in Biology

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http://bionumbers.hms.harvard.edu/

B10NUMB3R5

THE DATABASE OF USEFUL BIOLOGICAL NUMBERS

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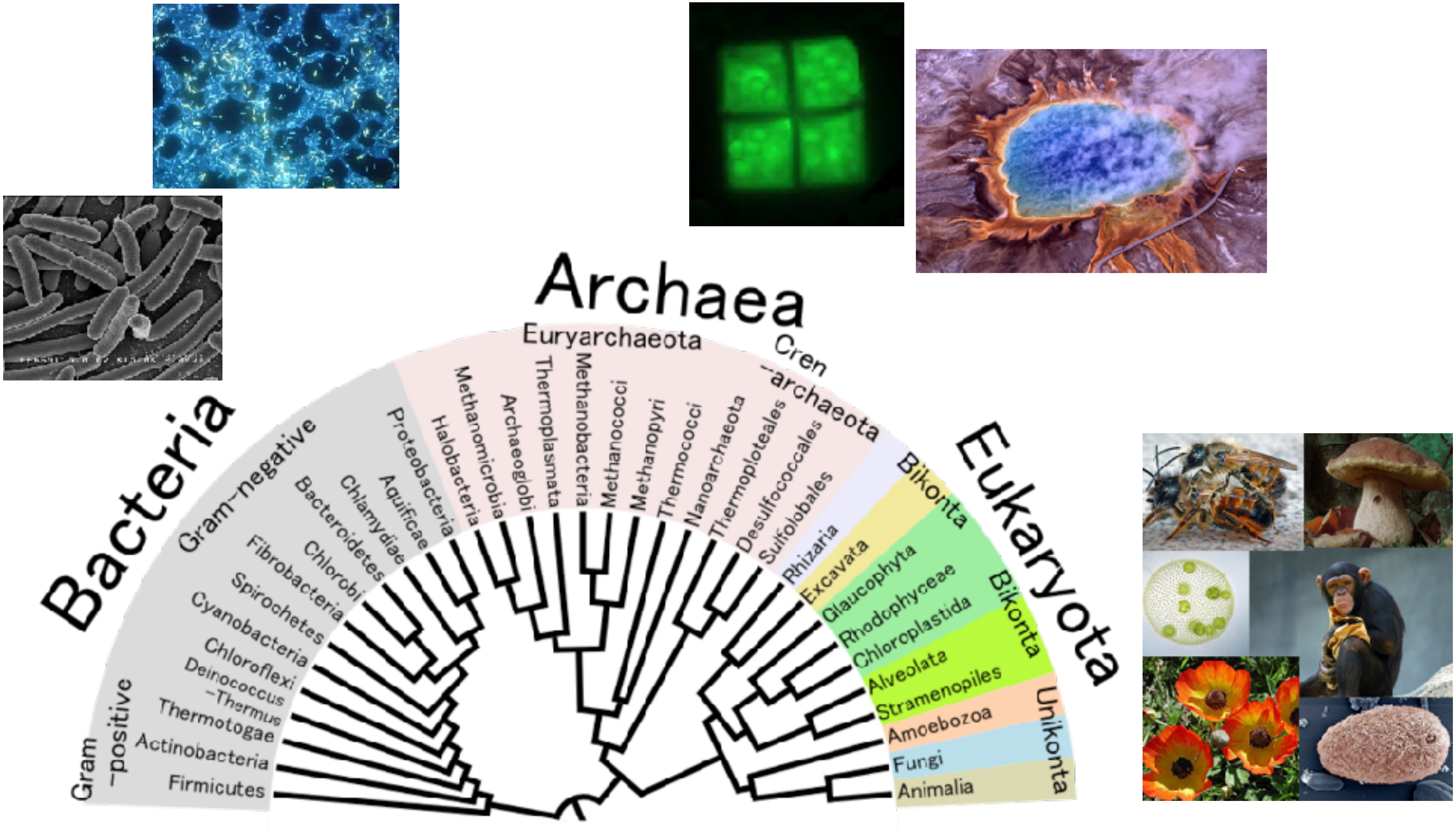
Phylogenetic tree



source: wiki

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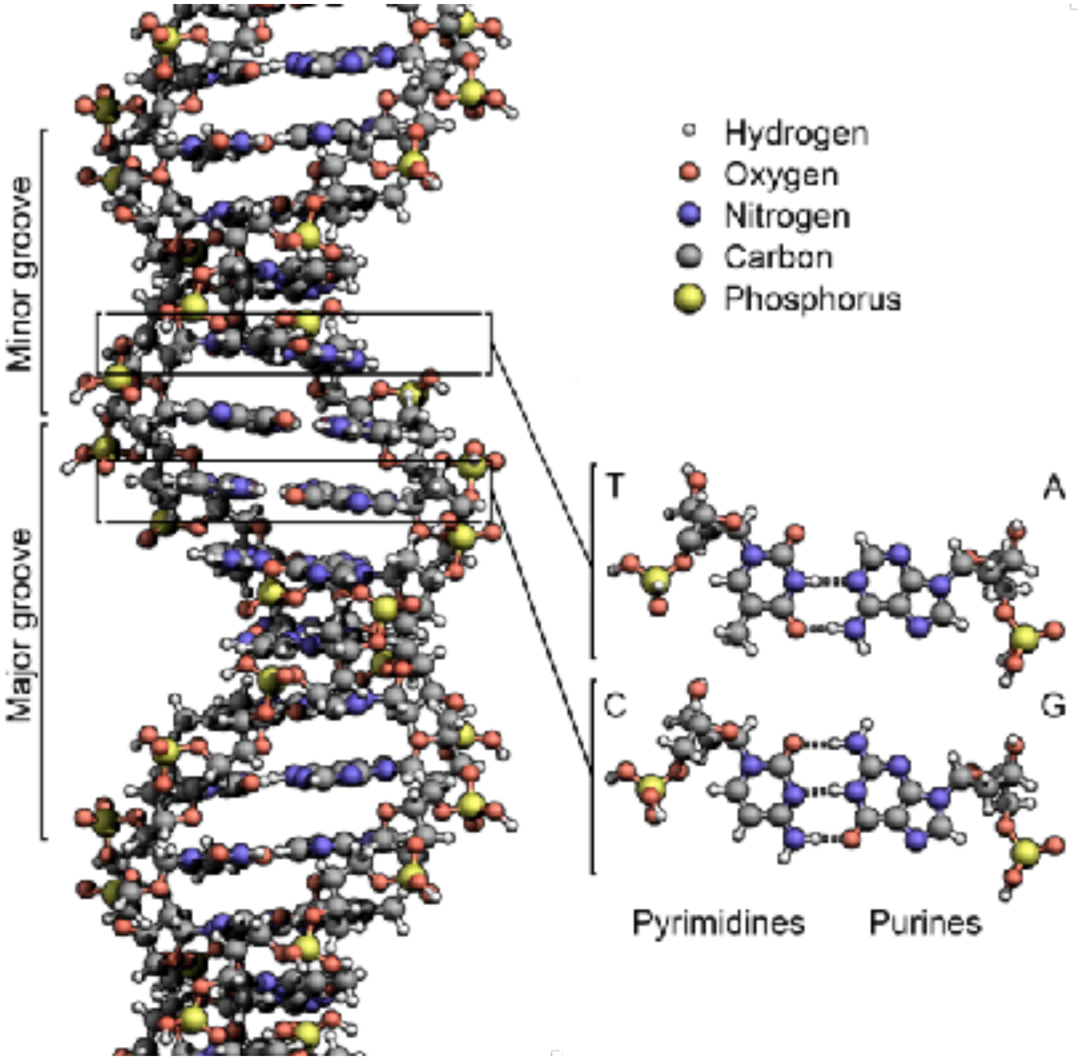
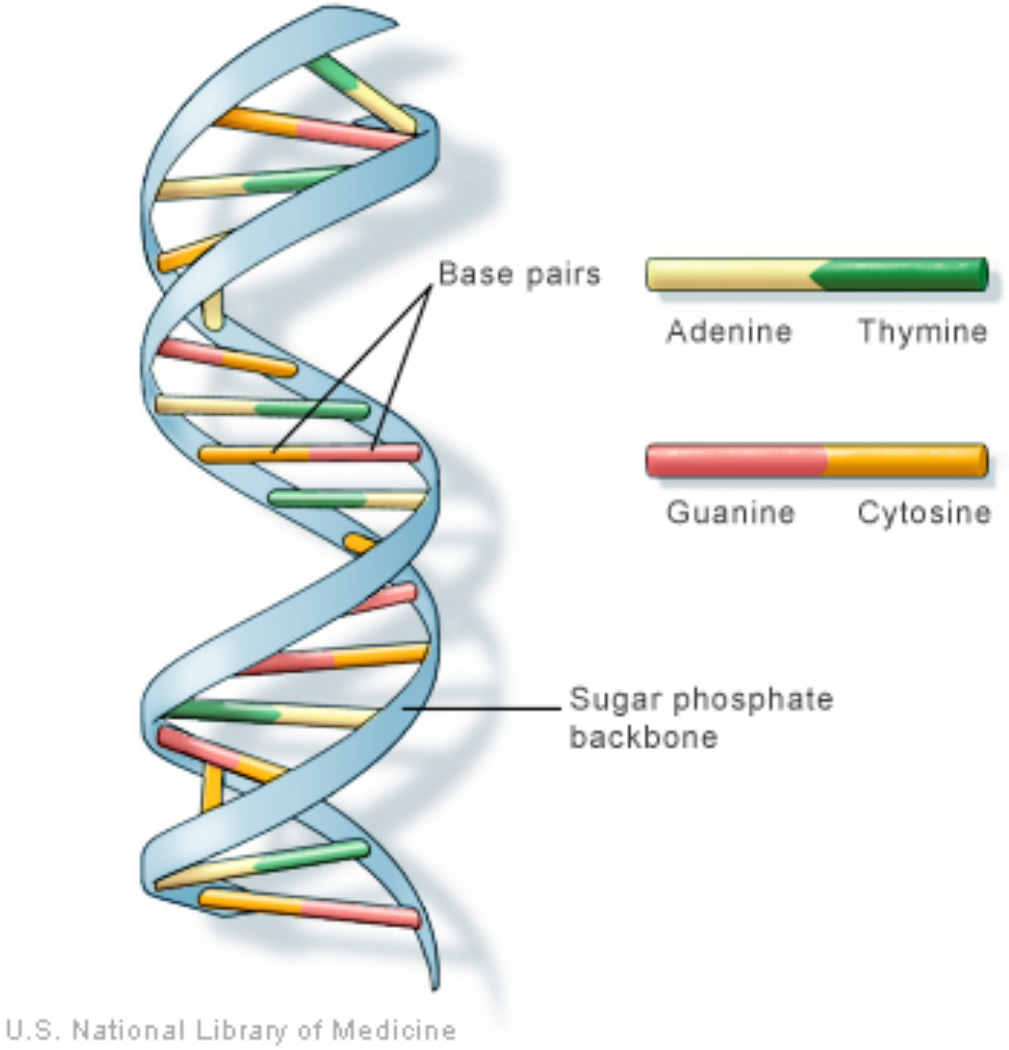
DNA





source: wiki

http://ghr.nlm.nih.gov/handbook/basics/dna

- DNA contour length in bacteria: ~1.5mm
- Length of DNA in nucleus of mammals: ~ 2-3m

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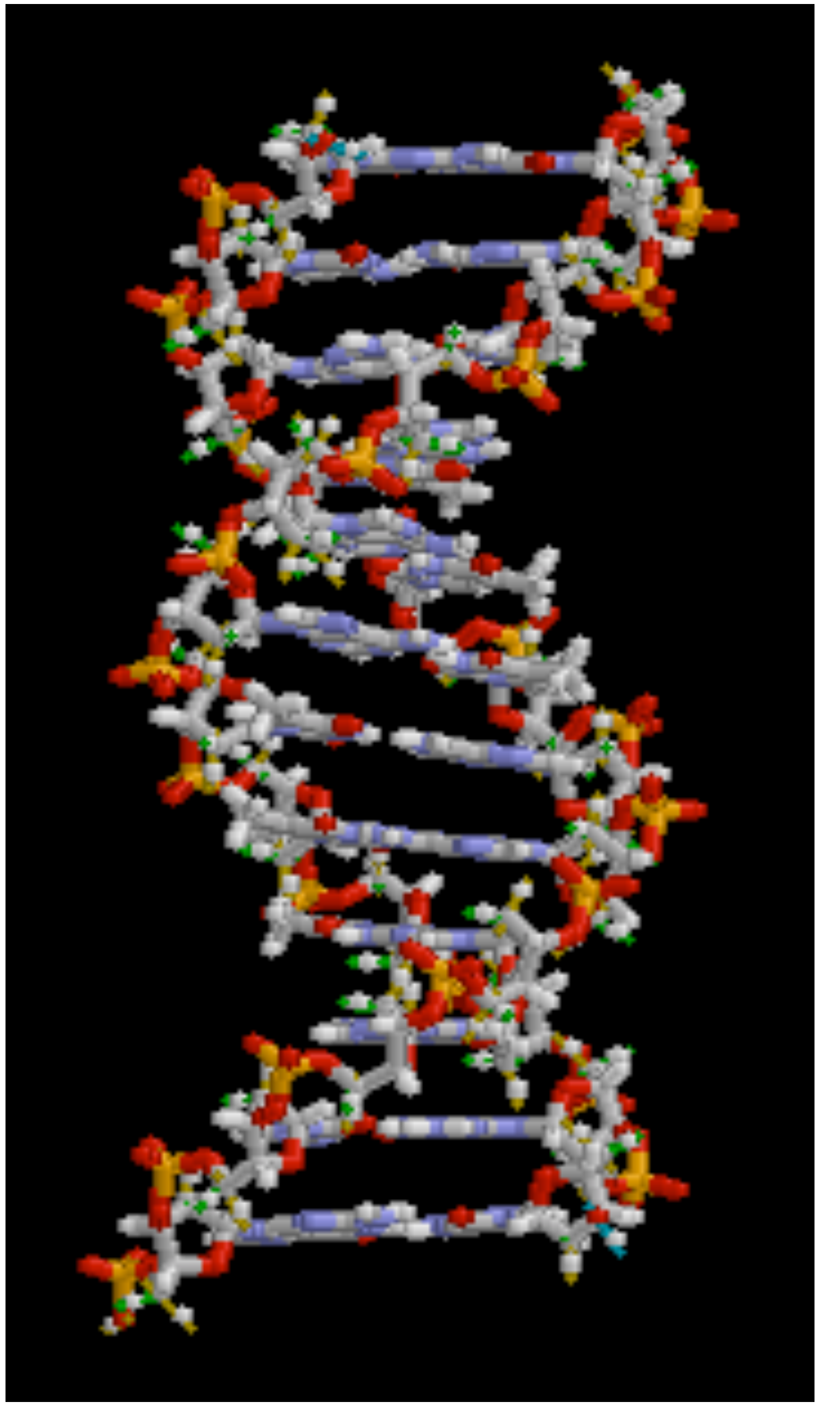
DNA = biopolymer pair



~ 3m per cell

~ 10^14 cells/human

> max. distance between Earth and Pluto (~50 AU = 7.5 x 10^12 m)

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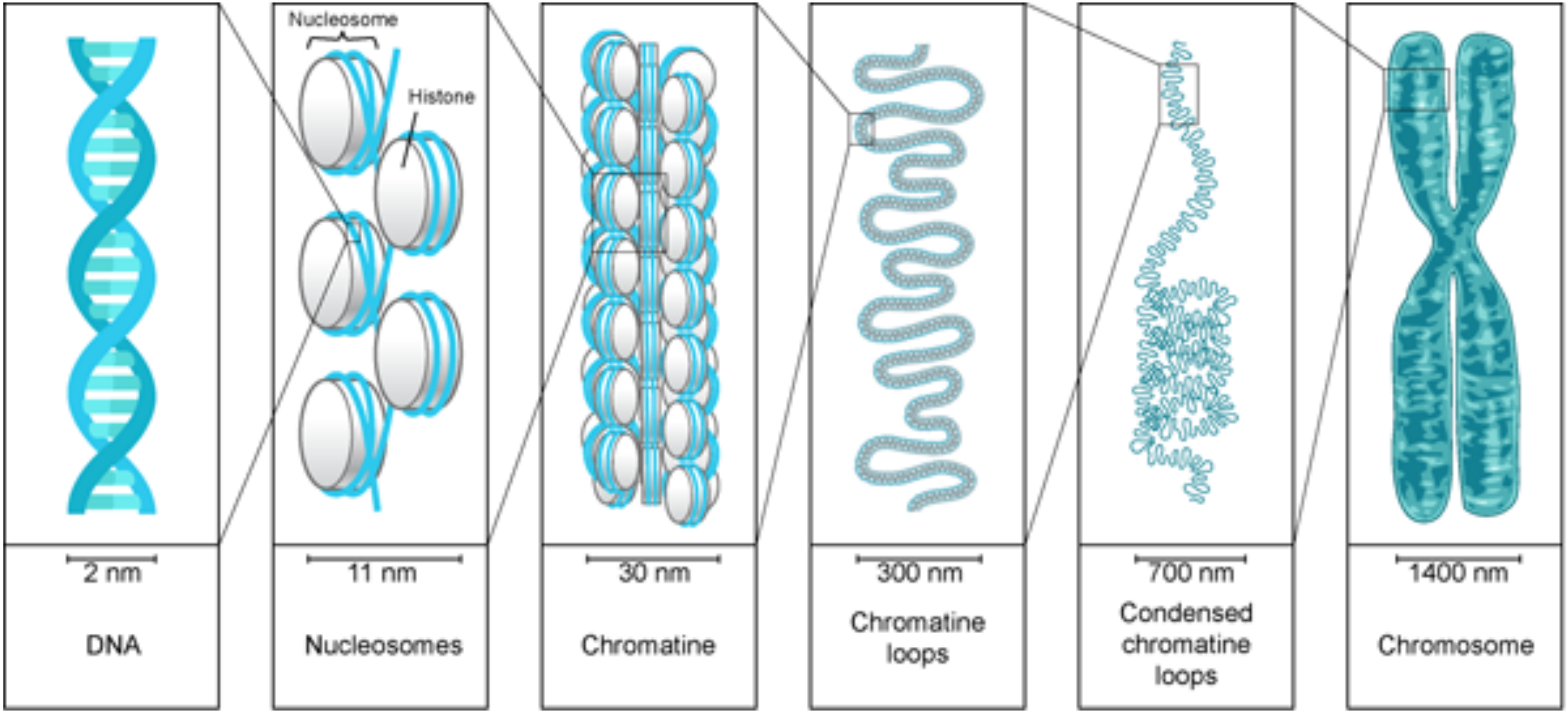
DNA packaging in eukaryotes



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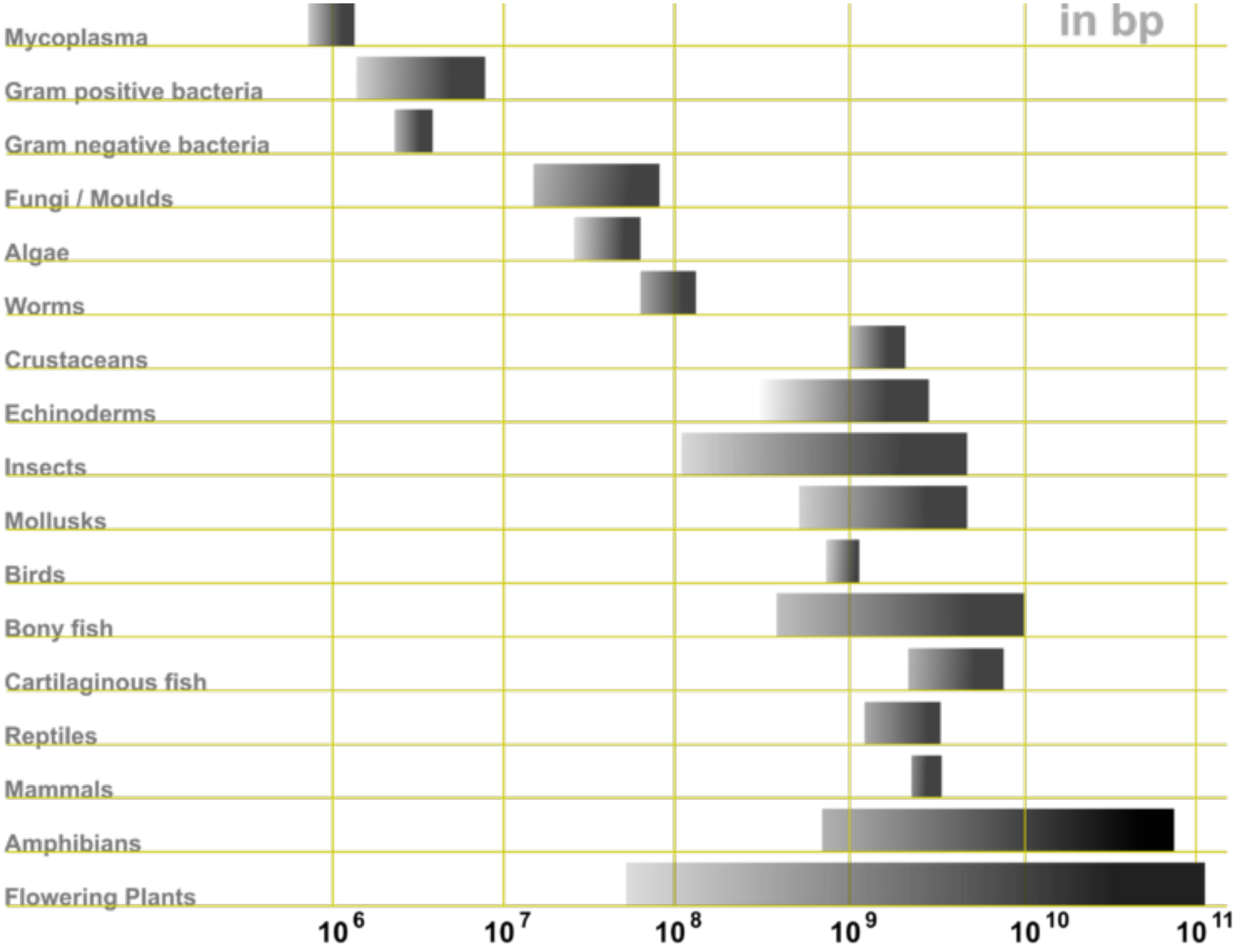

source: wiki

mass $1\mathrm{pg} = 978\mathrm{Mb}$

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source: wiki

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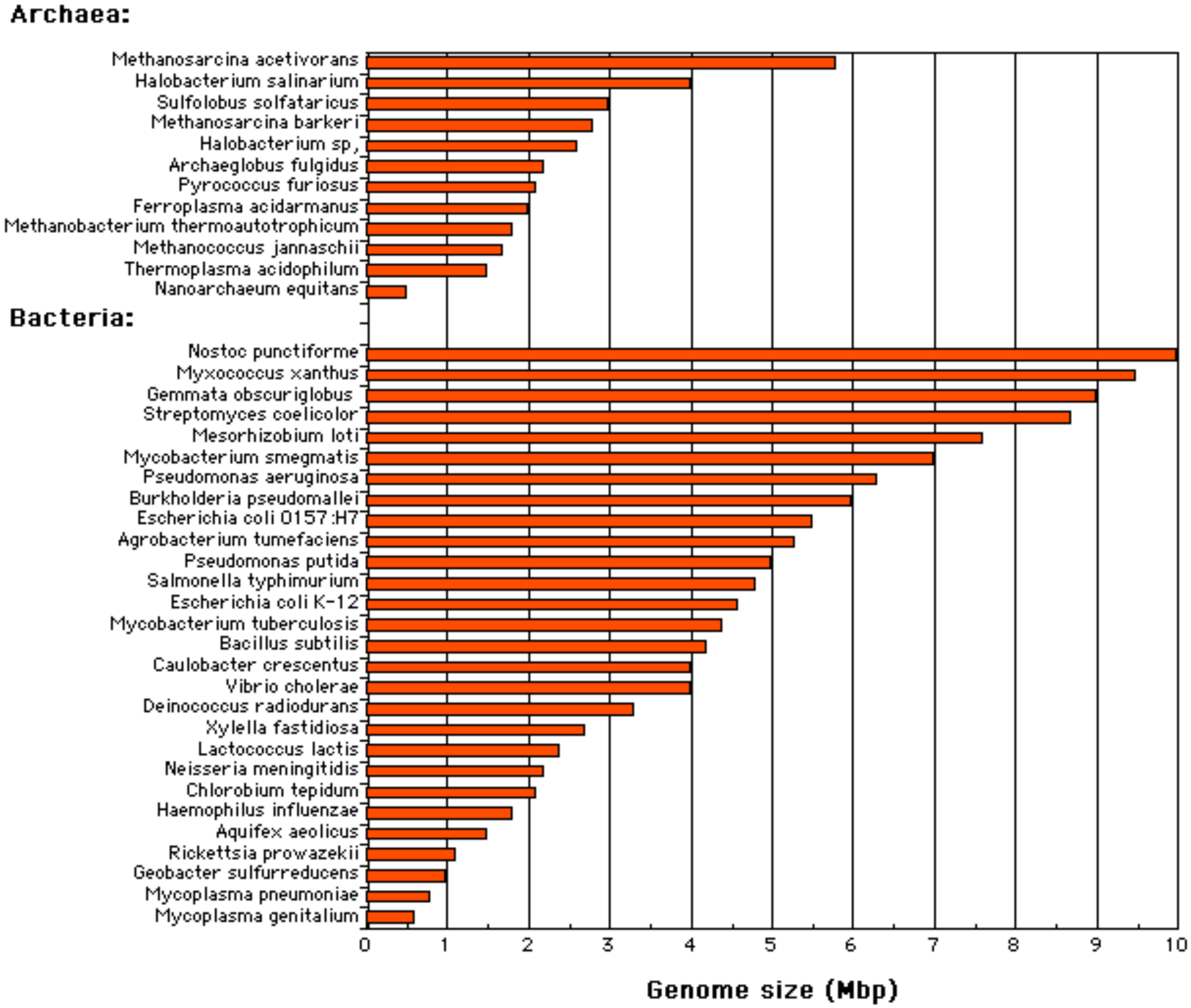
Prokaryotes



http://www.sci.sdsu.edu/~smaloy/MicrobialGenetics/topics/chroms-genes-prots/genomes.html

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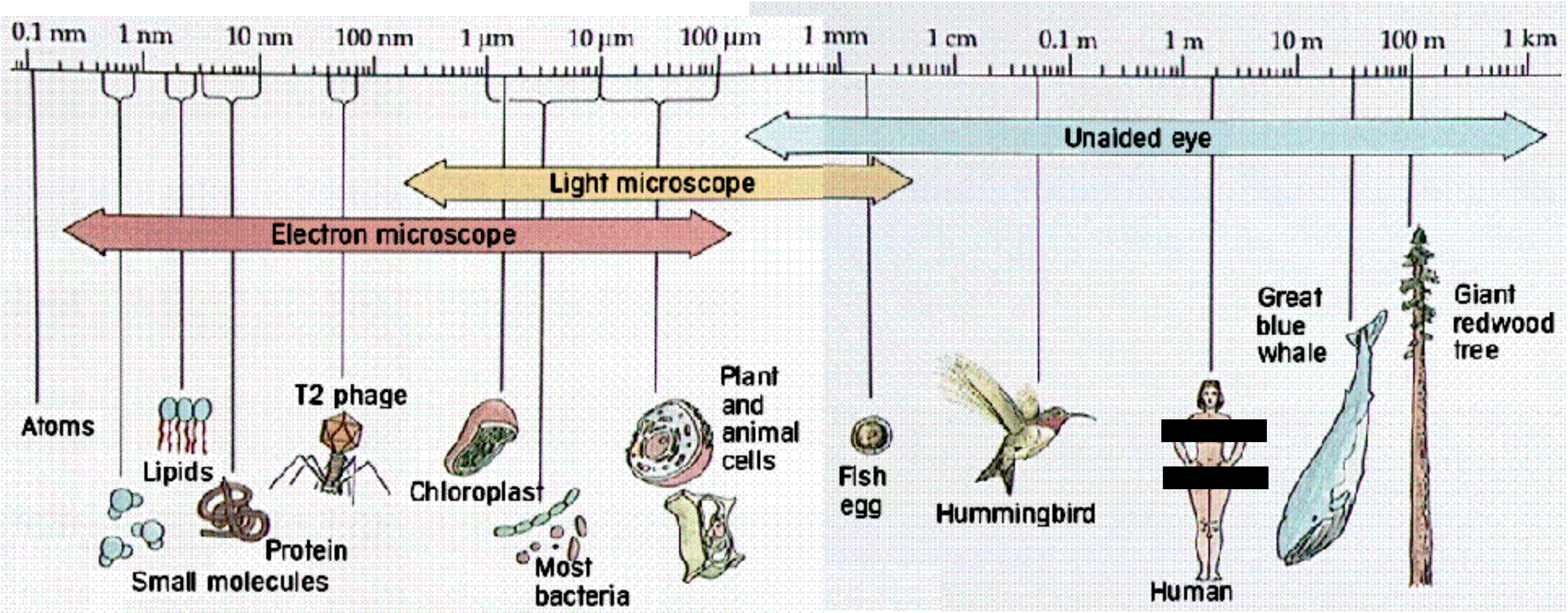
Typical length scales



[redacted]

[redacted]

http://www2.estrellamountain.edu/faculty/farabee/BIOBK/biobookcell2.html

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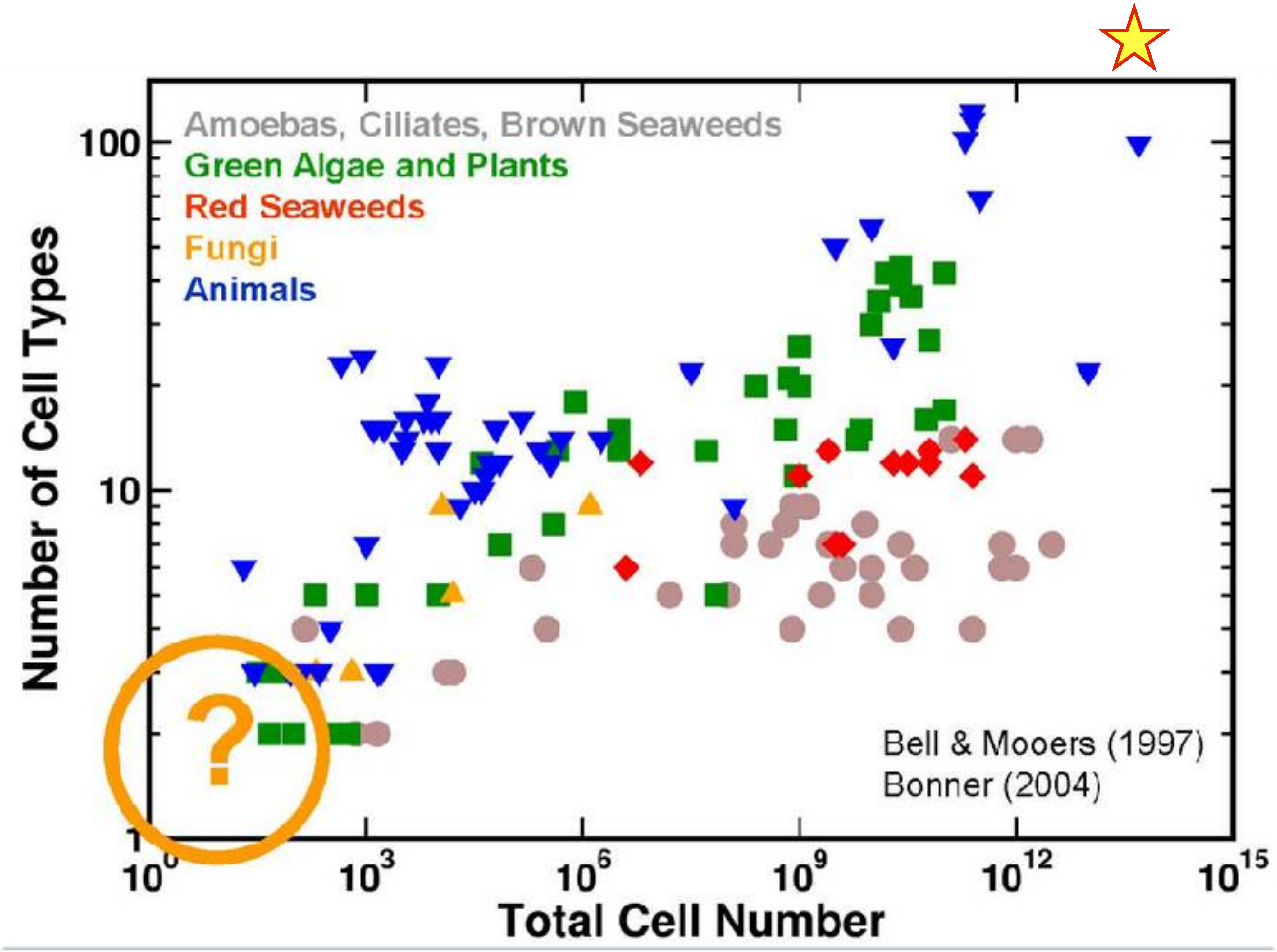
Species estimates

- estimated number of eukaryotic species on Earth: 8.7 million (Nature, 2011)
- undiscovered: 86% land spec & 91%marine spec
- ~ 300,000 plant species
- prokaryotic biomass ~ eukaryotic biomass
- oldest known fossilized prokaryotes from 3.5 billion years ago

Size-Complexity relation



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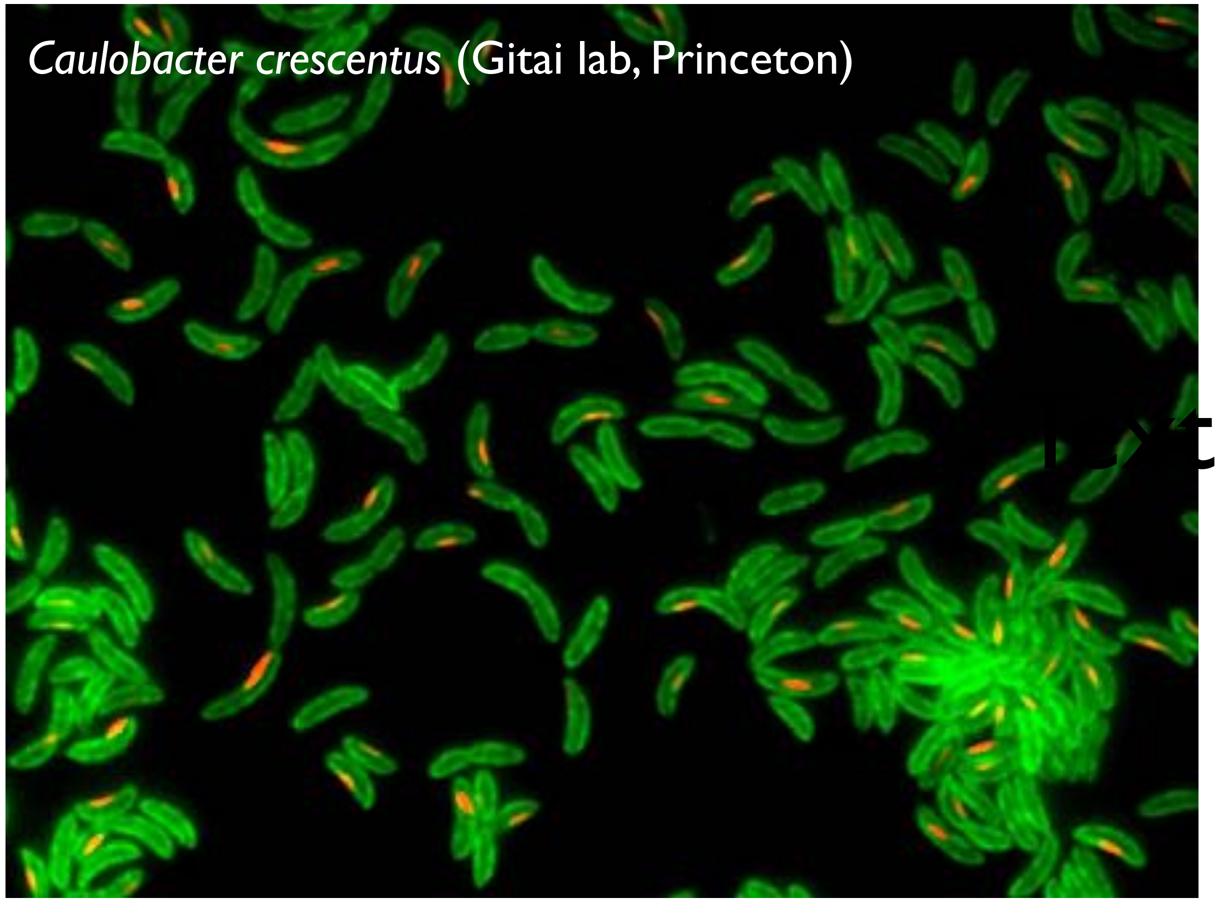
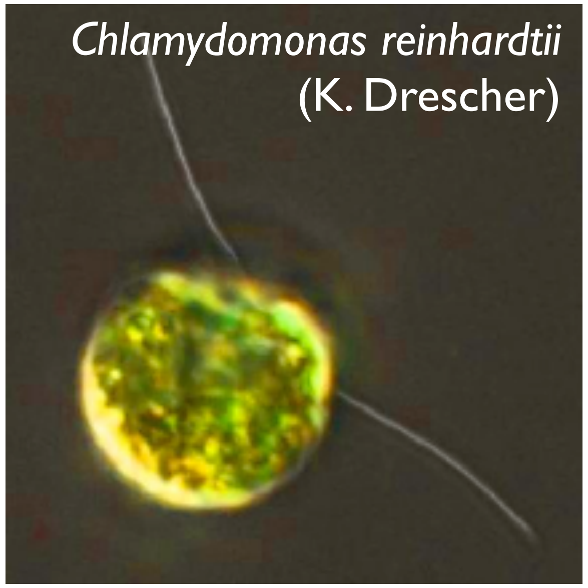
Unicellular organisms

Bacteria


Caulobacter crescentus (Gitai lab, Princeton)

size ~ 1μm
doubling time ~ 2h

Algae


Chlamydomonas reinhardtii
(K. Drescher)

size ~ 10μm
doubling time ~ 5-8h

Amoeba


size ~ 1mm
doubling time ~ 1d

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evolution from
unicellular to multicellular ?

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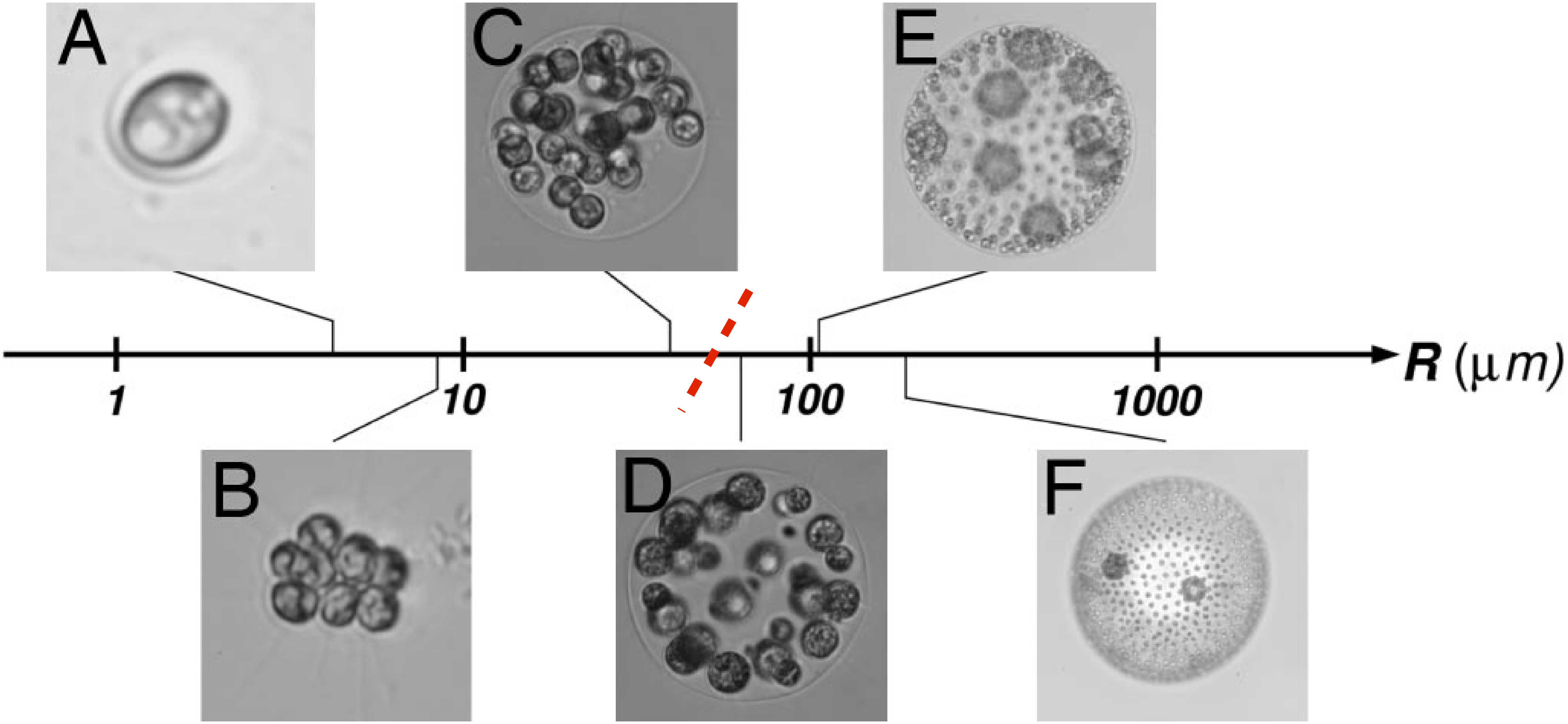
Evolution of multicellularity

Chlamydomonas reinhardtii

Eudorina elegans

Volvox carteri



Gonum pectorale

Pleodorina californica

Volvox aureus

Short et al, PNAS 2013

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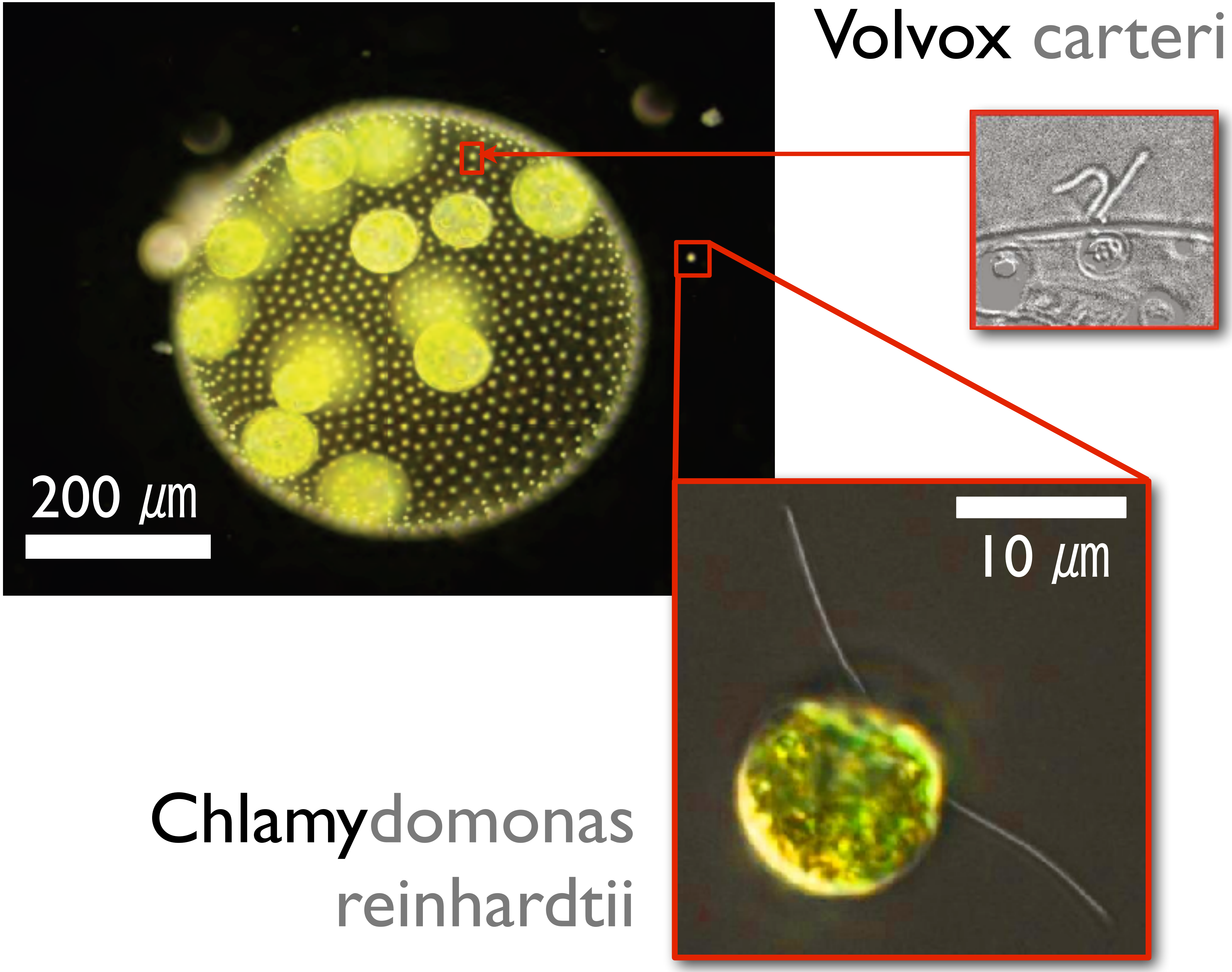
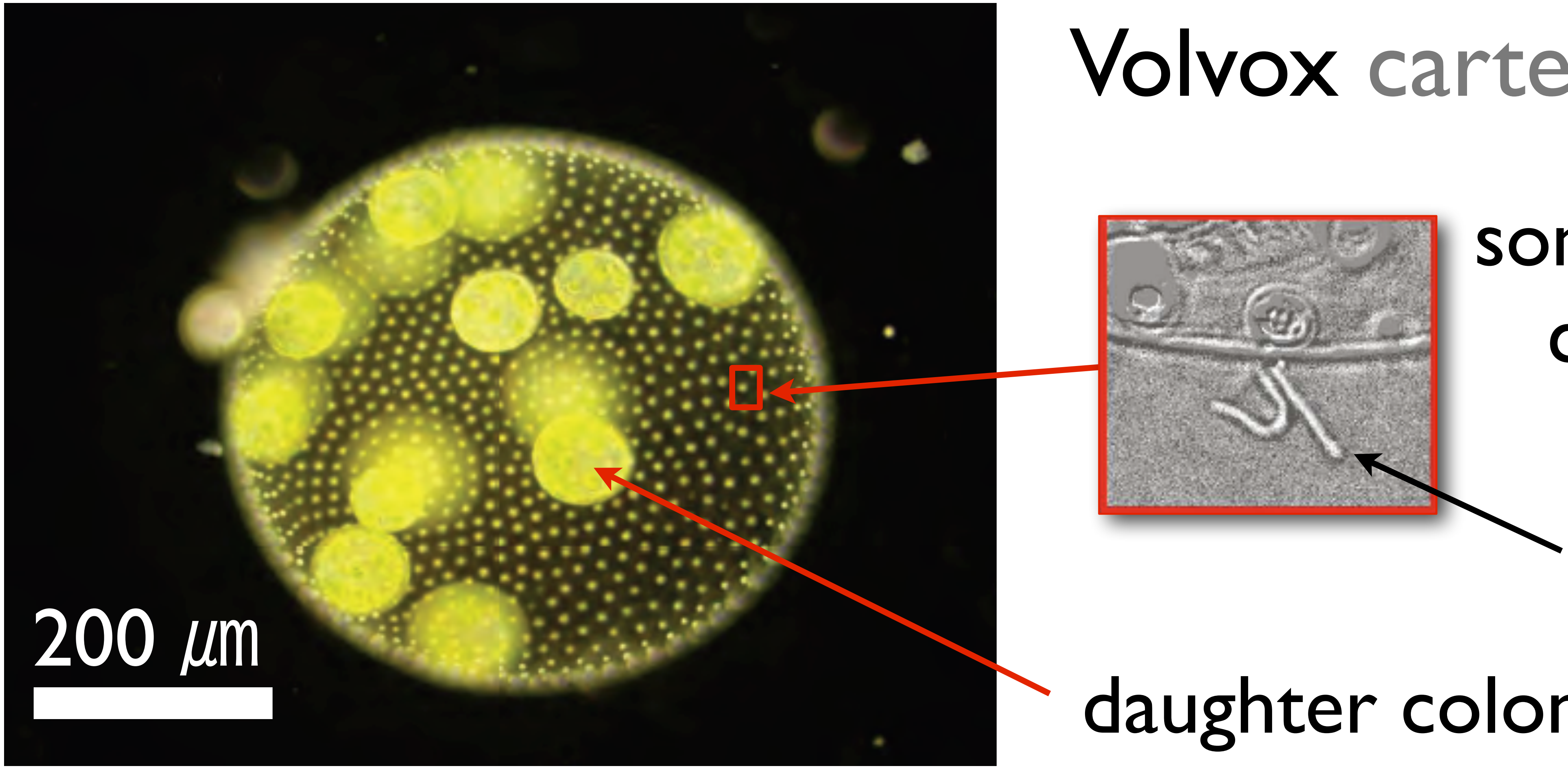
Volvox carteri



Chlamydomonas reinhardtii

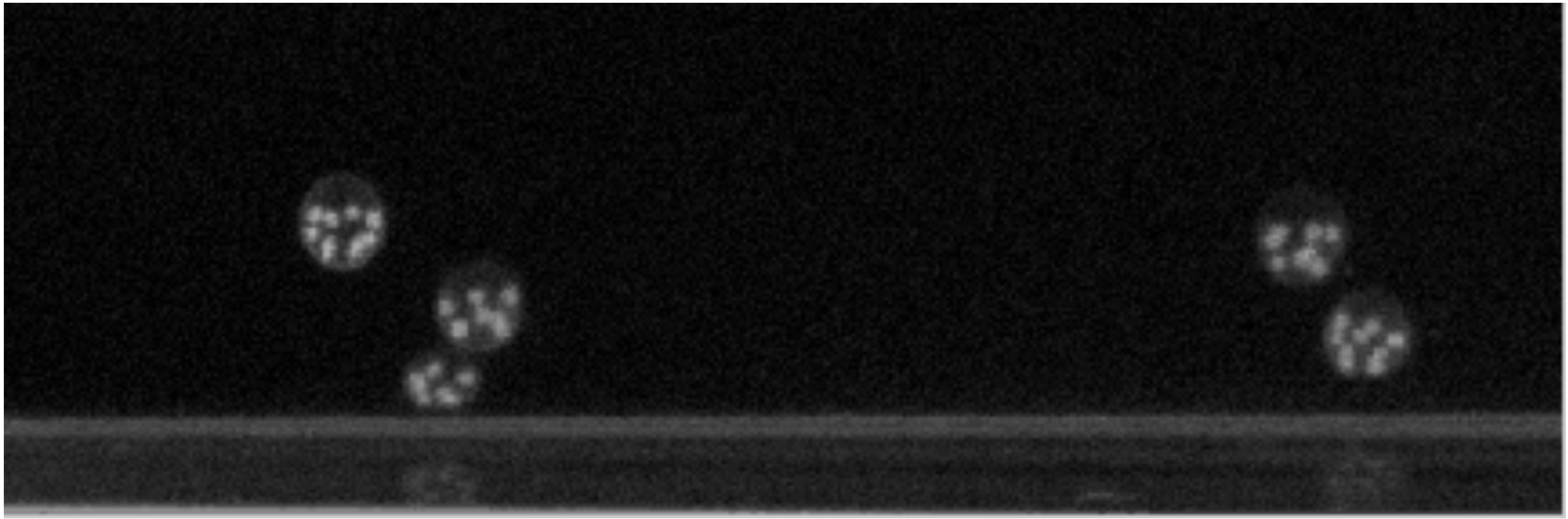
Volvox carteri



somatic cell

cilia

daughter colony



Drescher et al (2010) PRL

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how do organisms
achieve locomotion ?

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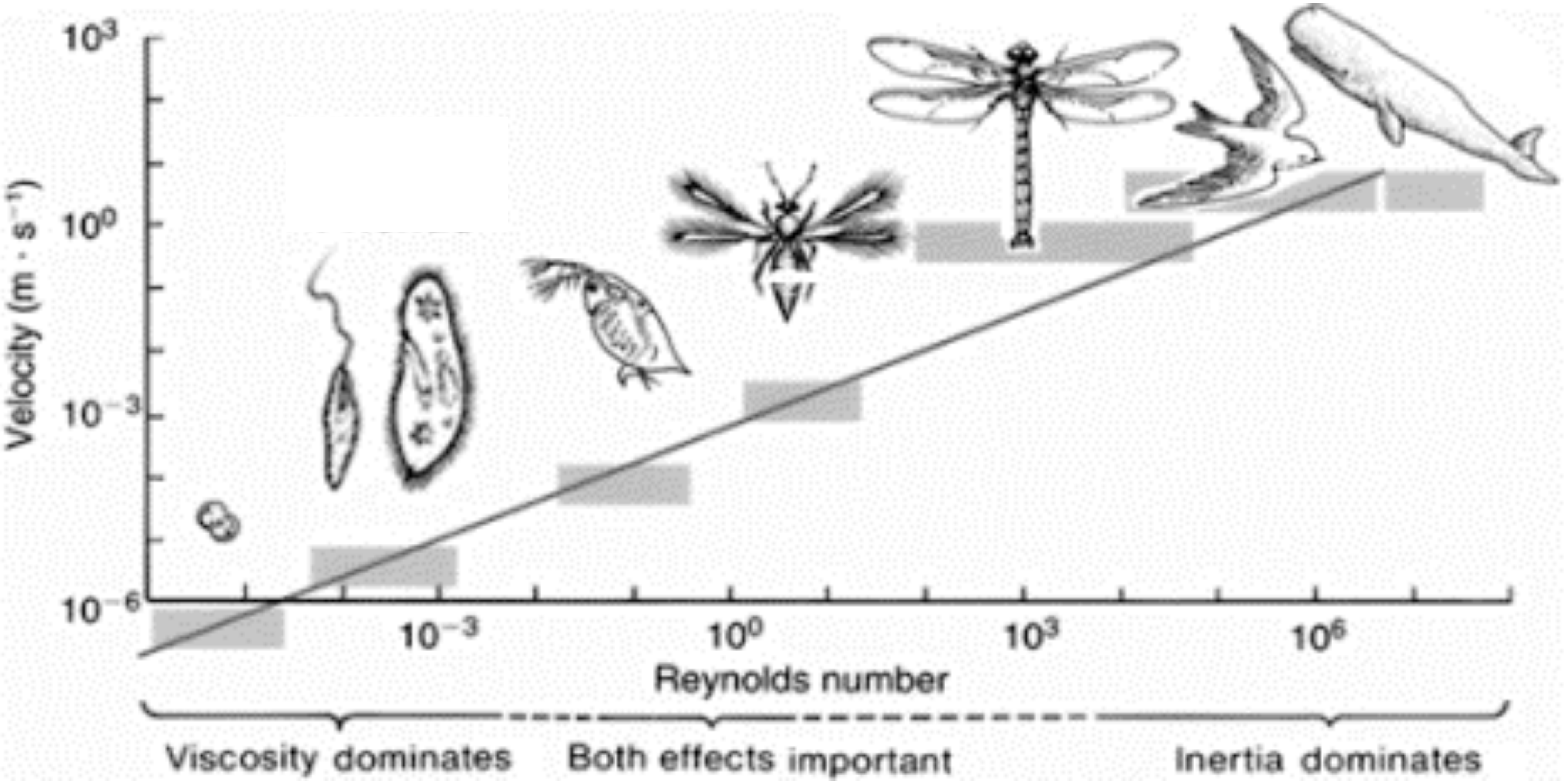
Reynolds numbers

$$Re = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$$



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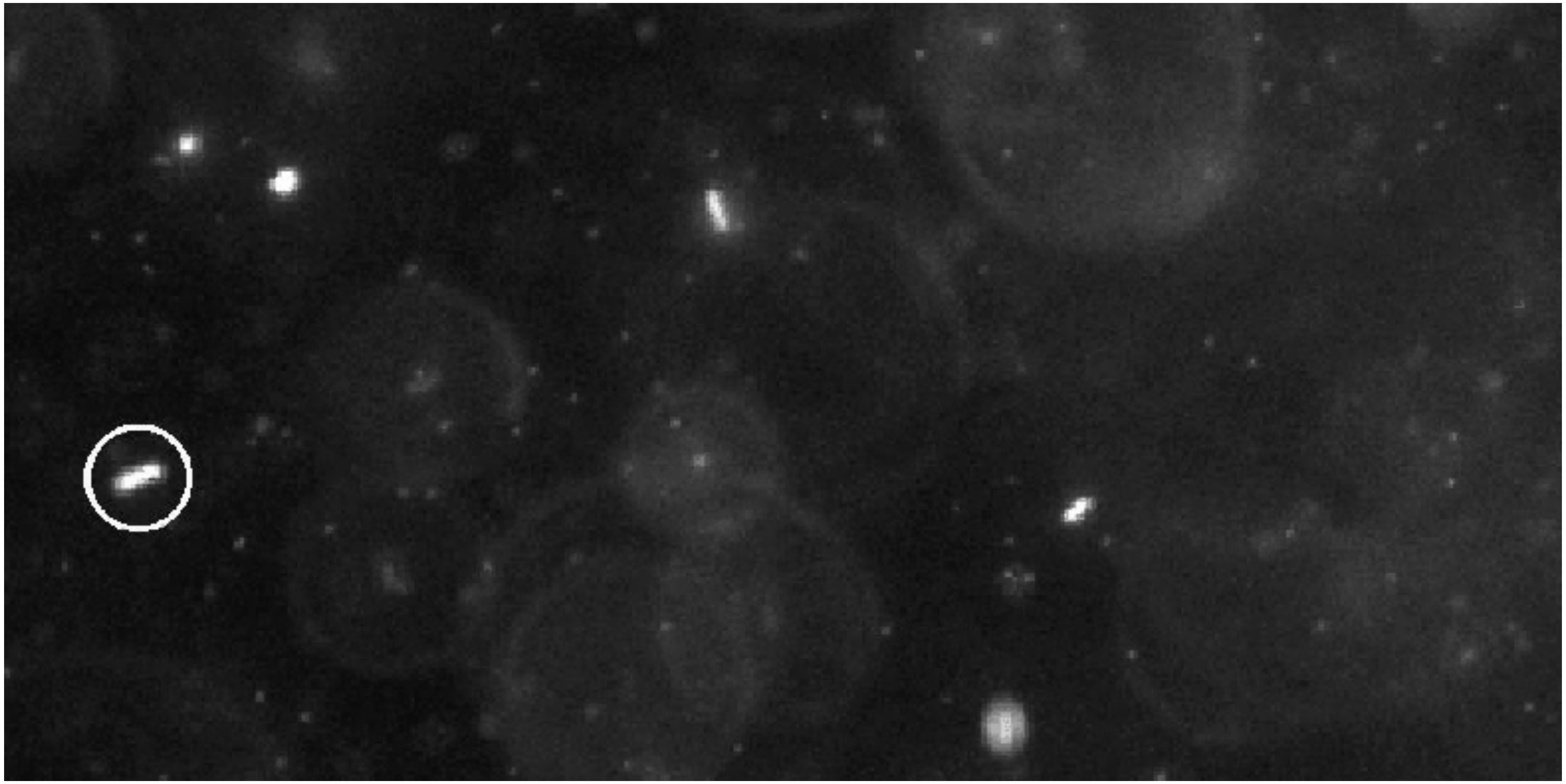
E.coli (non-tumbling HCB 437)



Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

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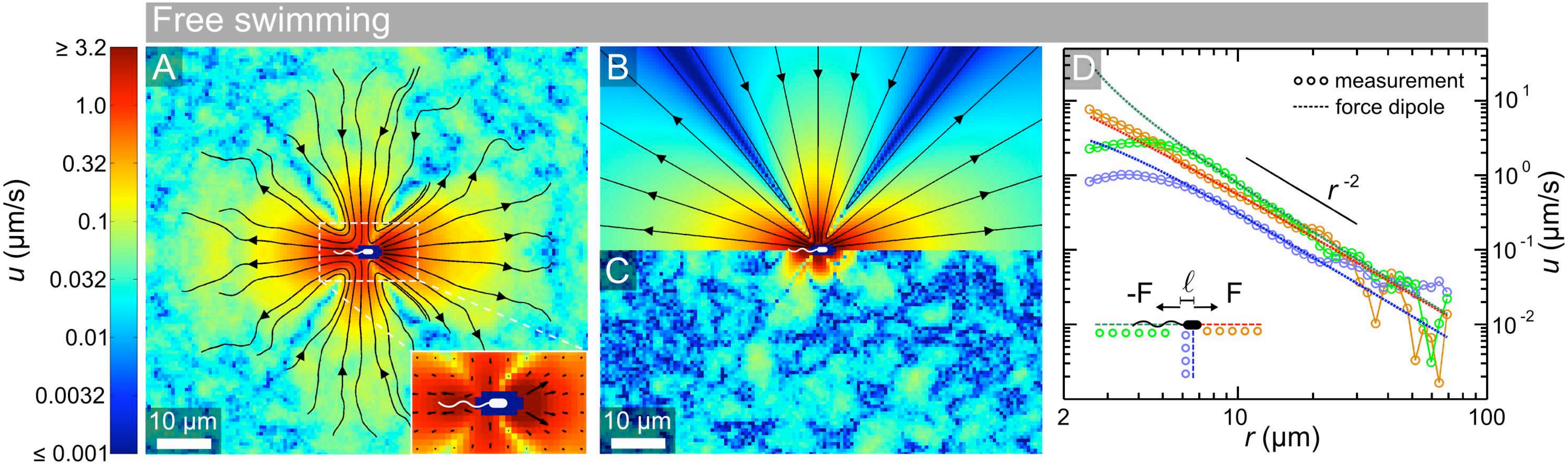
E.coli (non-tumbling HCB 437)



$$\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$V_0 = 22 \pm 5 \ \ \mu\text{m/s}$$

$$\ell = 1.9 \ \mu\text{m}$$

$$F = 0.42 \ \text{pN}$$

Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

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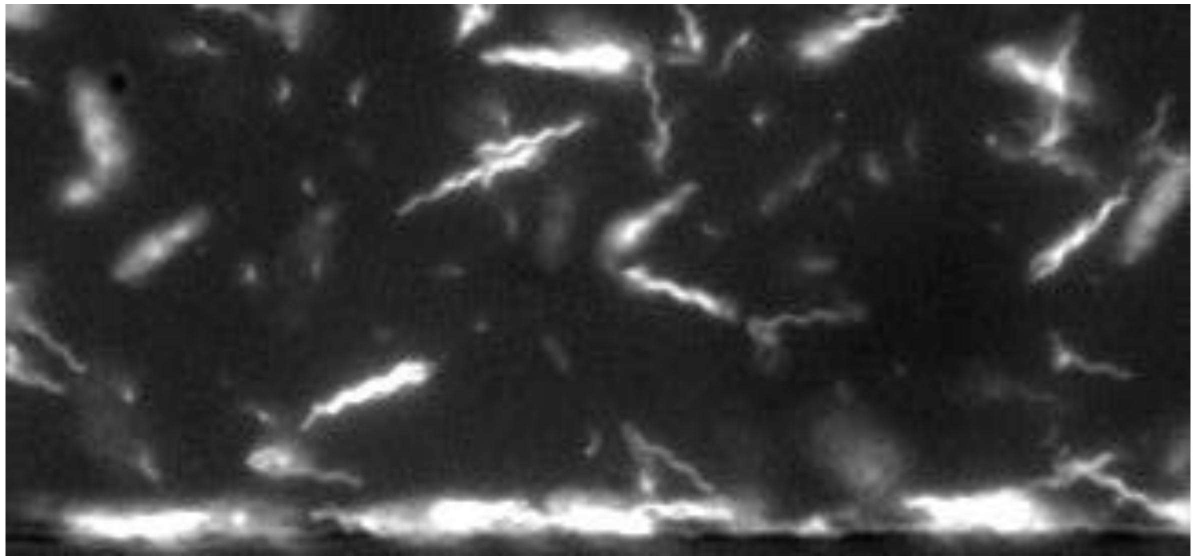
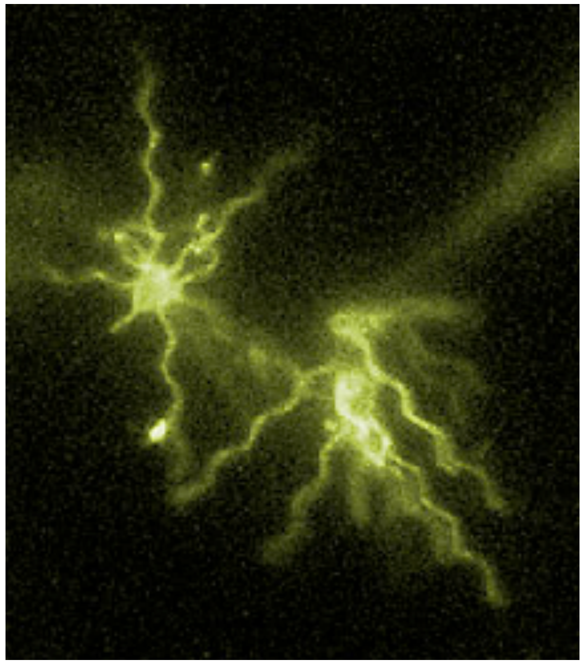
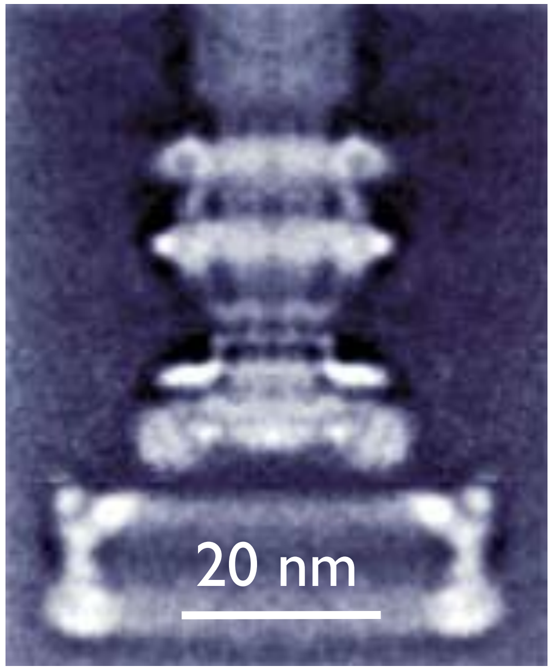
Bacterial motors

movie: V. Kantsler



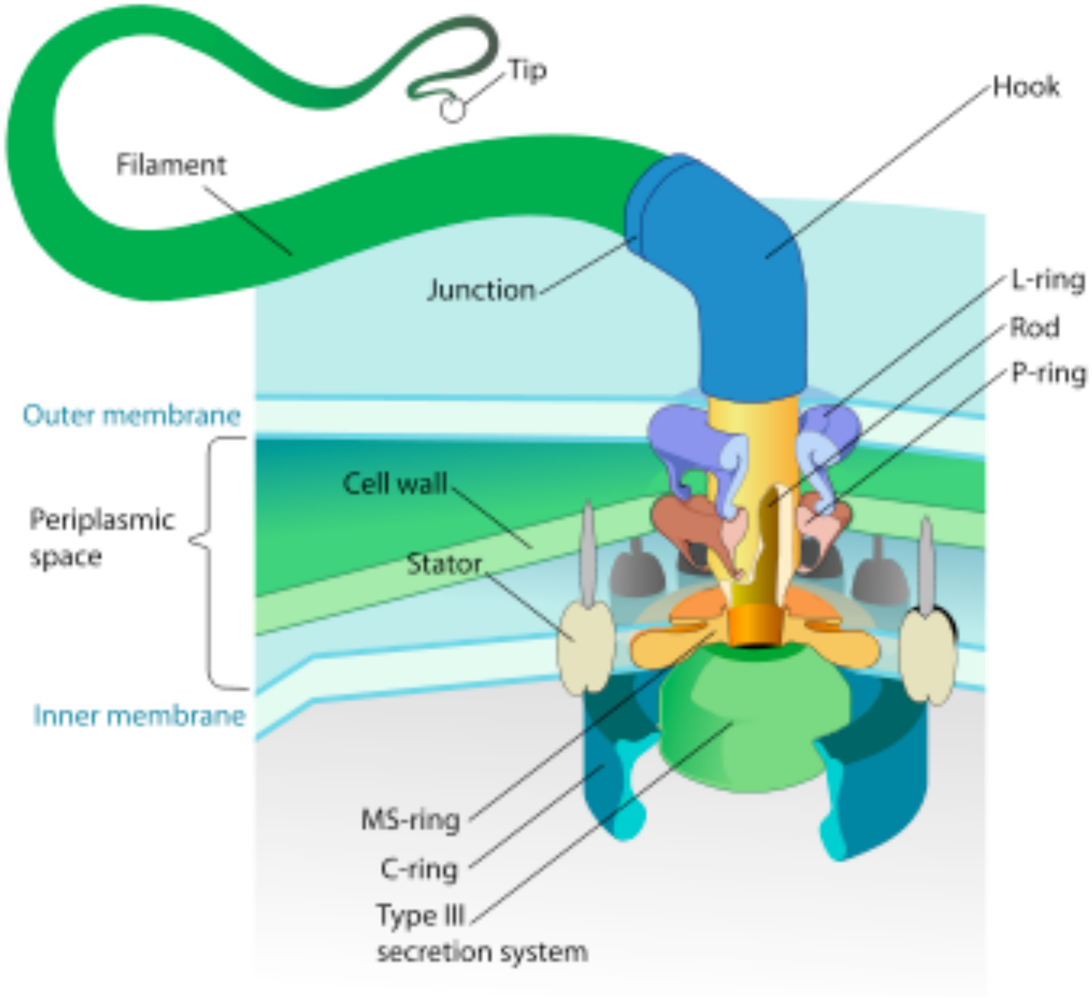


Berg (1999) Physics Today



Chen et al (2011) EMBO Journal

~20 parts



source: wiki

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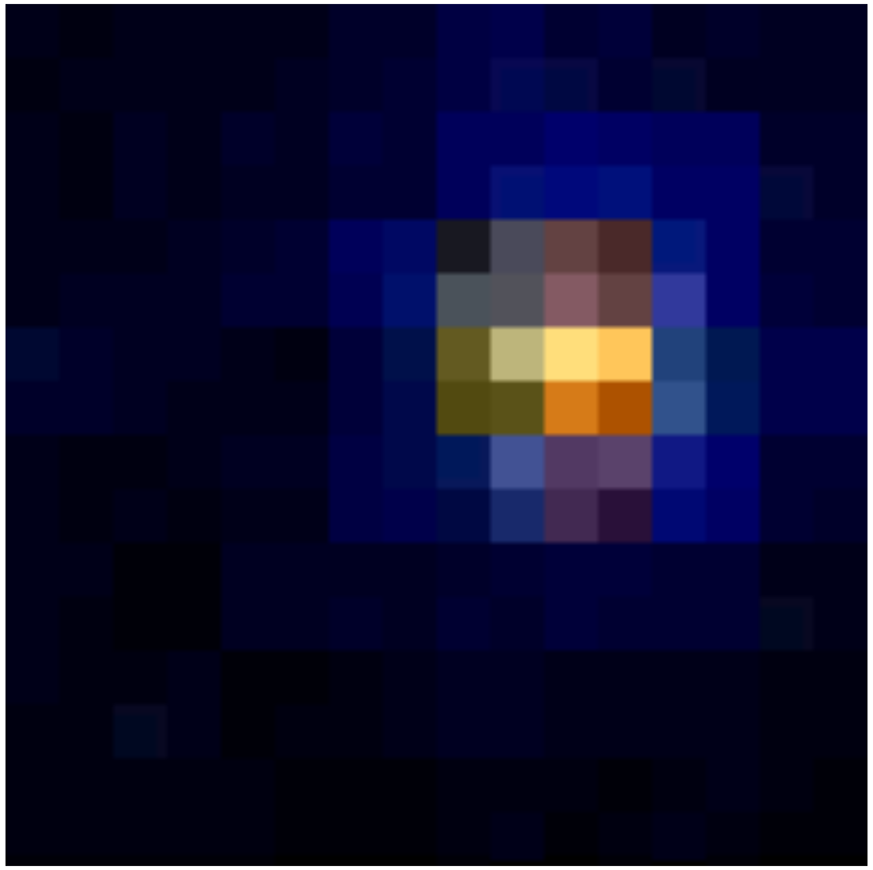
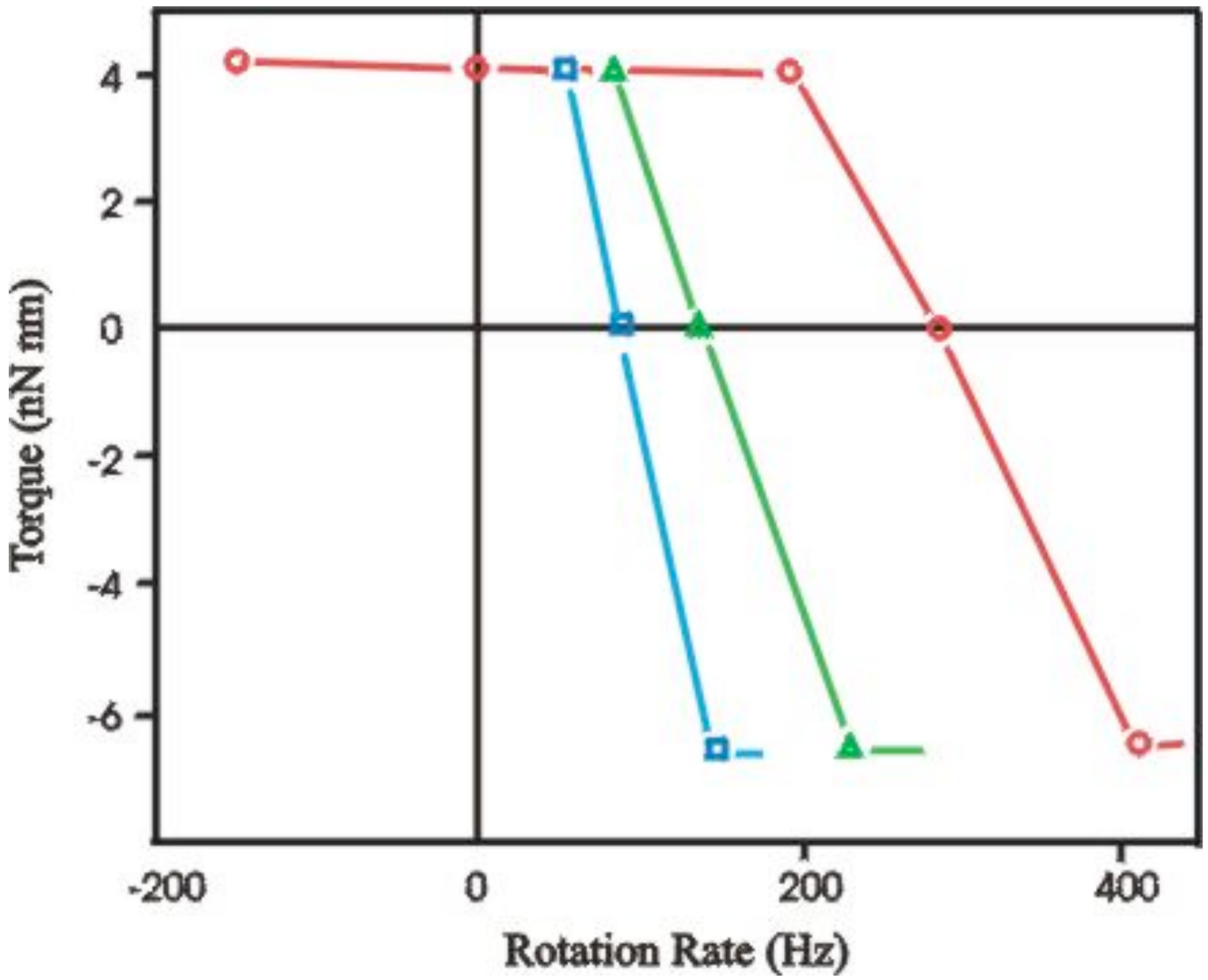
Torque-speed relation



200 nm fluorescent bead attached to a flagellar motor
26 steps per revolution
30x slower than real time
2400 frames per second
position resolution ~5 nm



Berry group, Oxford

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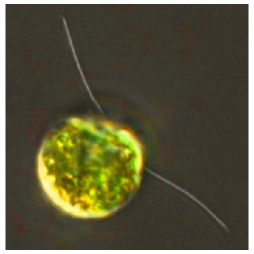
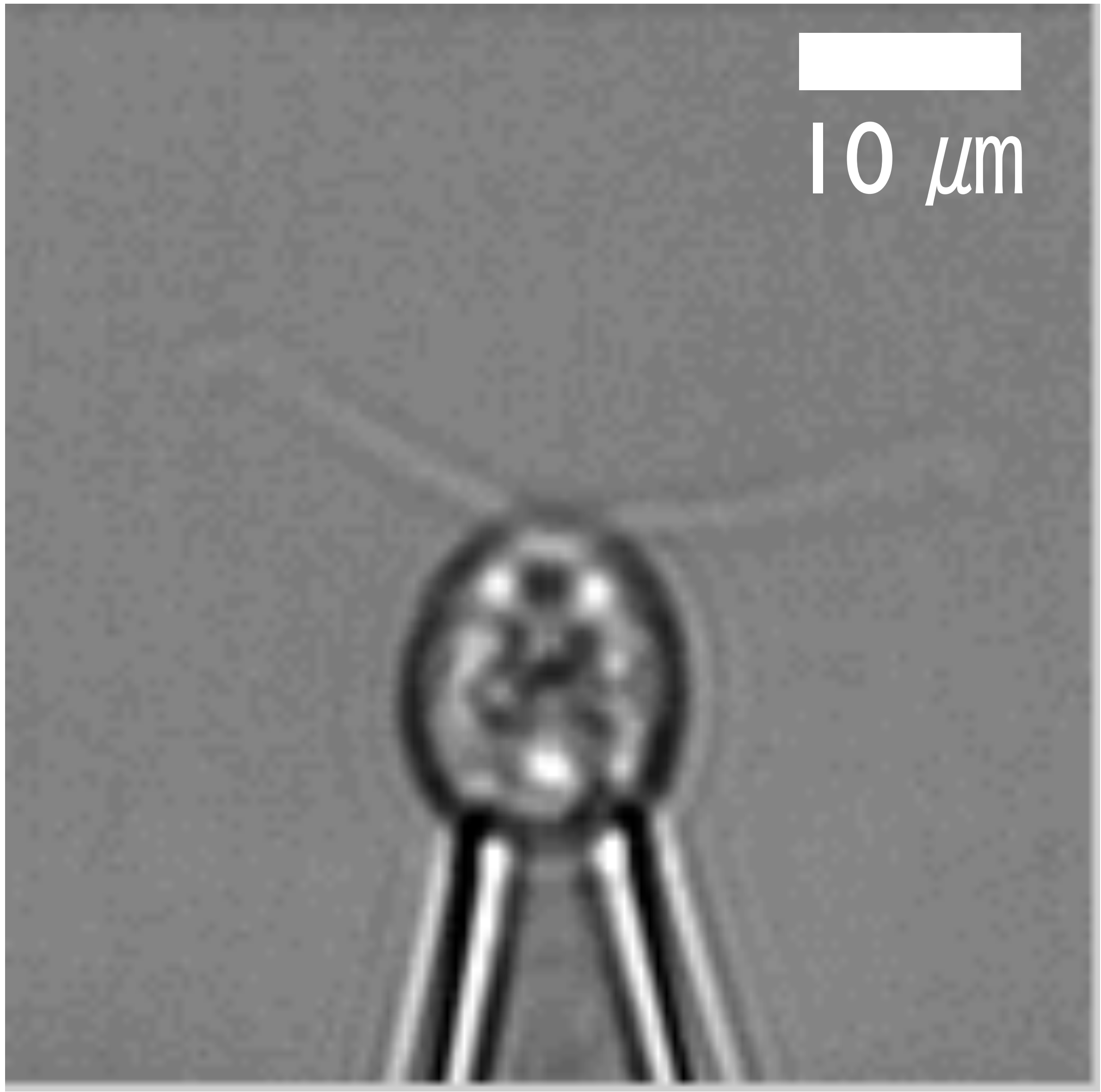
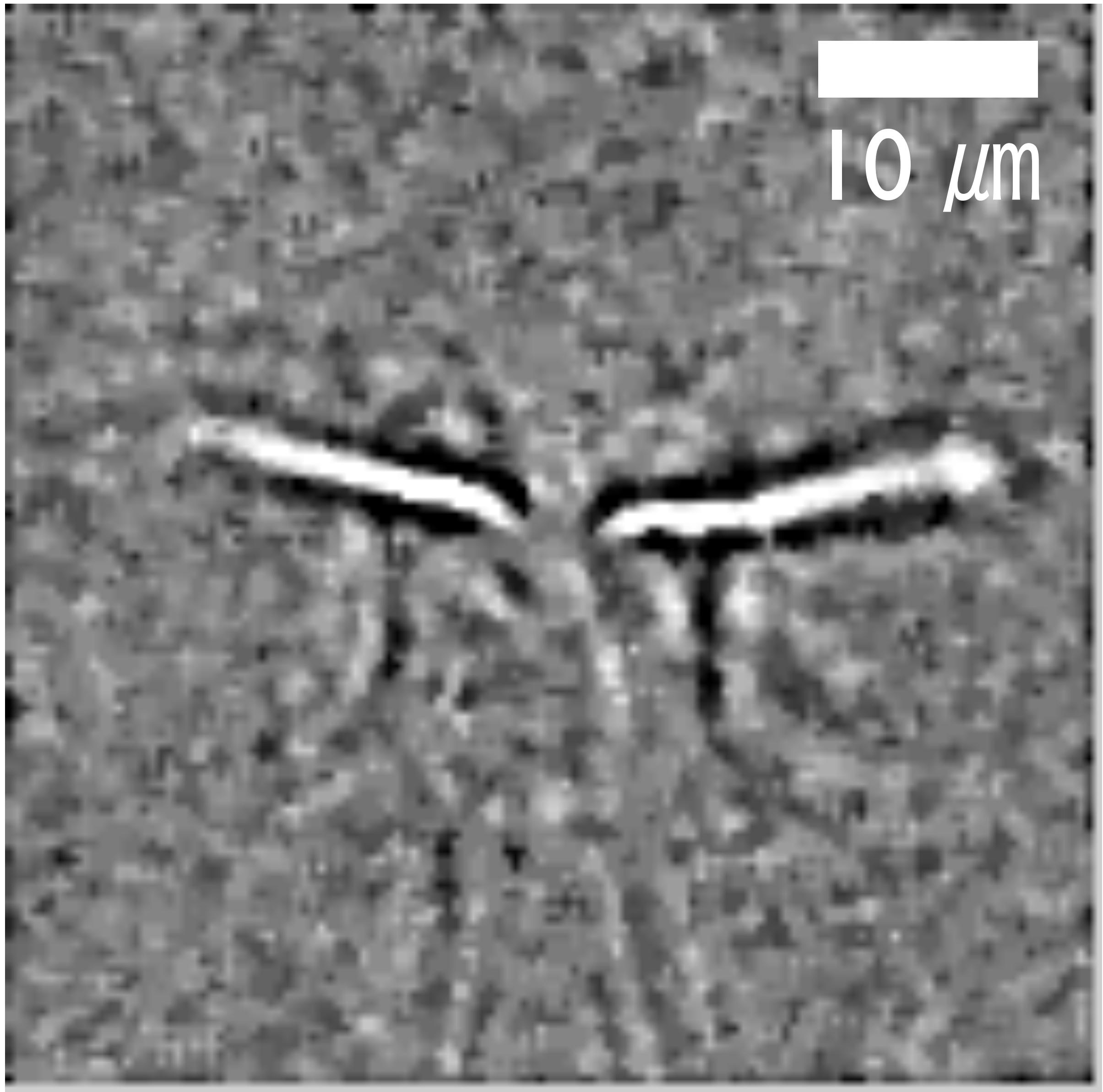
Chlamydomonas alga





~ 50 beats / sec



speed ~100 μm/s

Goldstein et al (2011) PRL

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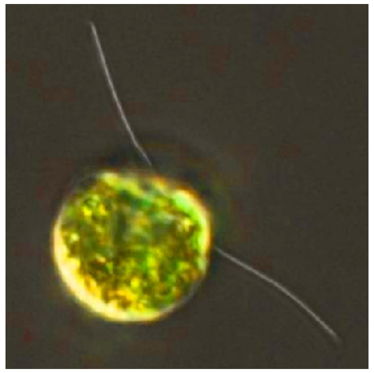
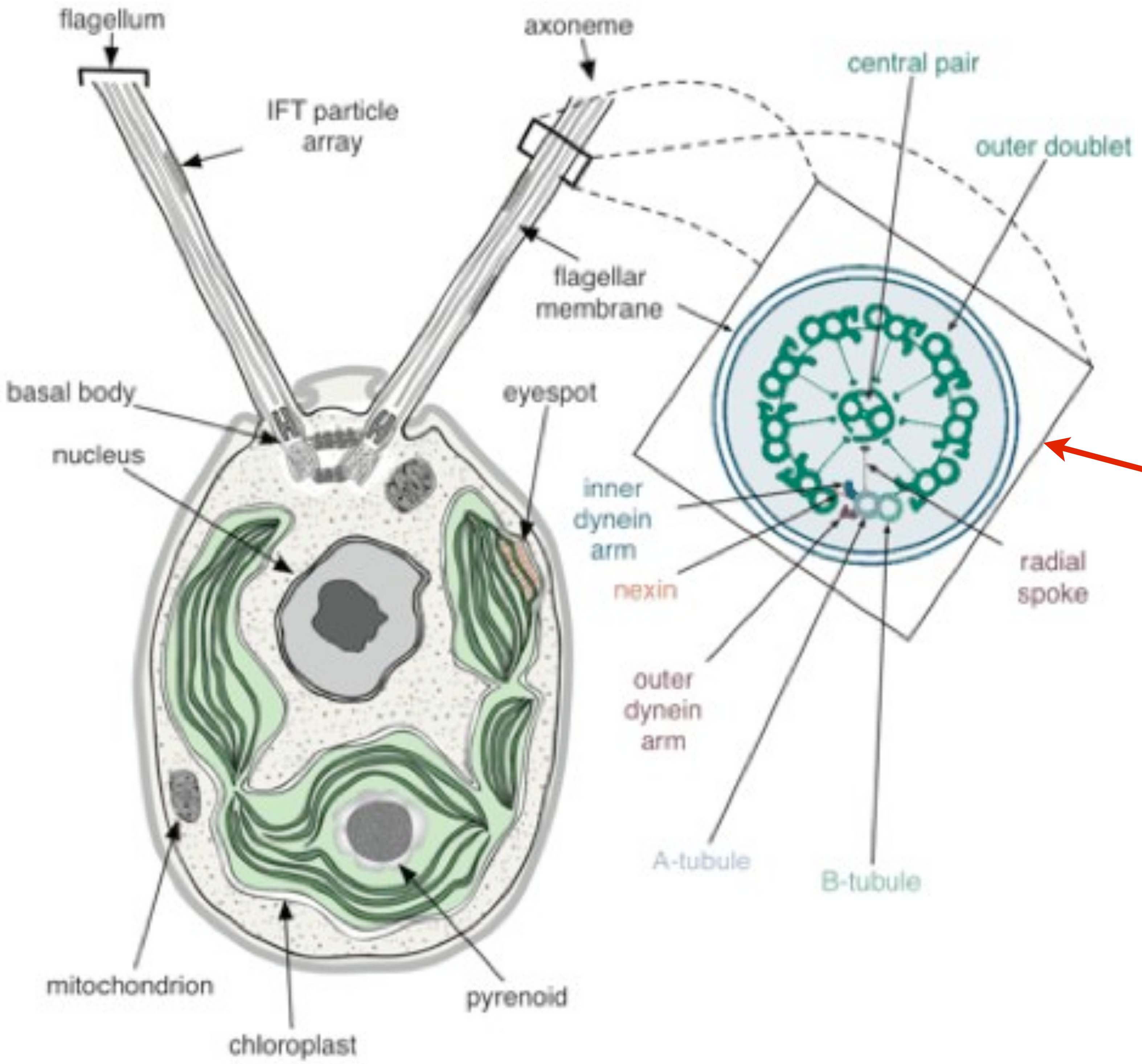
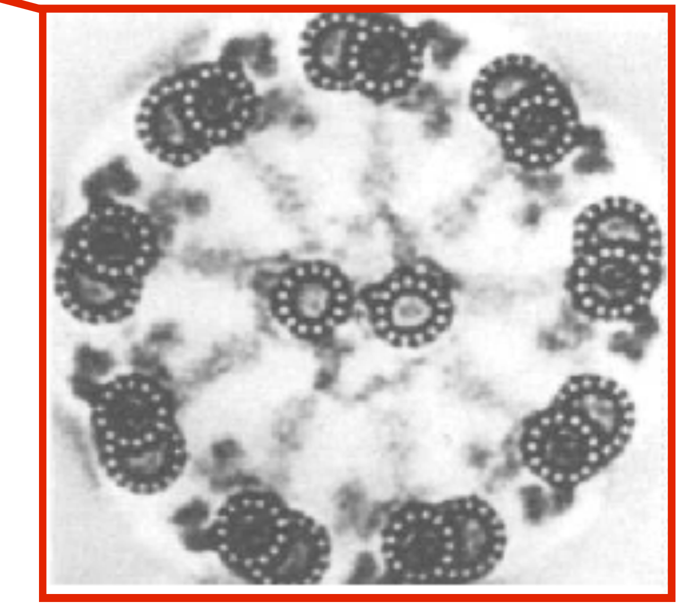
Chlamy

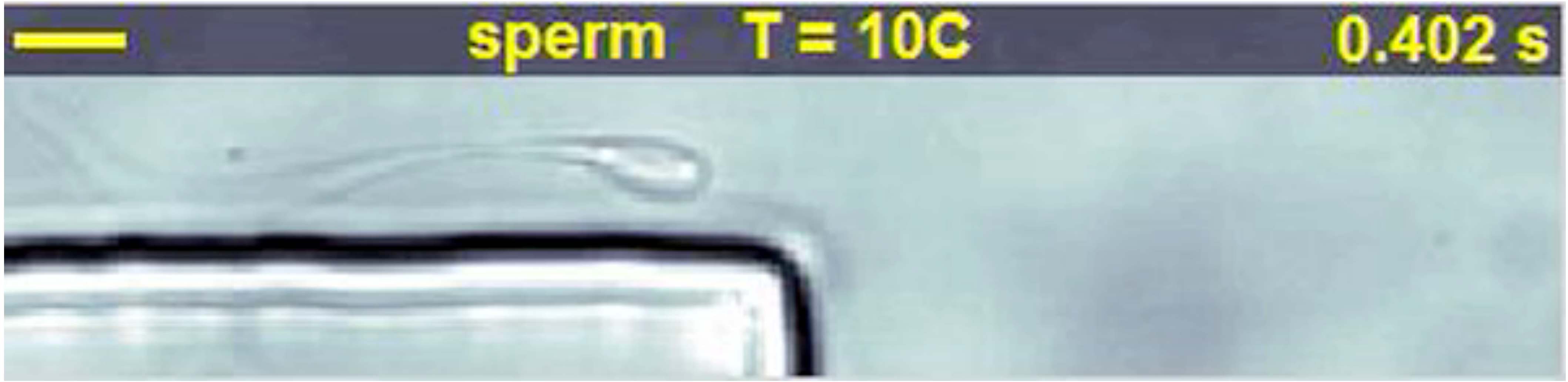


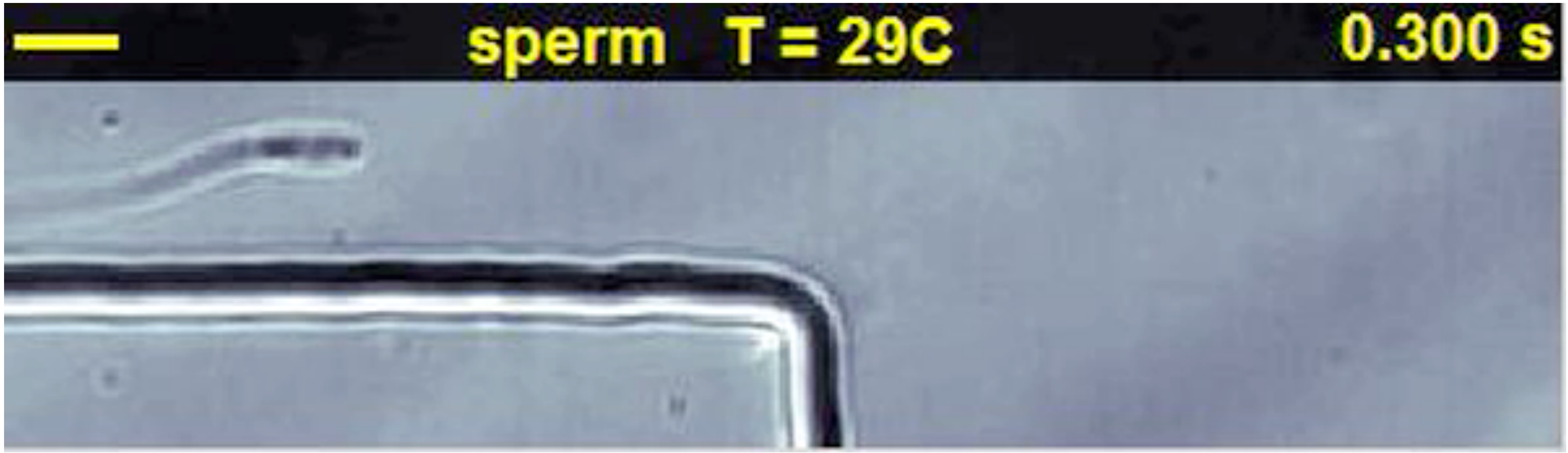


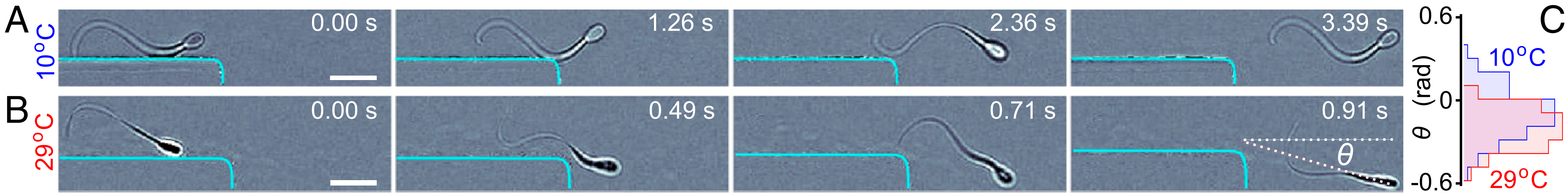


Merchant et al (2007) Science

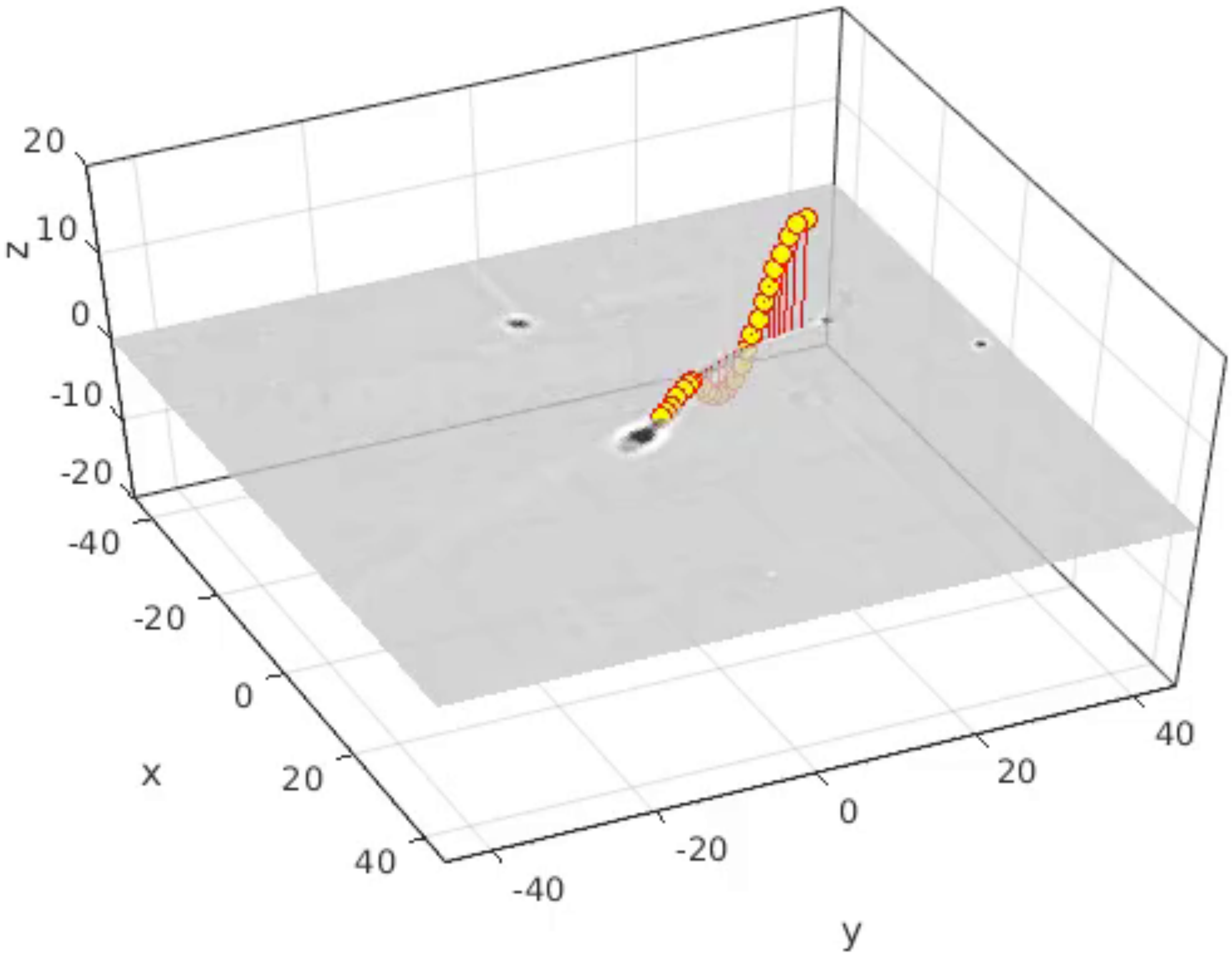
Sperm near surfaces







Kantsler, Dunkel, Polin, Goldstein (2012) PNAS



PNAS 2015

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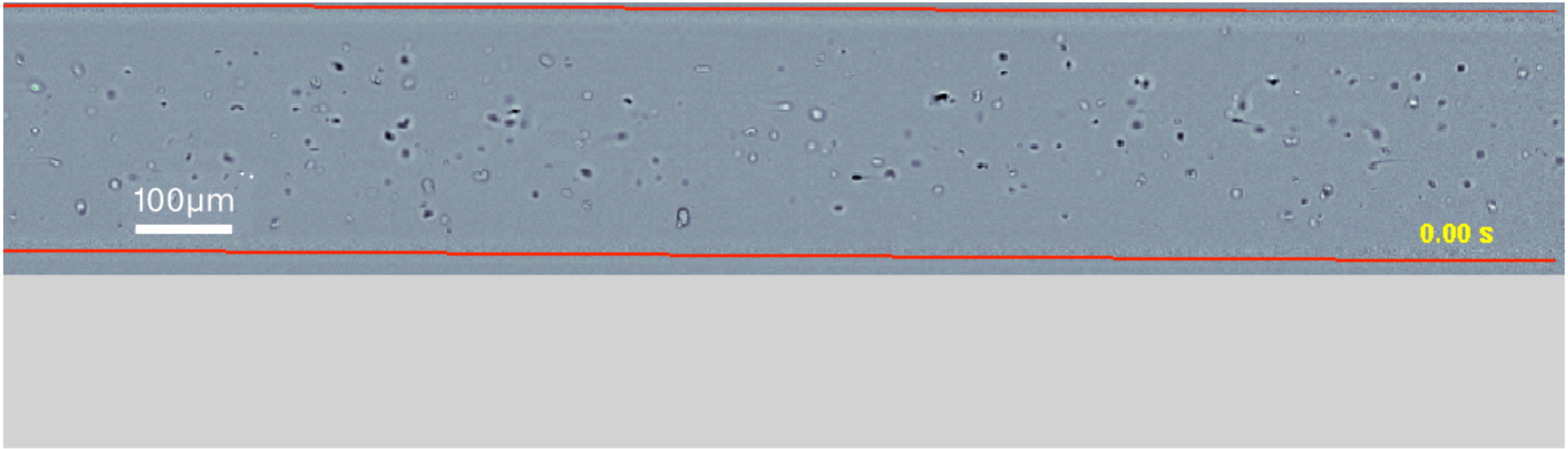
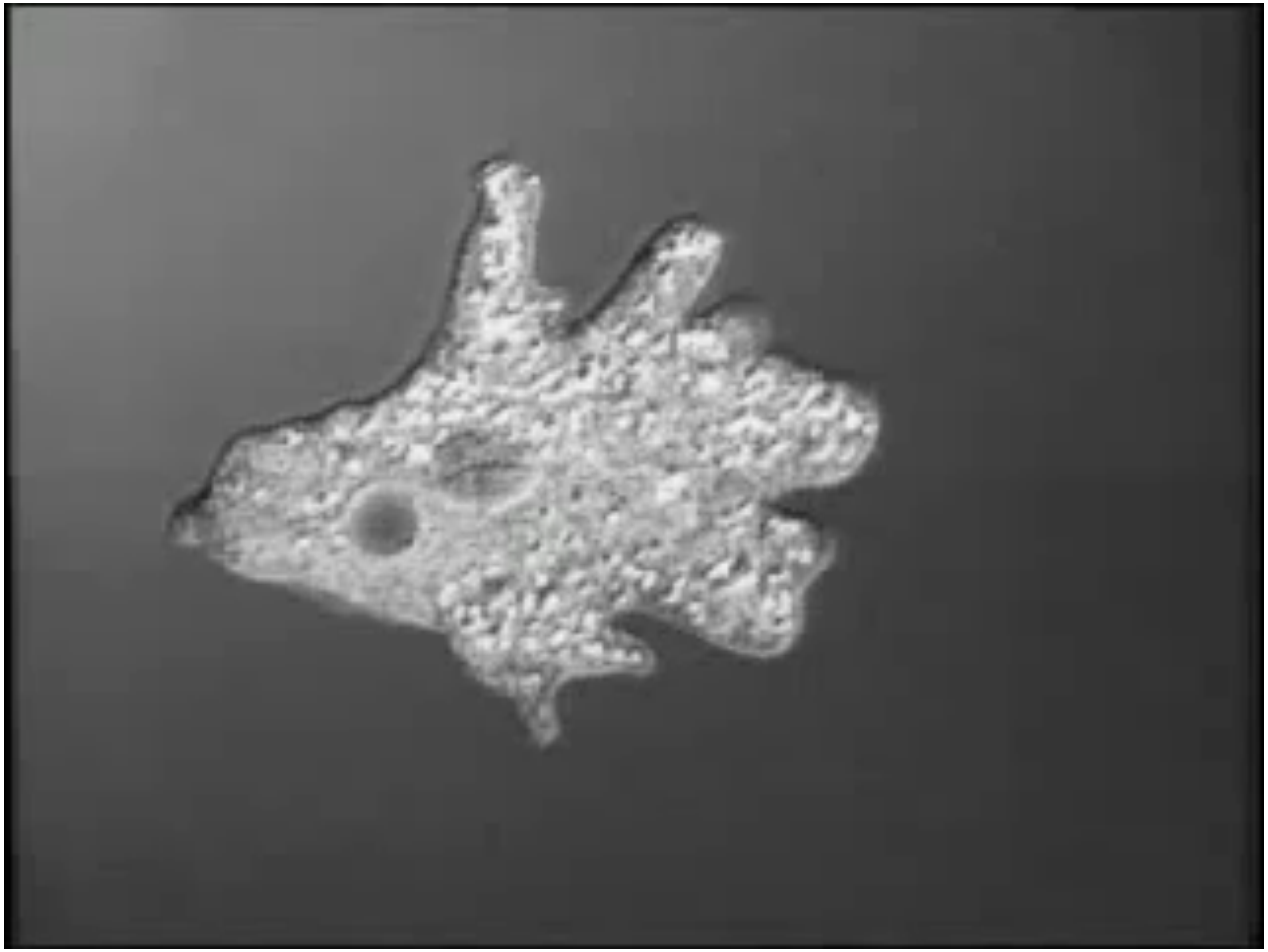
Surface + shear flow

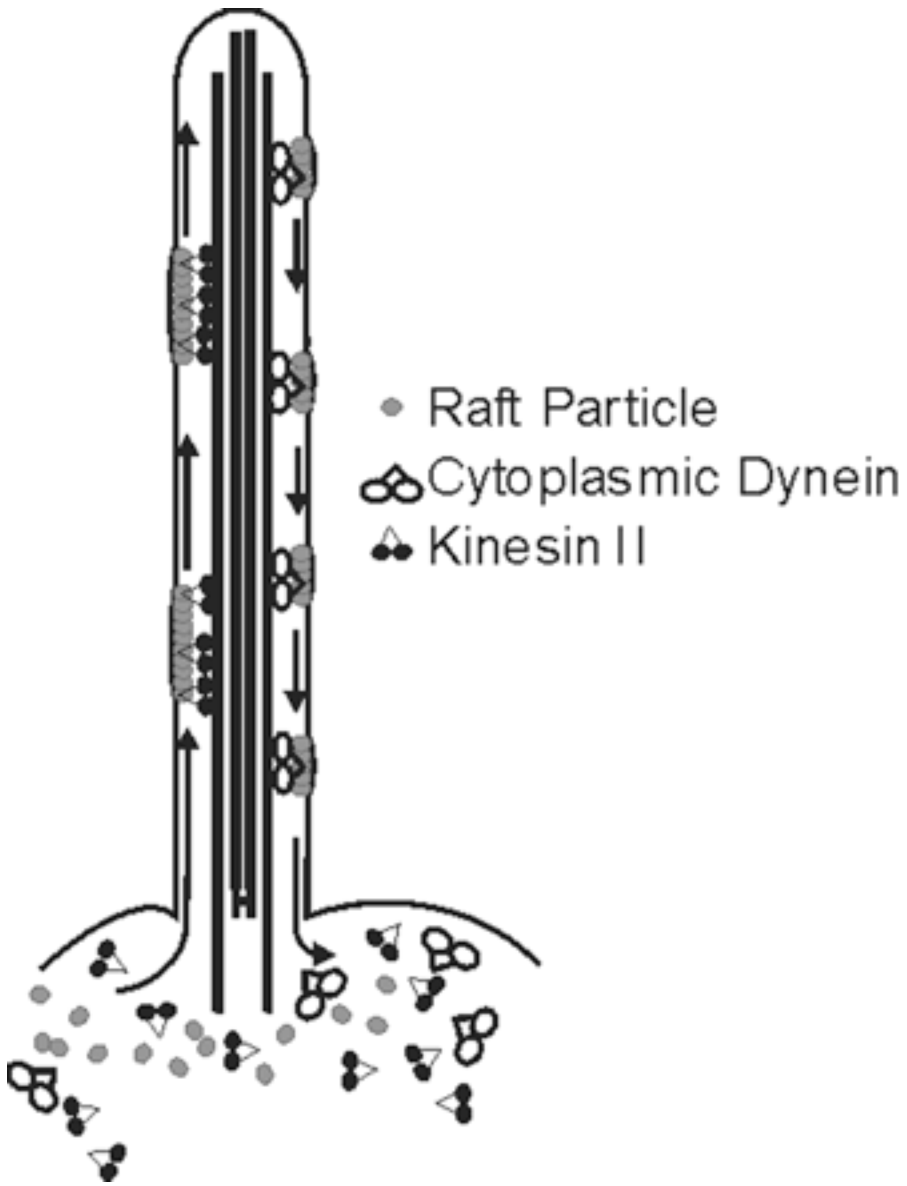


Kantsler et al 2014 (elife)

Amoeba

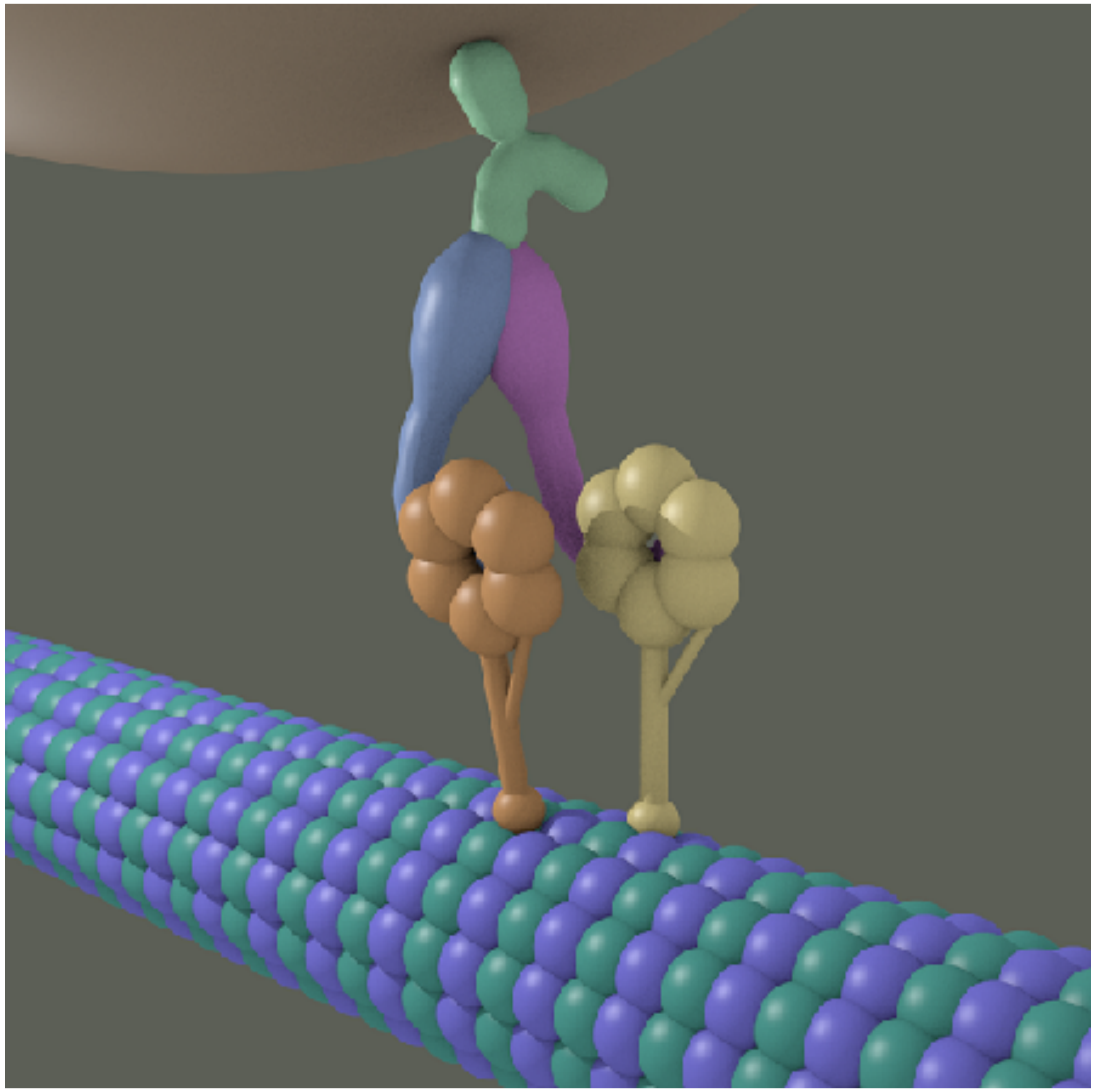


Eukaryotic motors



http://www.plantphysiol.org/content/127/4/1500/F4.expansion.html

Sketch: dynein molecule carrying cargo down a microtubule



Yildiz lab, Berkeley

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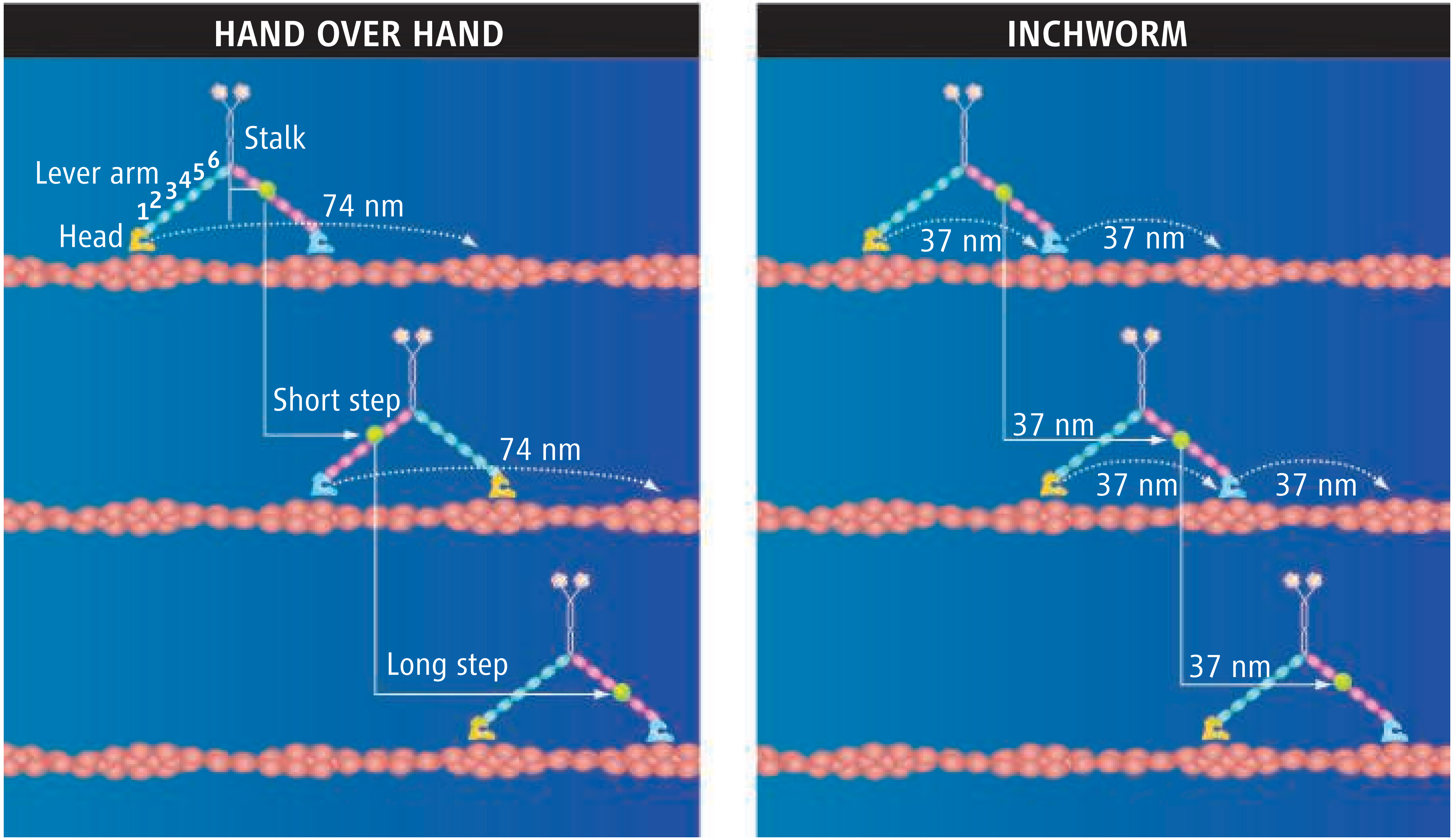
Walking modes



Myosin V: Walking or inchworming? Predicted movement for the heads and a dye molecule label (green dot) on the lever arm in the hand-over-hand model **(left)** and the inchworm model (**right**). The FIONA assay has revealed that myosin V, along with kinesin and myosin VI, walks hand-over-hand.

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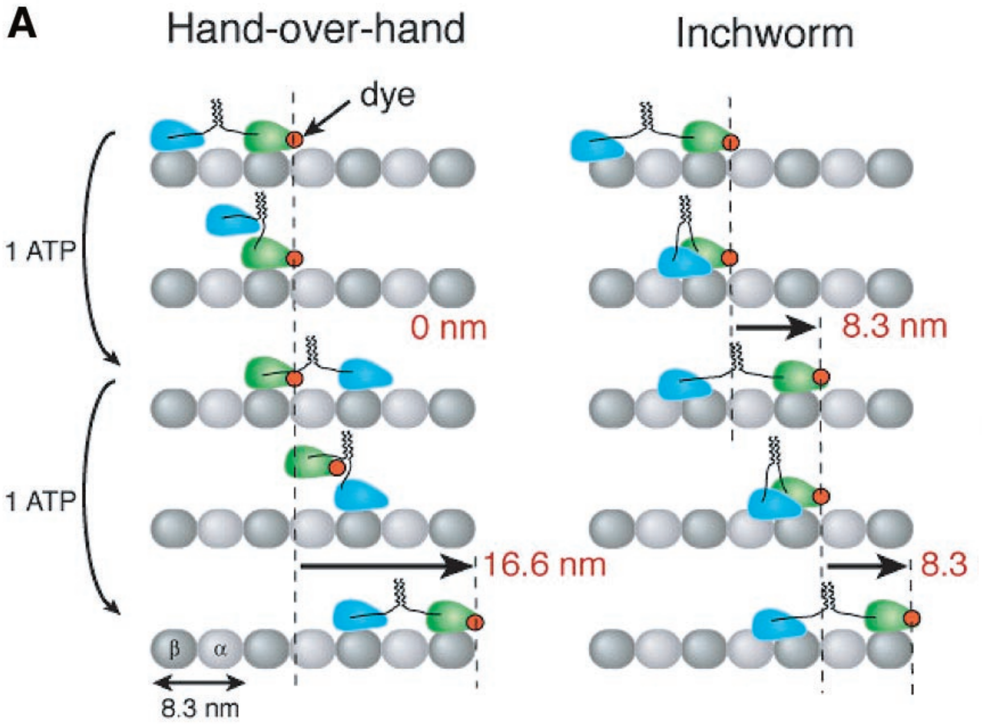
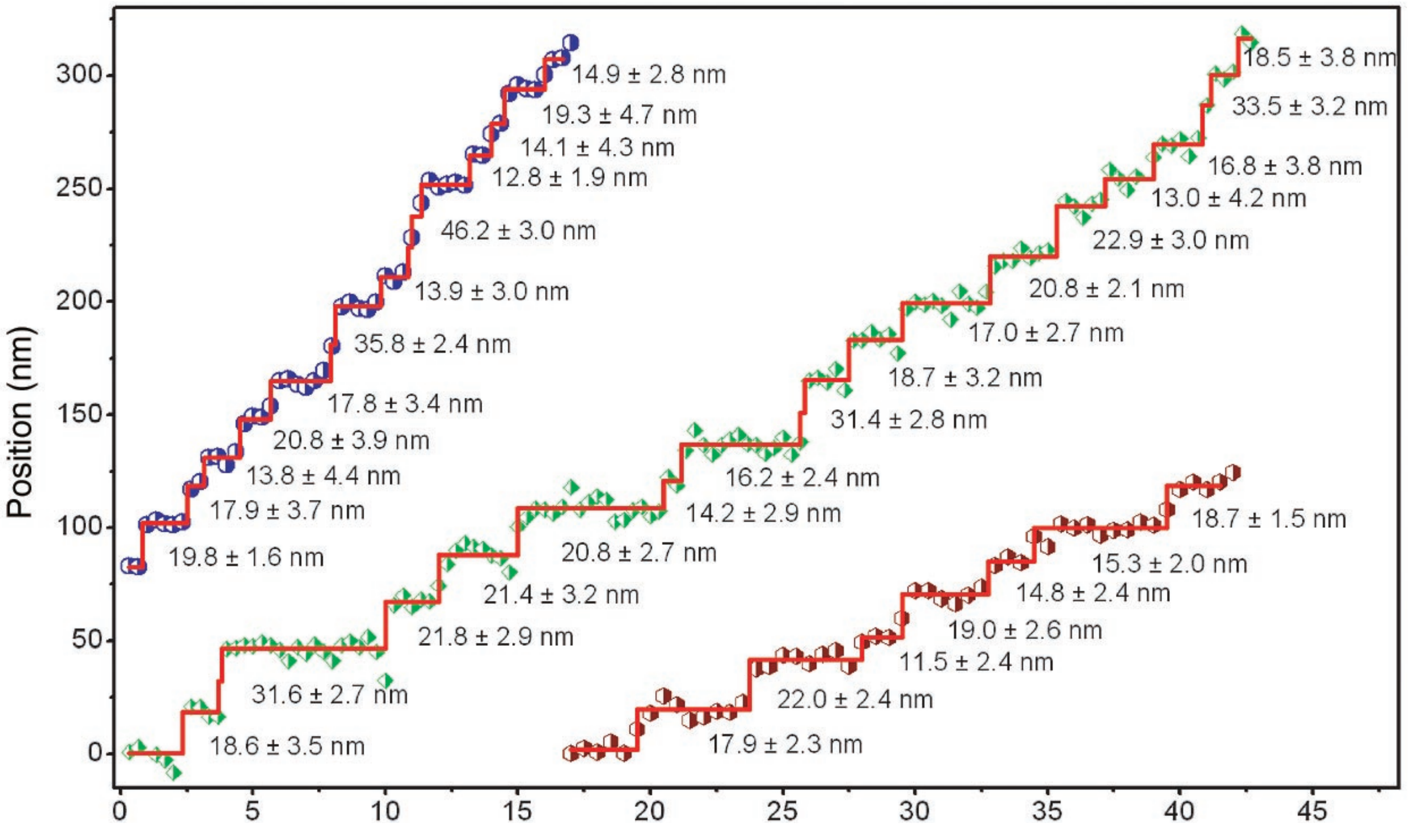
Kinesin walks hand-over-hand





Yildiz et al (2005) Science

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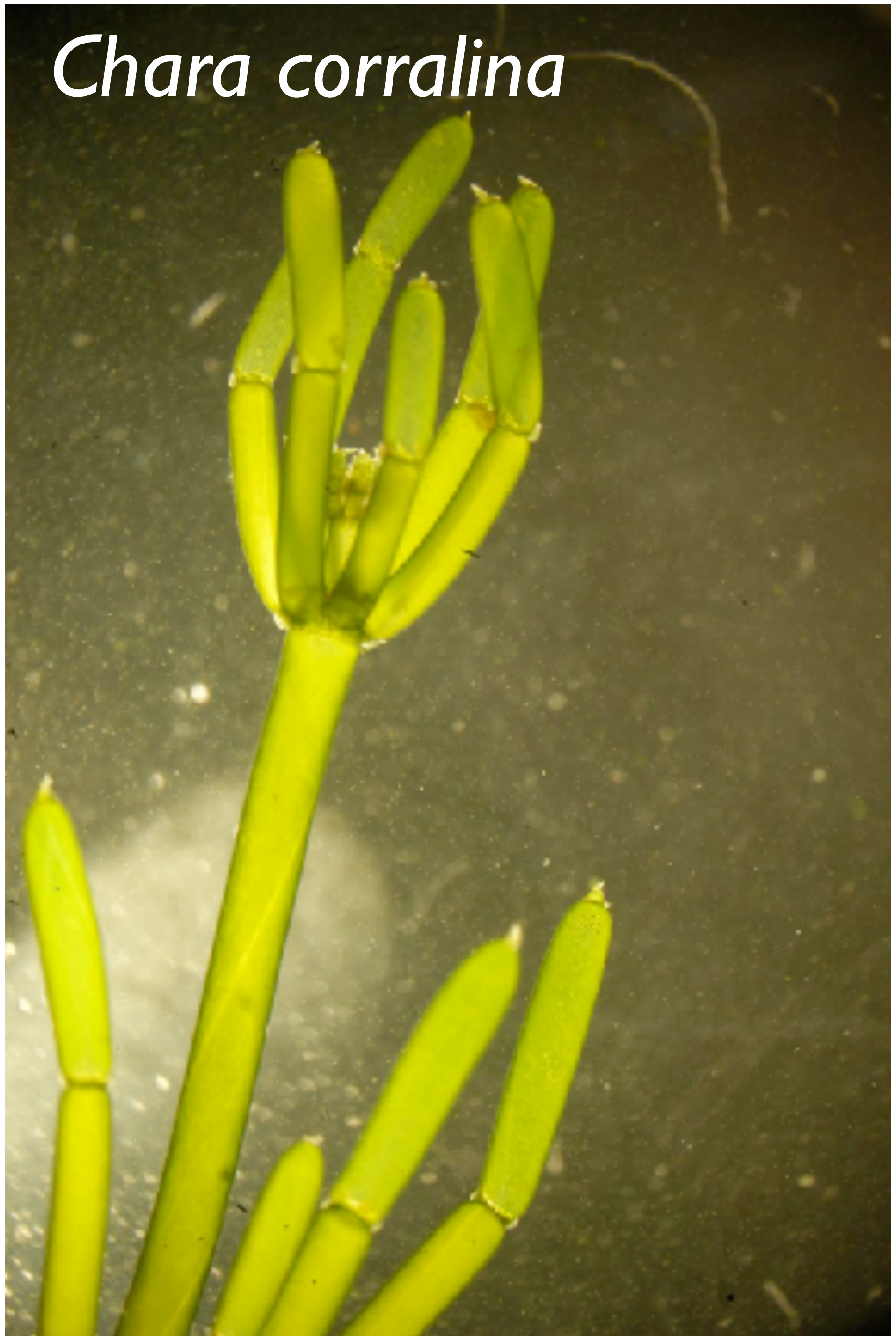
Intracellular transport



http://damtp.cam.ac.uk/user/gold/movies.html

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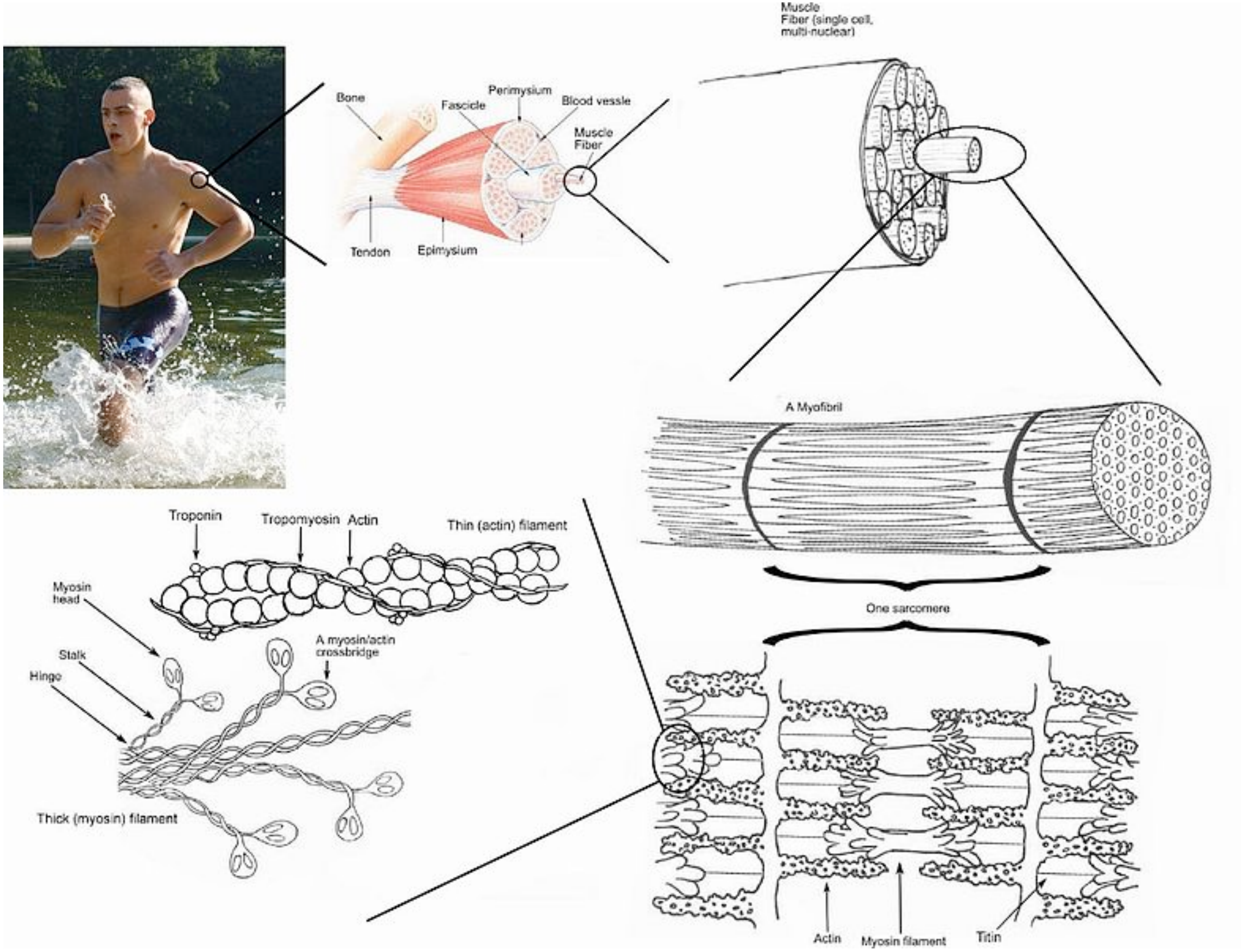

wiki

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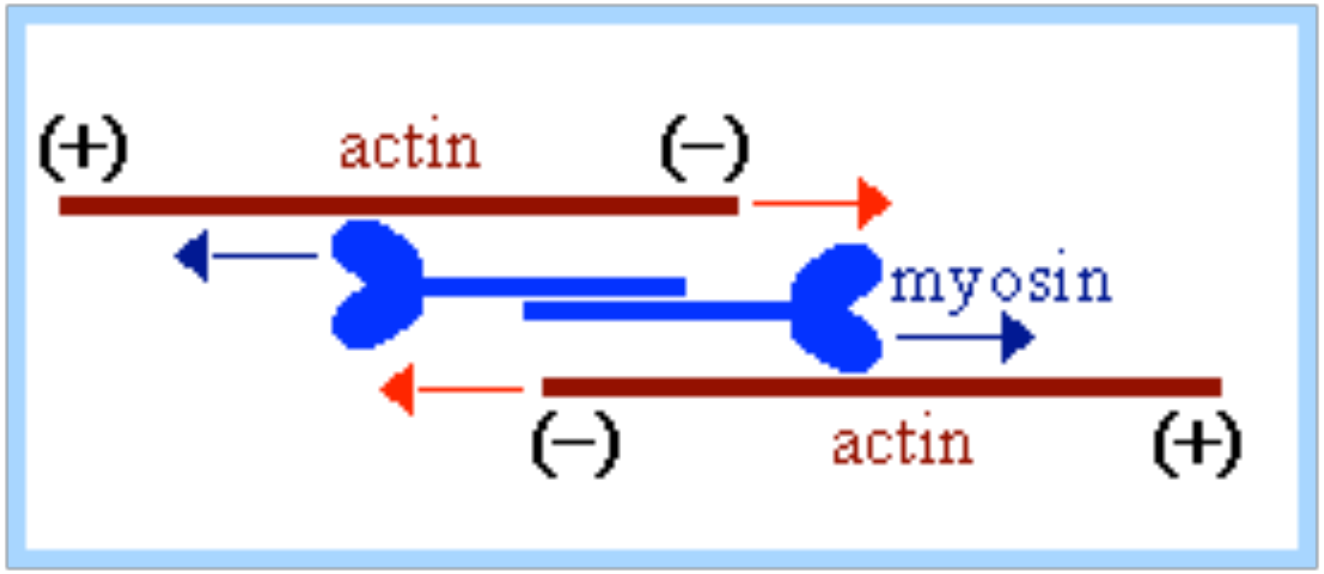
Actin-Myosin

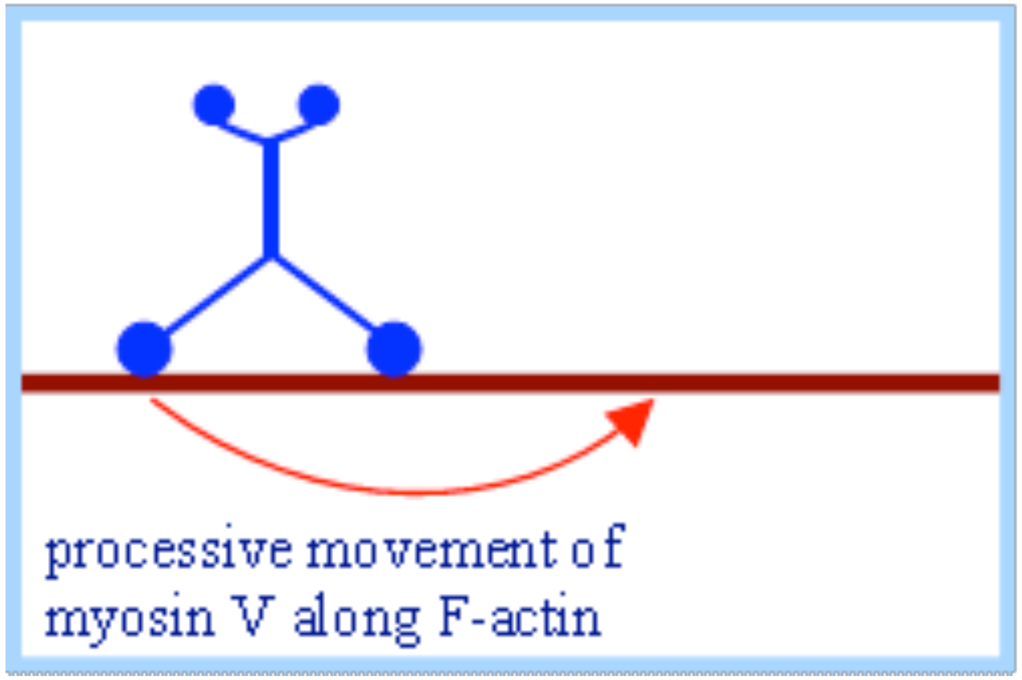
Myosin

F-Actin

helical filament



myosin-II



myosin-V

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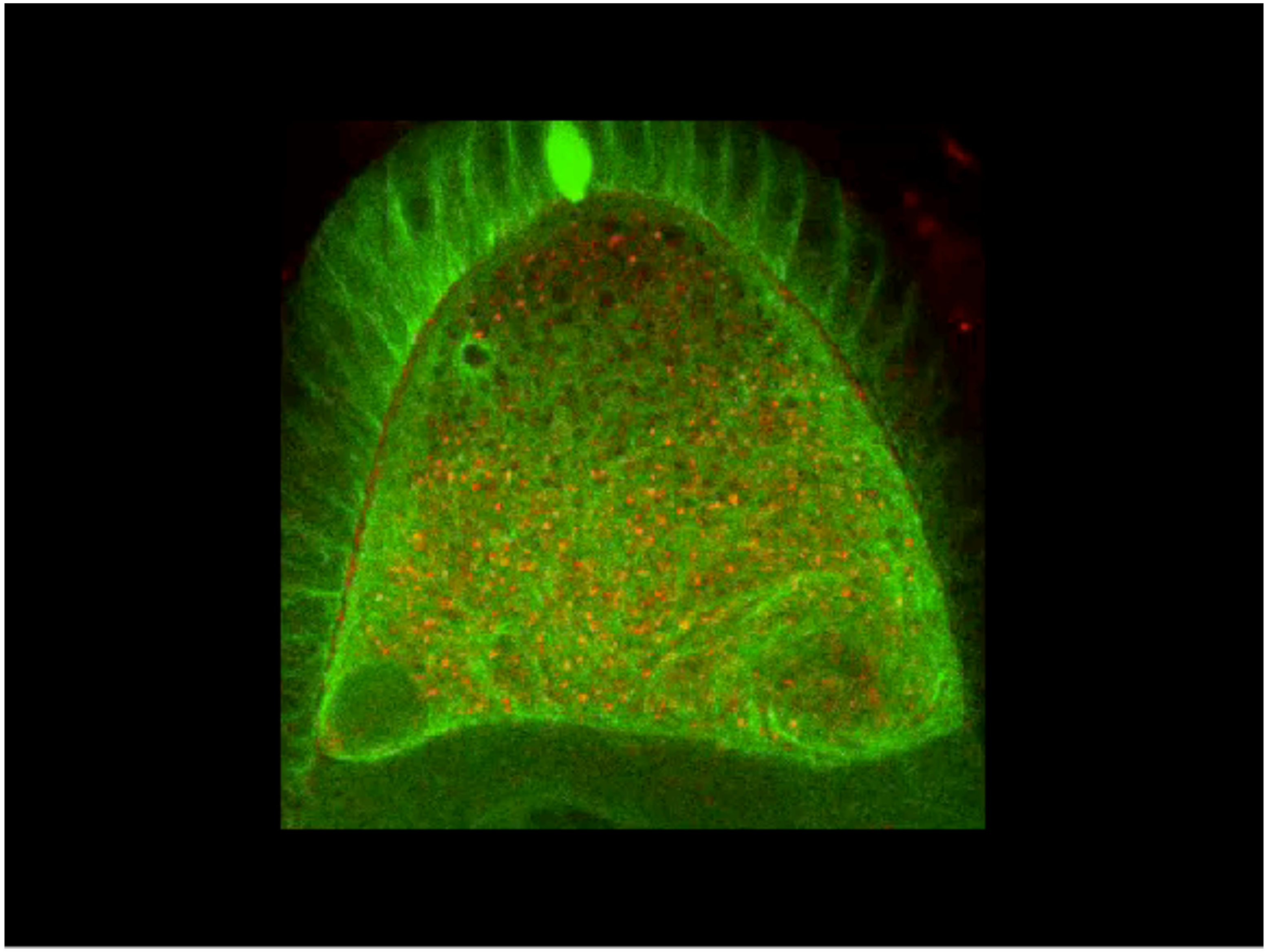
our lecture course:

generic models of micro-motors

Polymers & filaments (D=1)



Drosophila oocyte

Dogic Lab, Brandeis

Physical parameters (e.g. bending rigidity) from fluctuation analysis

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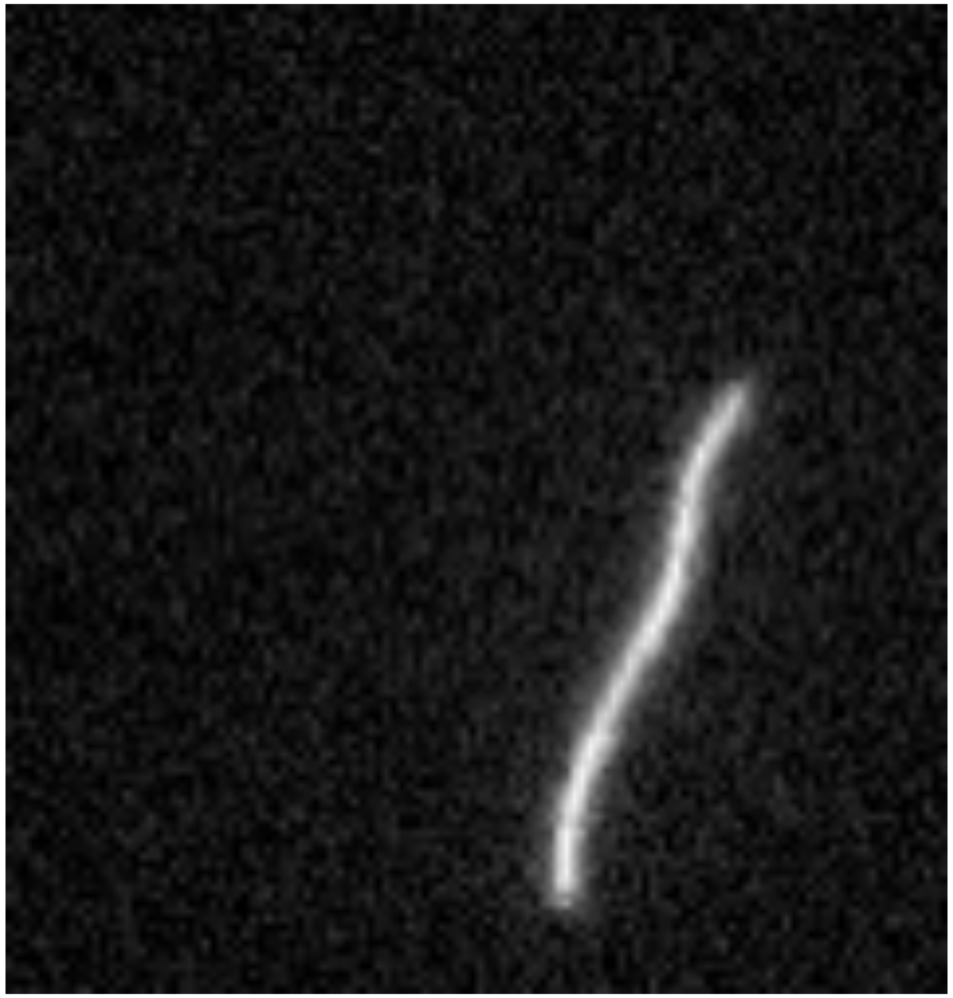
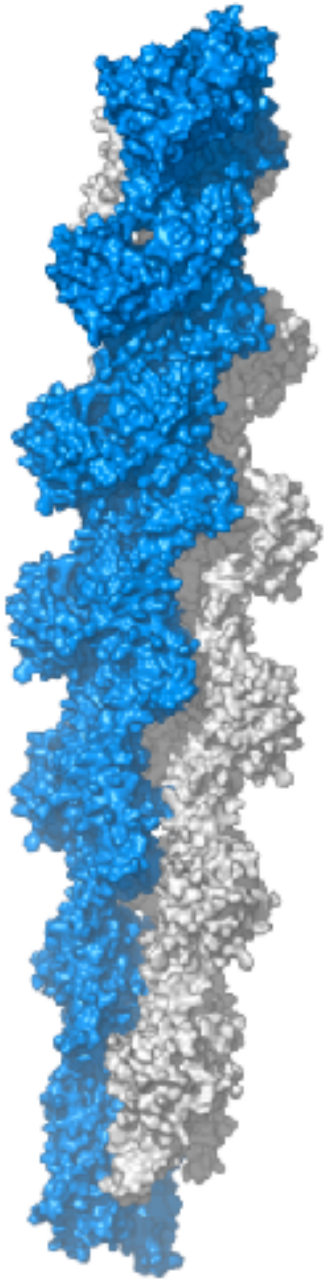
Actin in 2D



Dogic Lab (Brandeis)



F-Actin

helical filament

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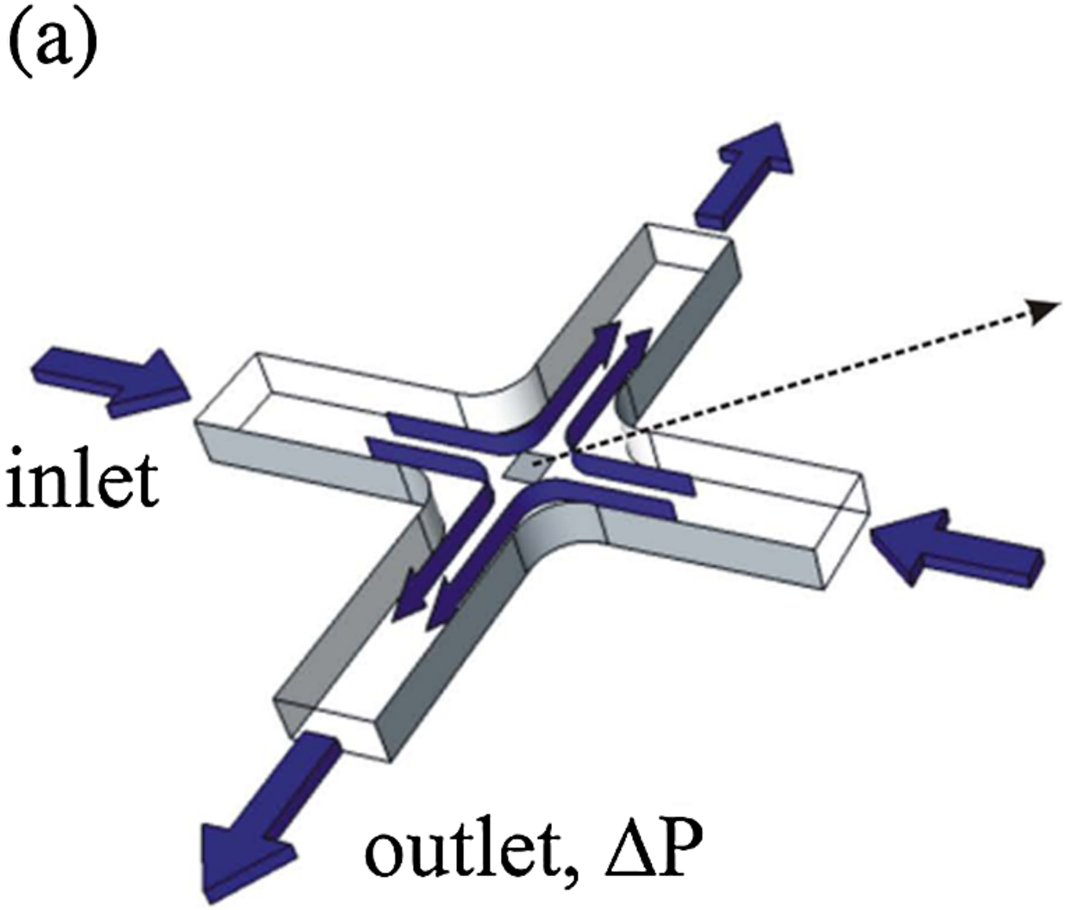
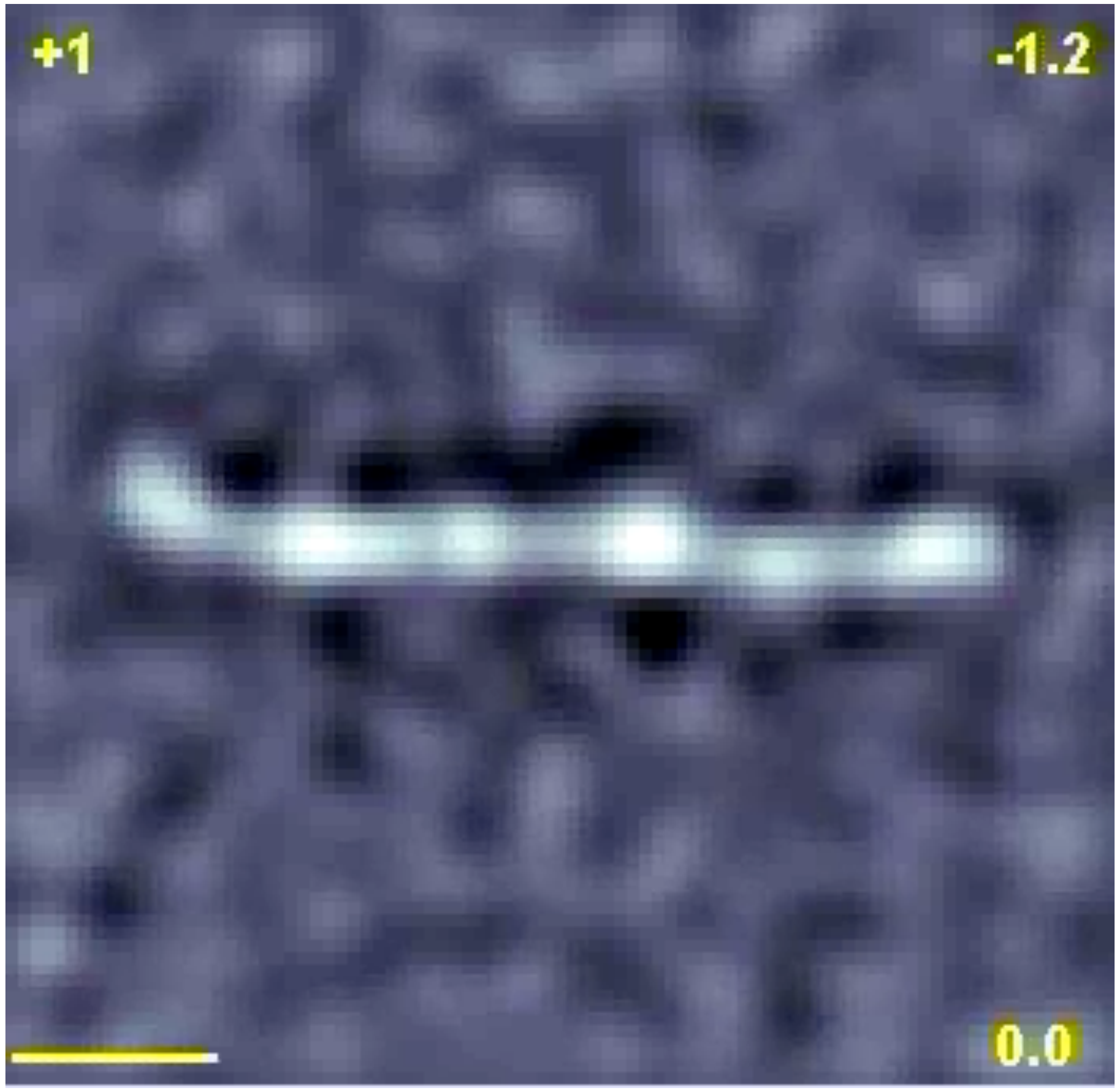
Actin in flow





Kantsler & Goldstein (2012) PRL

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our lecture course:

- polymer models
- how to relate fluctuations to mechanical properties

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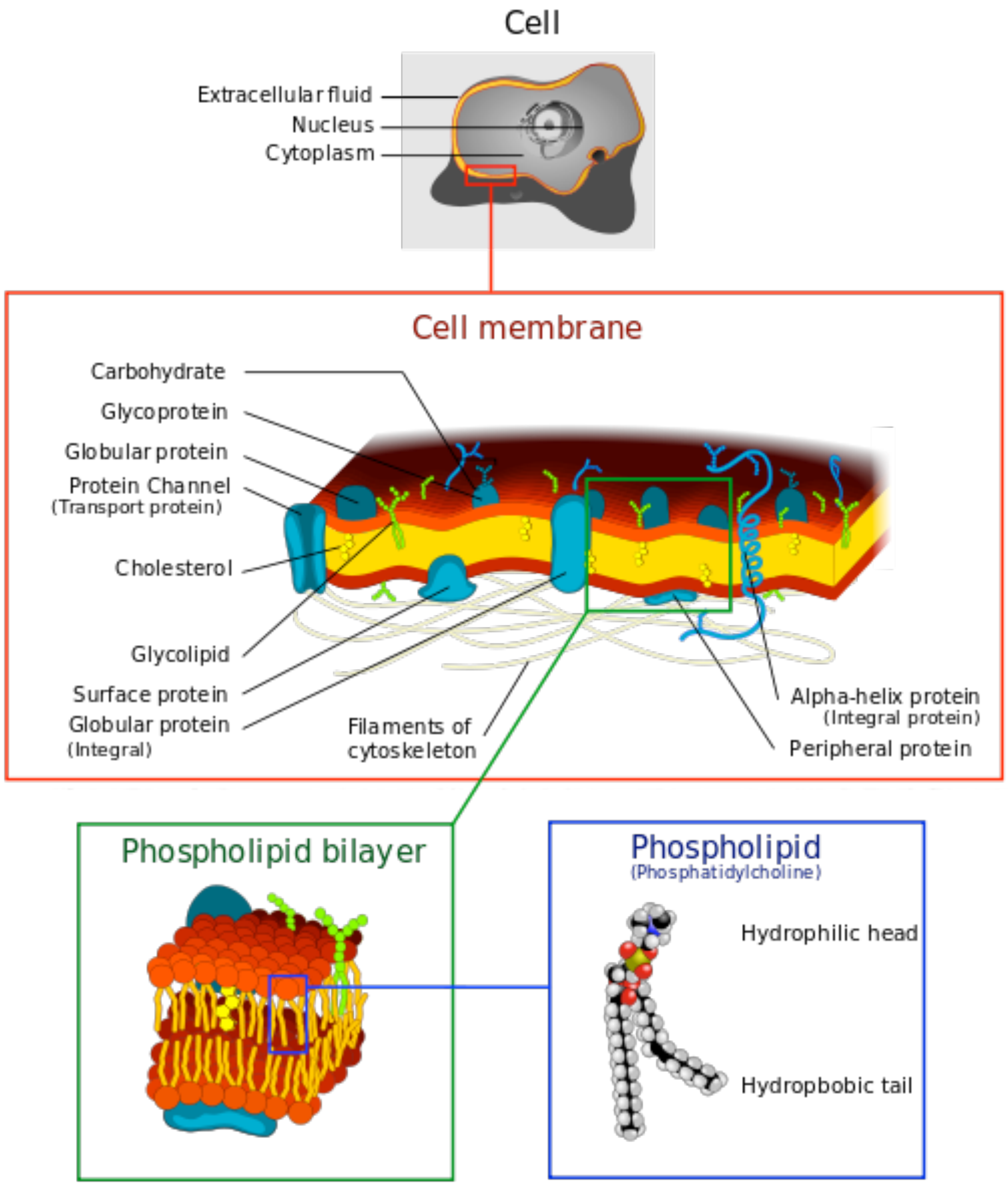
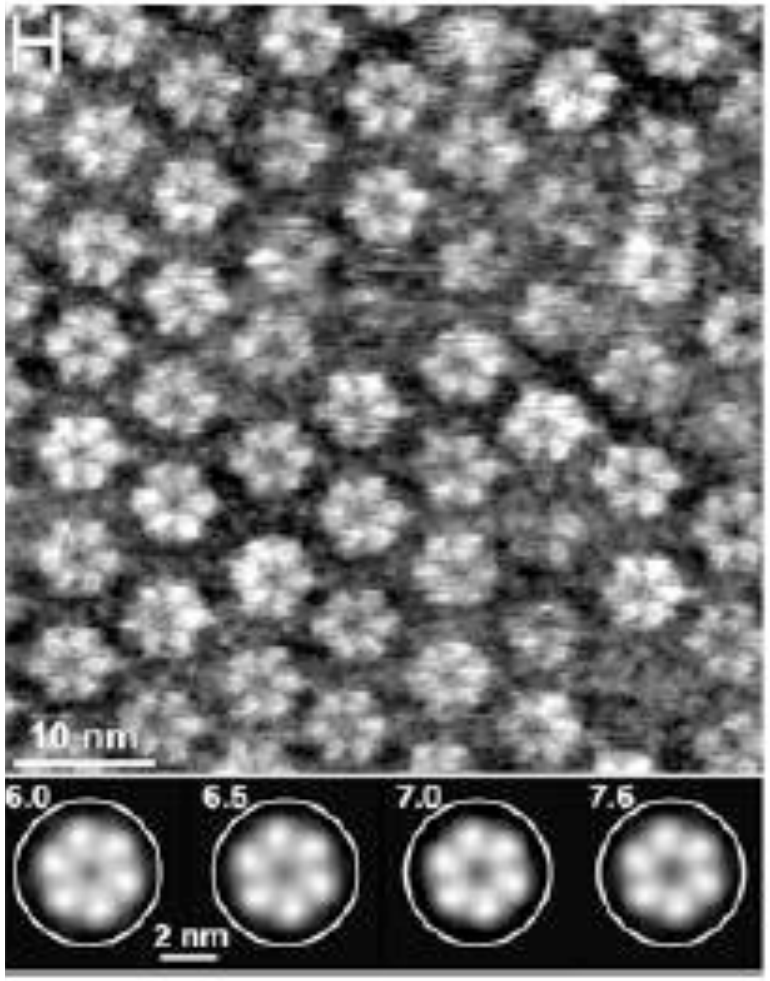
Cell membranes (D=2)





http://www.sbmp-itn.eu/sbmps/research_method/

source: wiki

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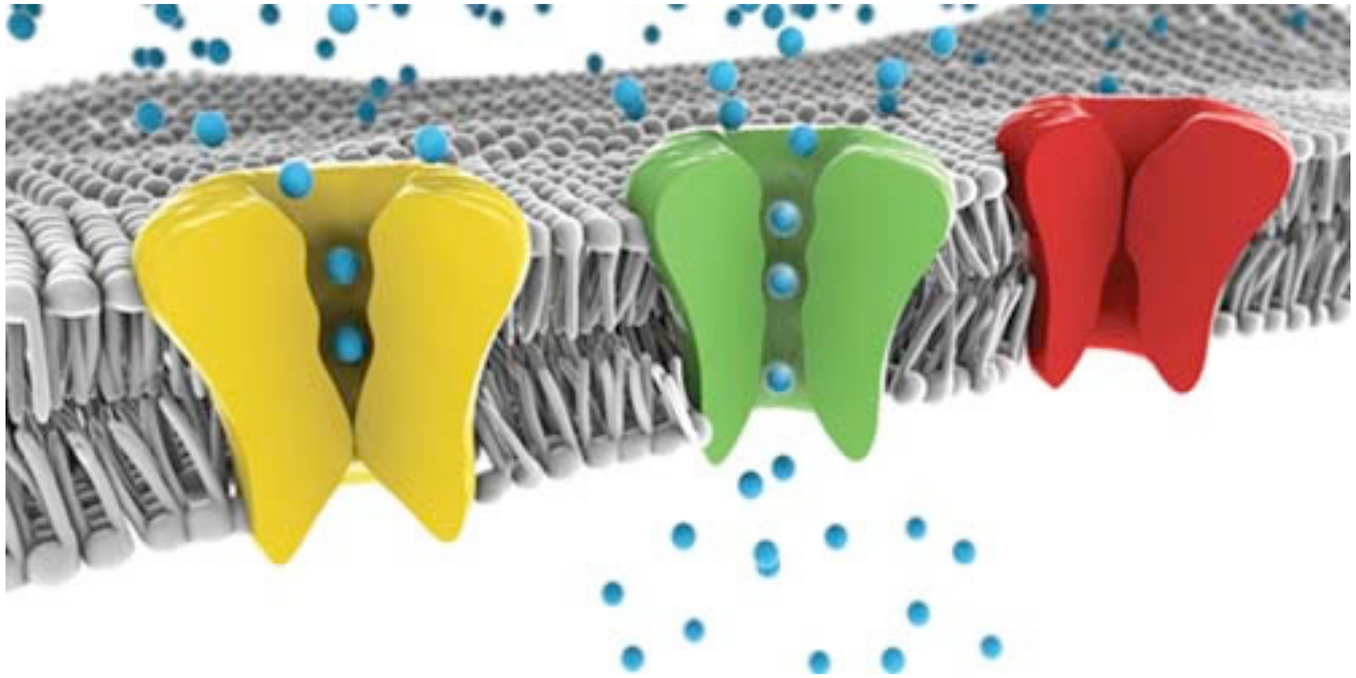
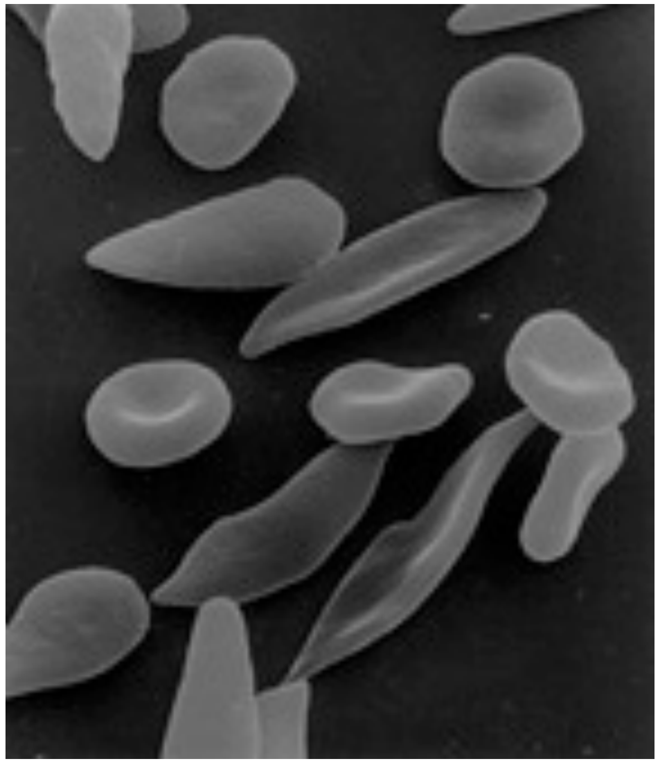
Cell membranes (D=2)

transport:
stochastic
escape problems

Illustration by J.P. Cartailler. Copyright 2007, Symmation LLC.



shape:
differential
geometry



red blood cells
affected by
sickle-cell disease

source: wiki

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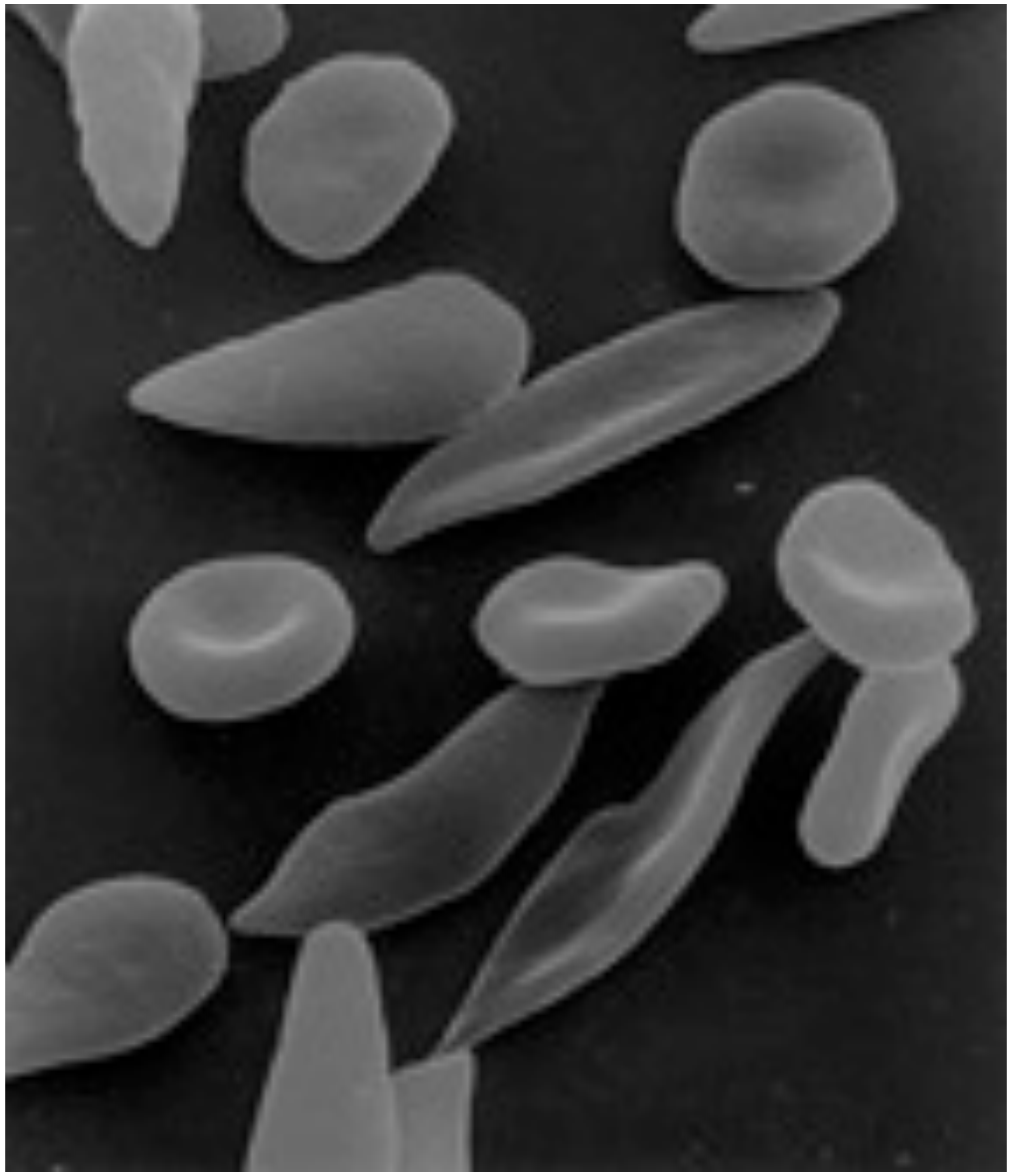
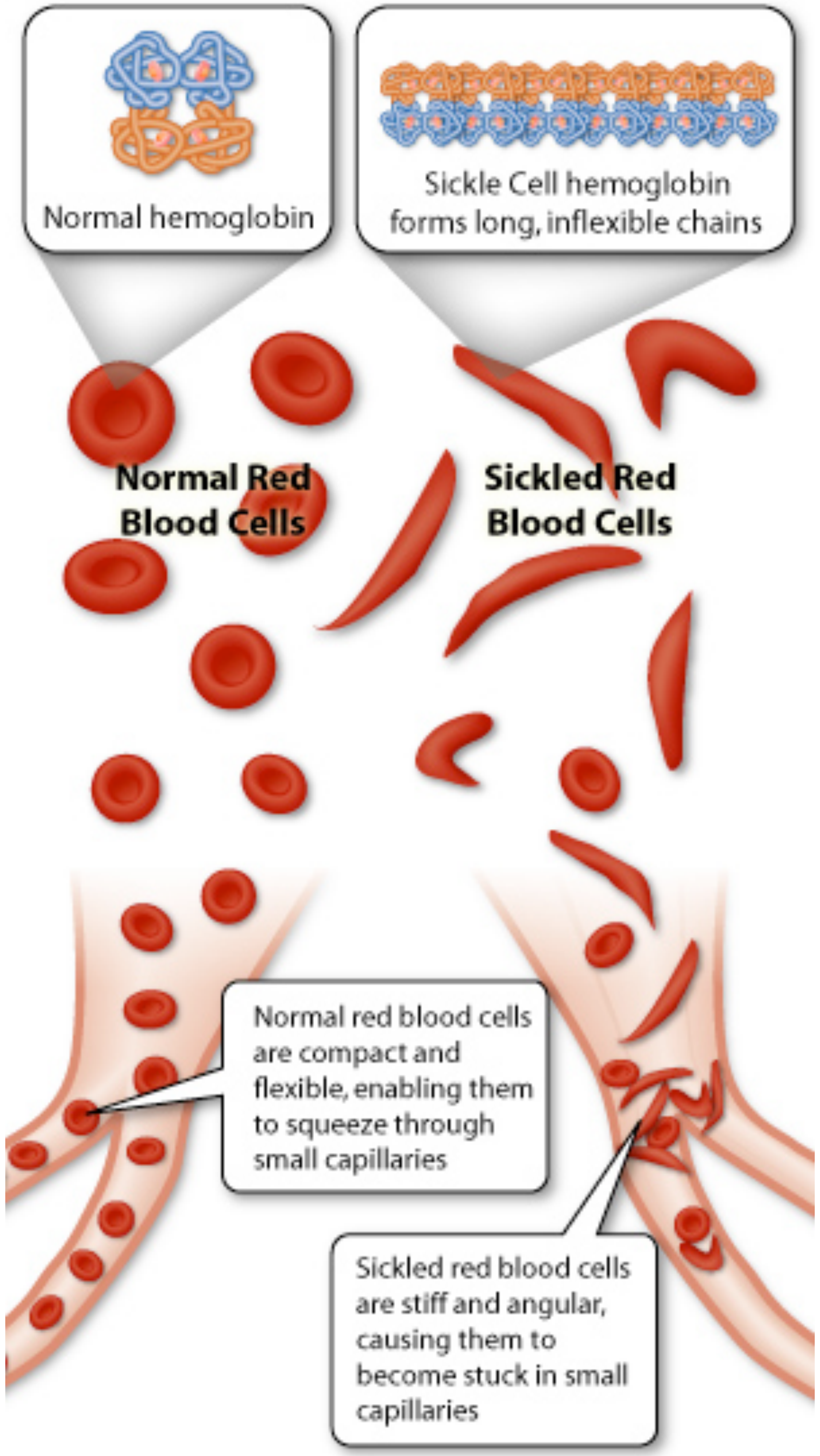
Blood cells: shape & function

source: wiki



red blood cells affected by sickle-cell disease



Normal red blood cells are compact and flexible, enabling them to squeeze through small capillaries

Sickled red blood cells are stiff and angular, causing them to become stuck in small capillaries

http://learn.genetics.utah.edu/

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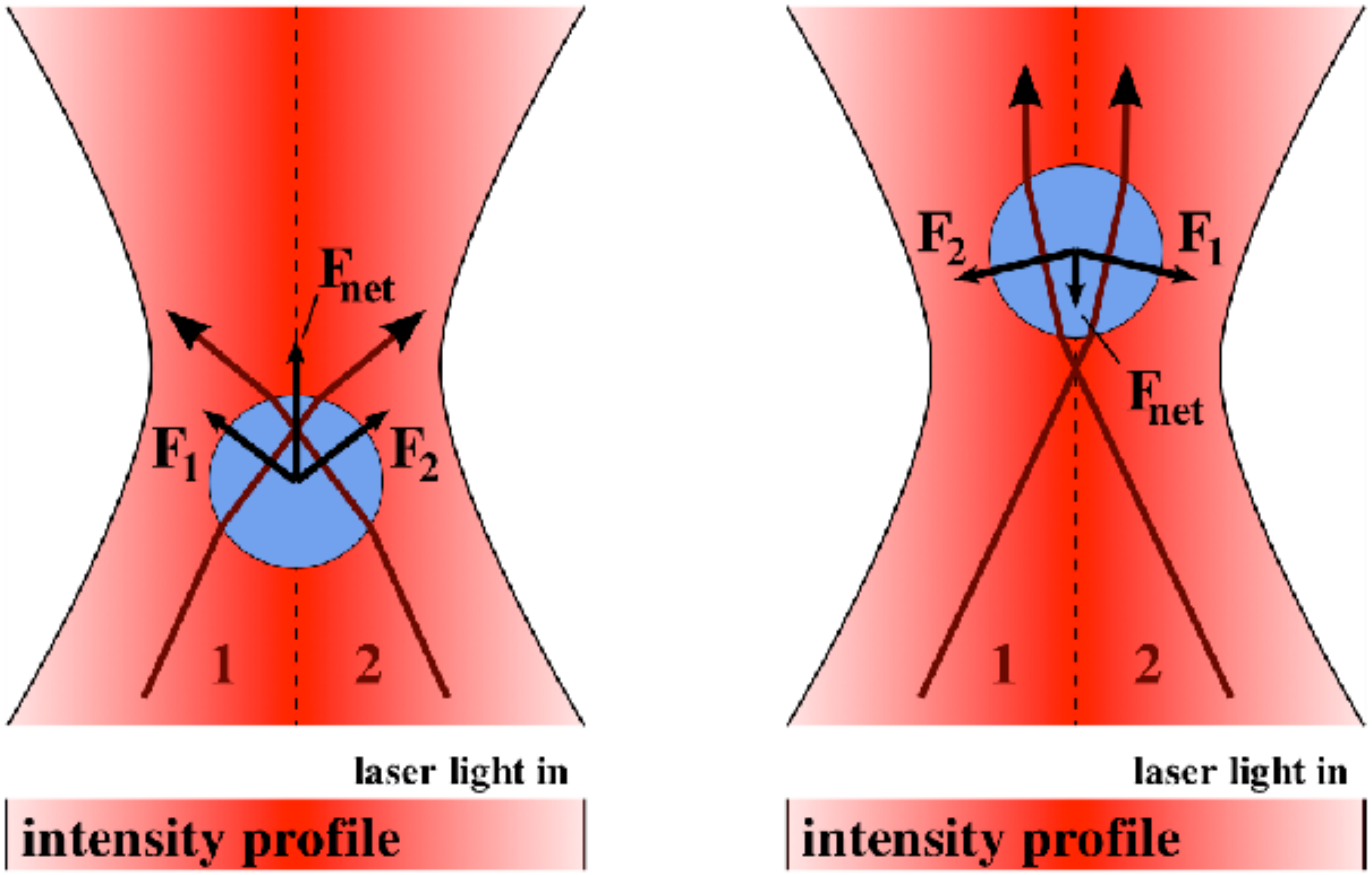
Optical tweezer

source: wiki



http://www.nature.com/ncomms/journal/v4/n4/extref/ncomms2786-s1.swf

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Dynamics of a vesicle in general flow

J. Deschamps, V. Kantsler, E. Segre, and V. Steinberg[1]

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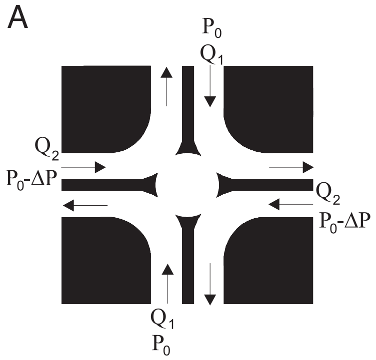
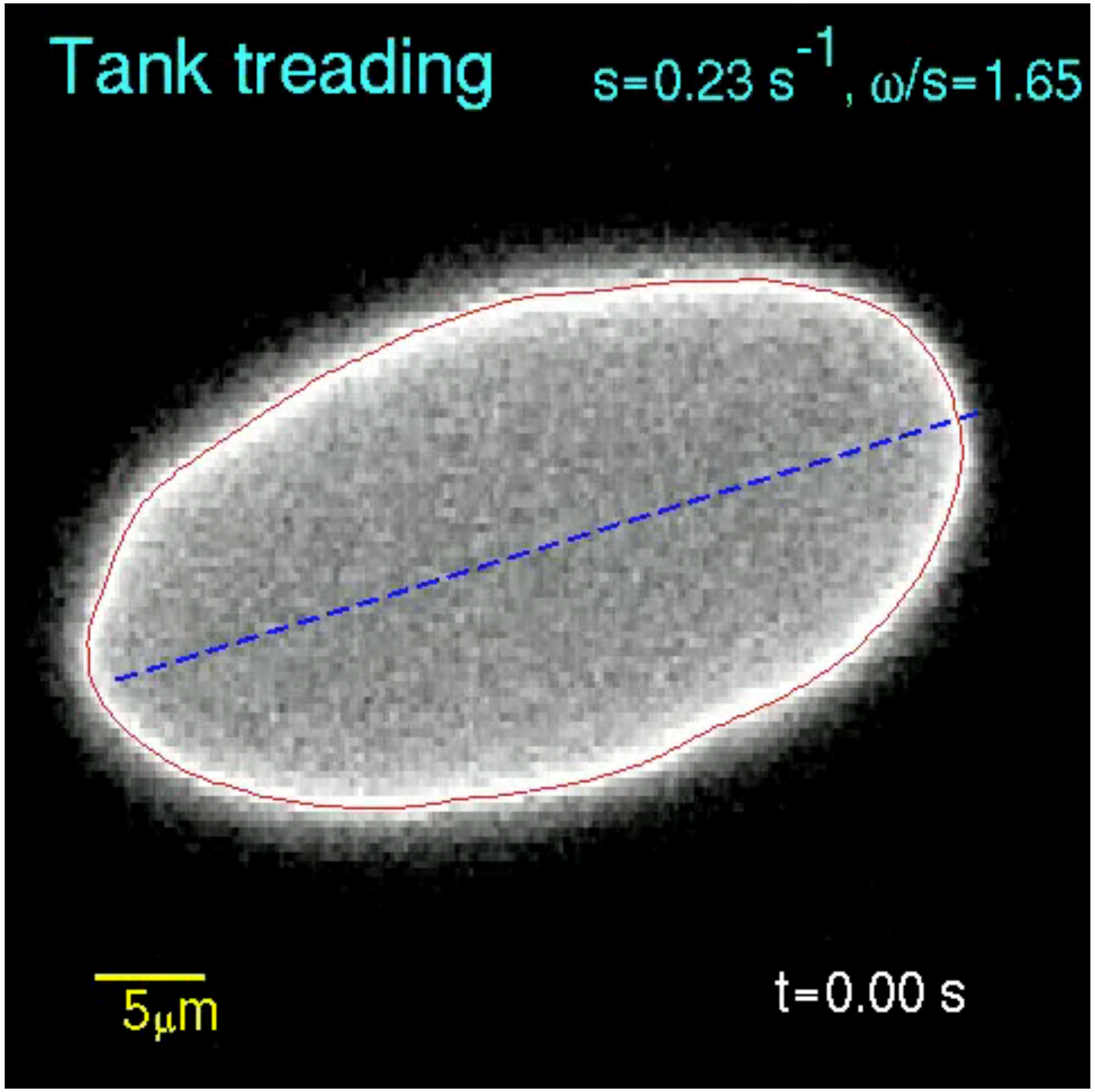
11444–11447 | PNAS | July 14, 2009 | vol. 106 | no. 28





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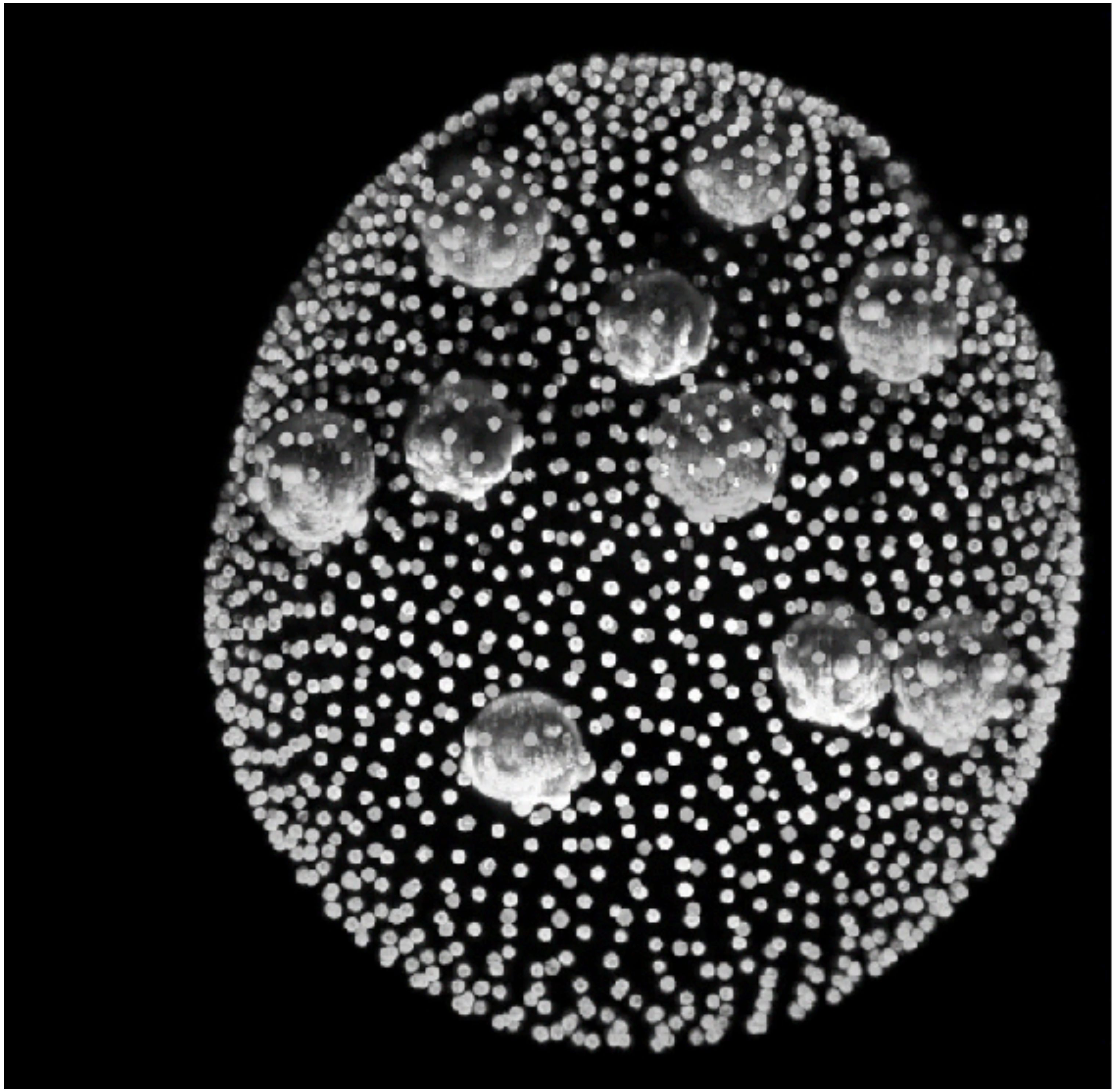
Volvox inversion



http://www.damtp.cam.ac.uk/user/gold/movies.html

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our lecture course:

- ‘differential geometry’ of membranes

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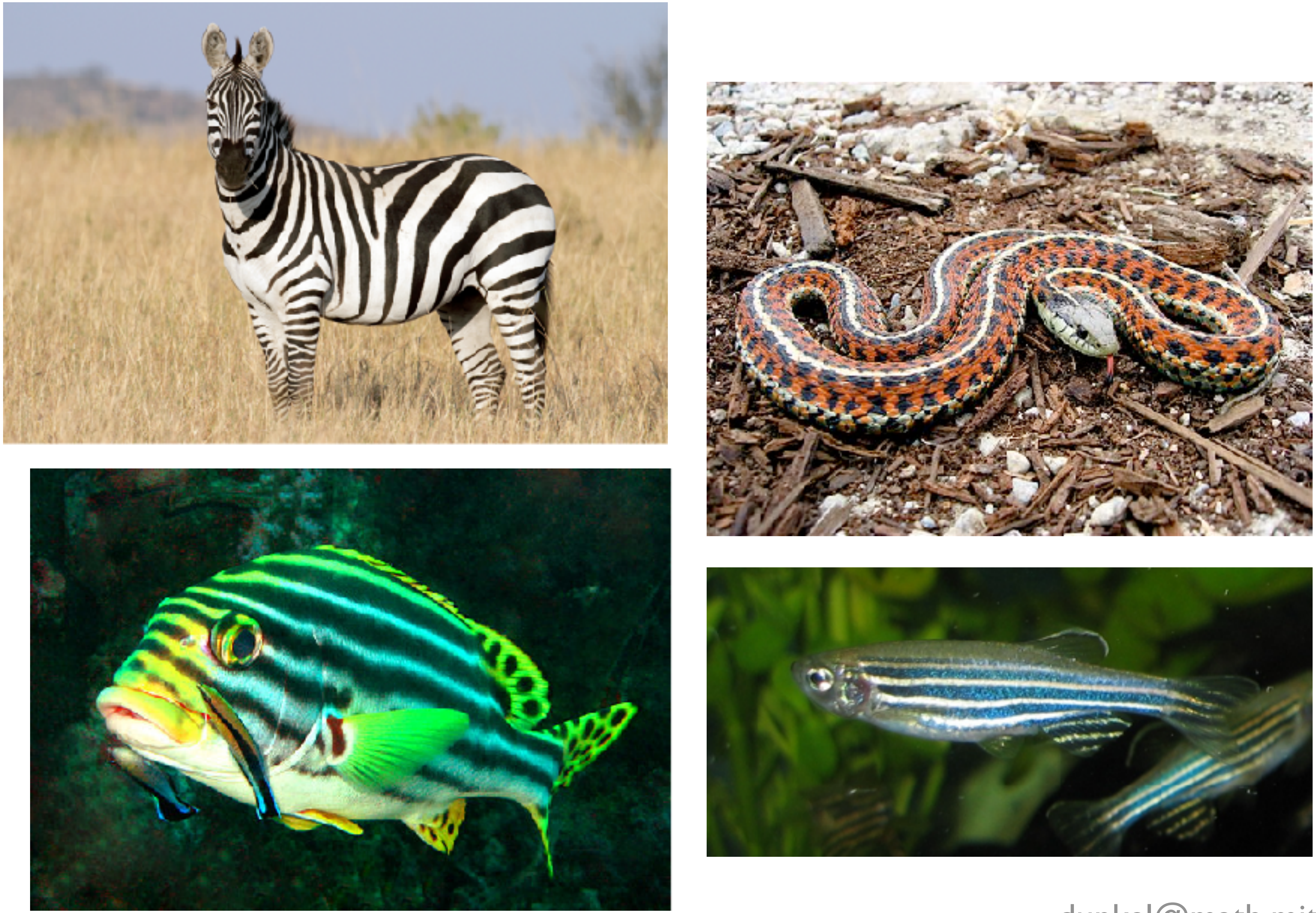
Stationary patterns



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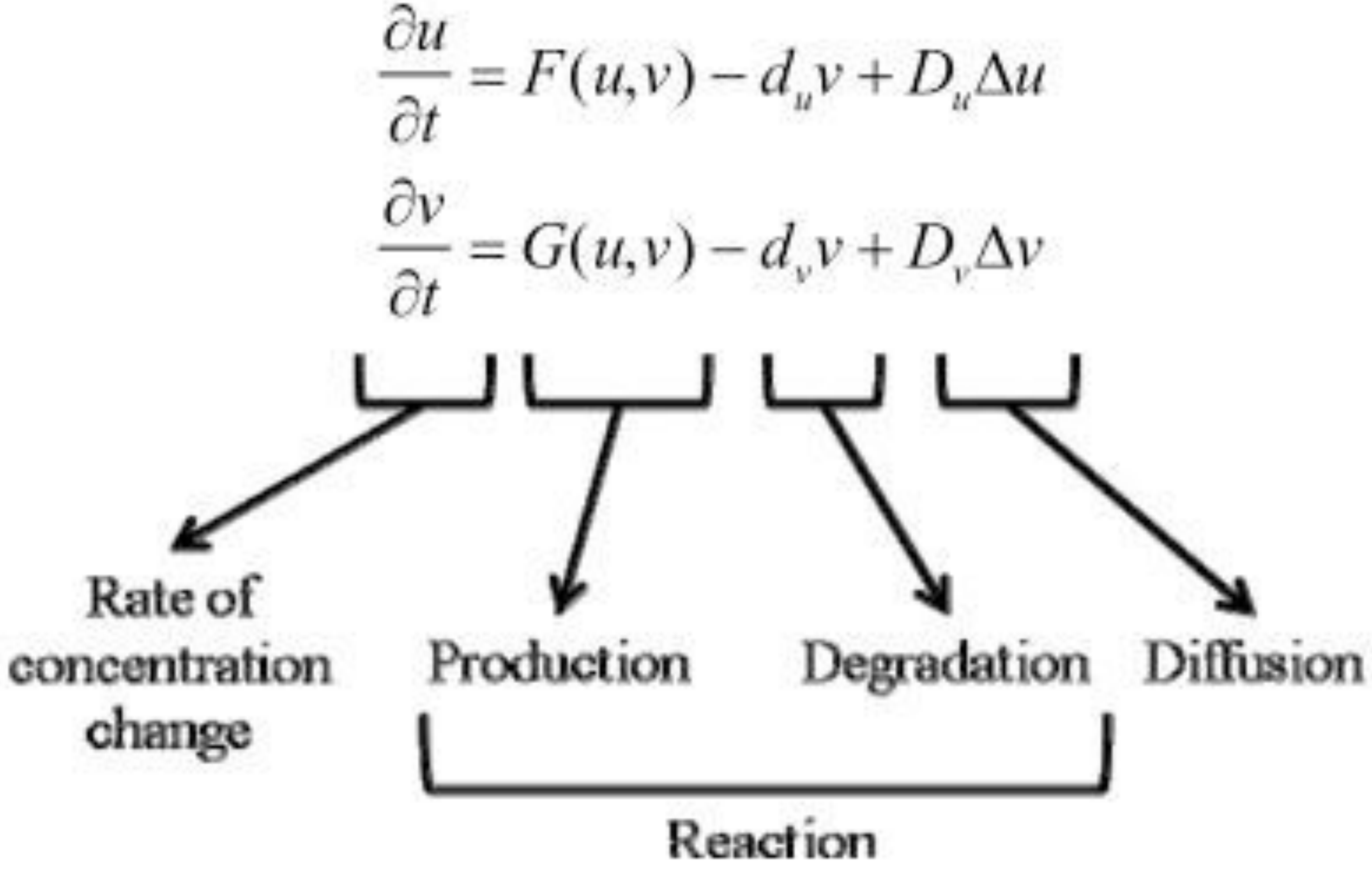
Turing model



$$\frac{\partial u}{\partial t} = F(u,v) - d_u v + D_u \Delta u$$

$$\frac{\partial v}{\partial t} = G(u,v) - d_v v + D_v \Delta v$$

A. M. Turing. **The chemical basis of morphogenesis.** *Phil. Trans. Royal Soc. London. B* **327**, 37–72 (1952)

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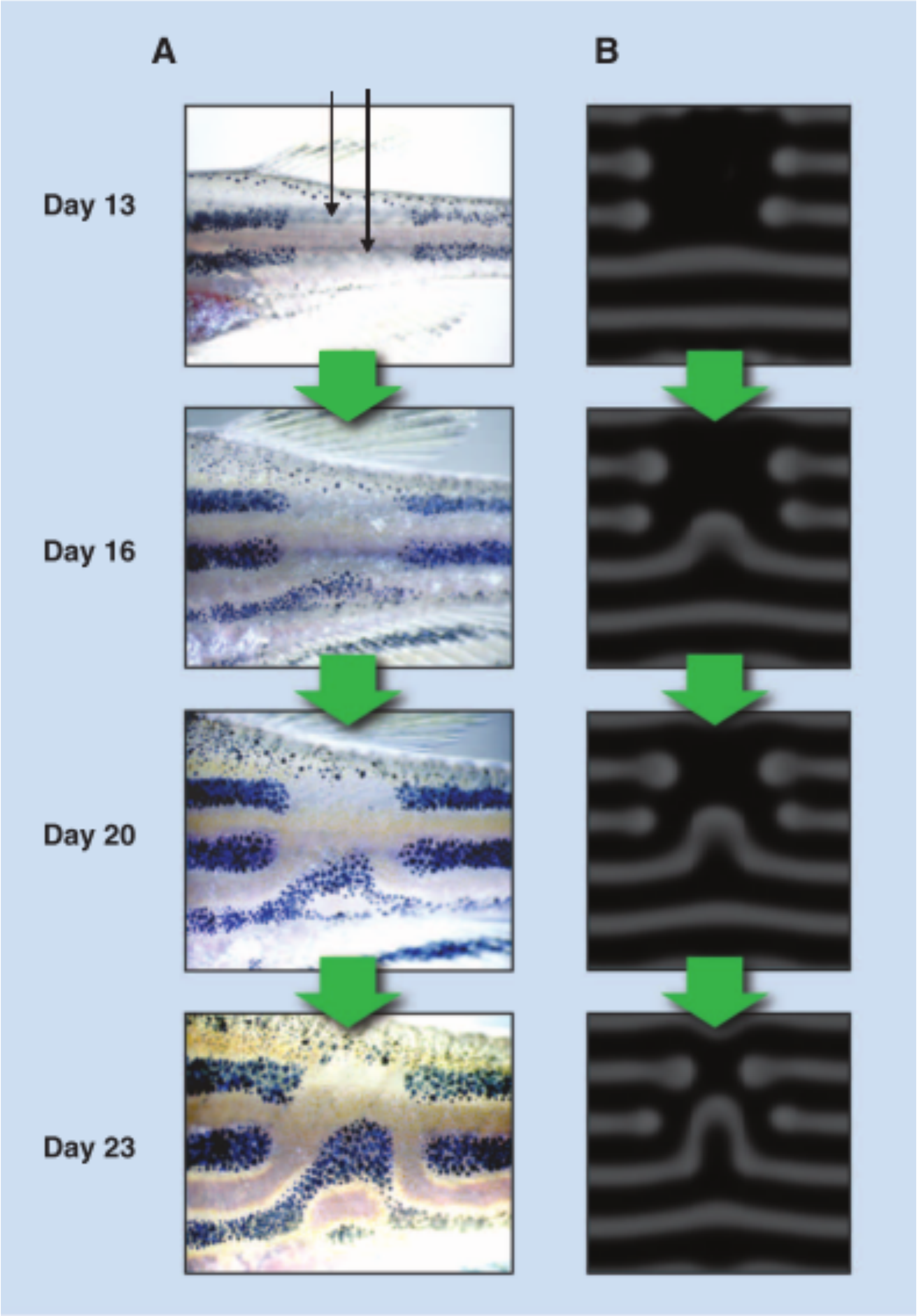
wiki

The matching of zebrafish stripe formation and a Turing model



Kondo S, & Miura T (2010). Reaction-diffusion model as a framework for understanding biological pattern formation. *Science, 329* (5999), 1616-20

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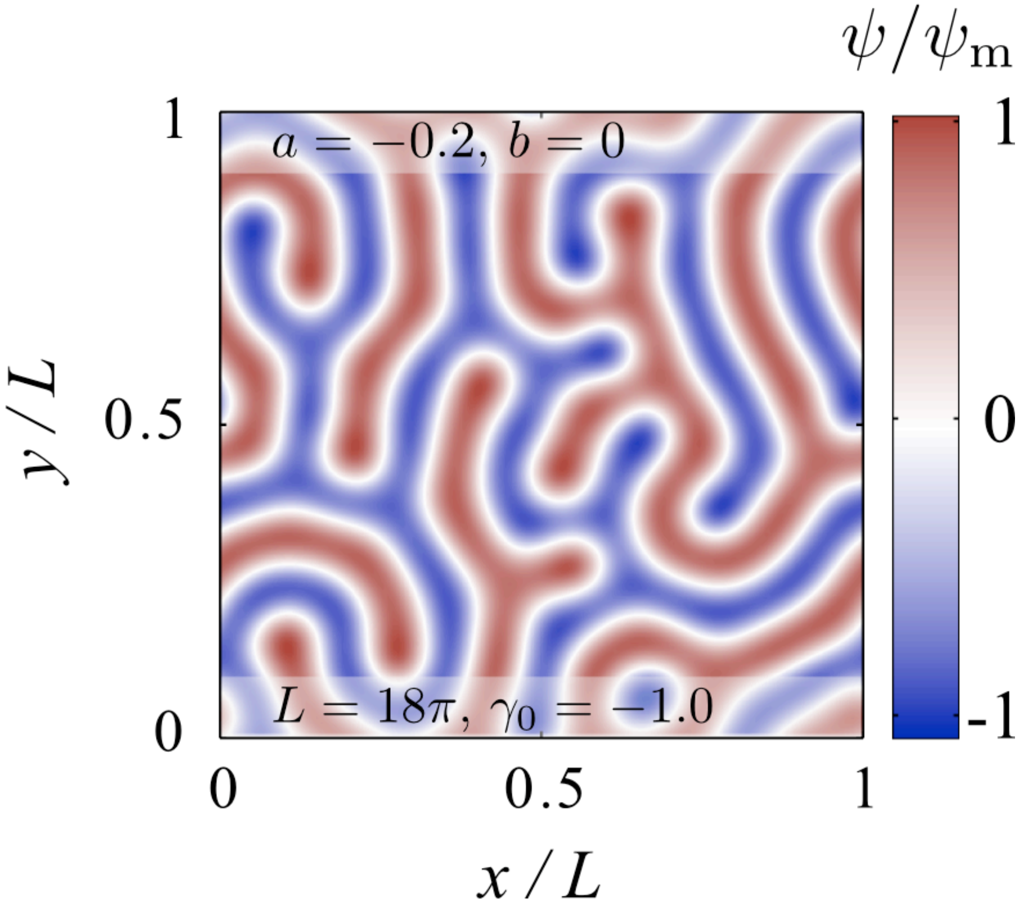
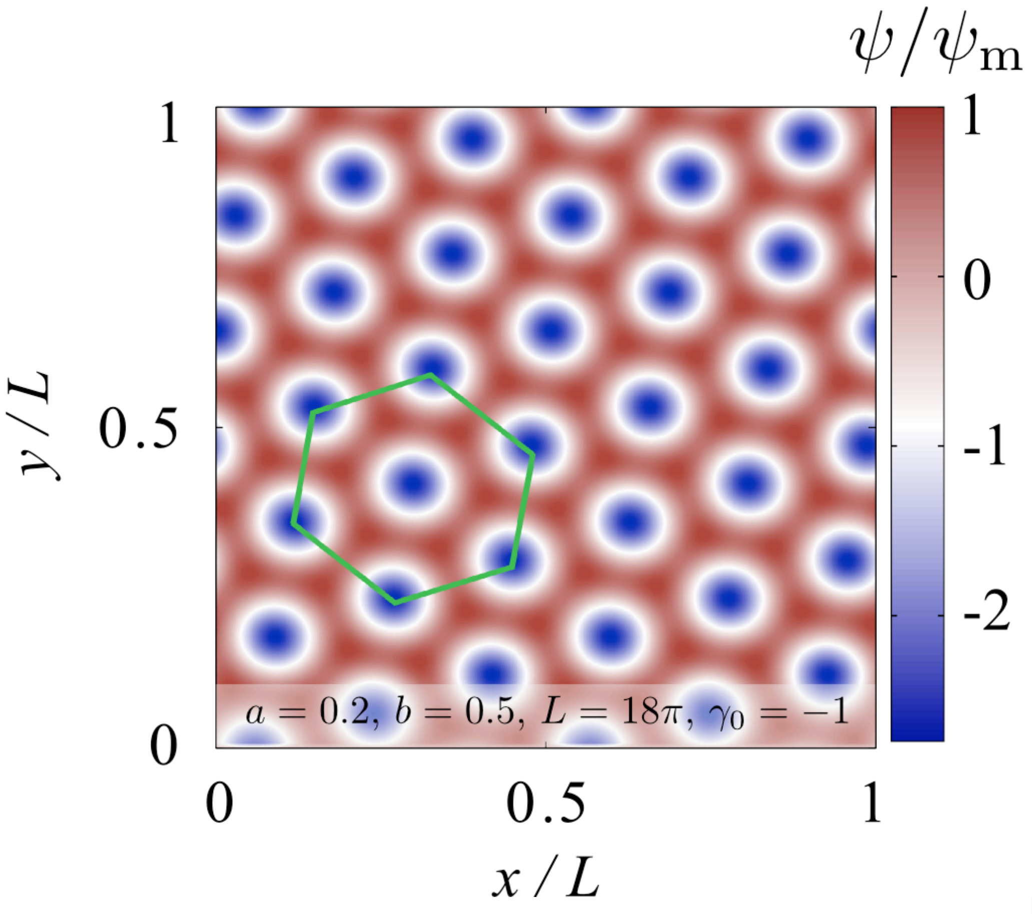
Scalar field theory

2d Swift-Hohenberg model

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

$$U(\psi) = \frac{a}{2}\psi^2 + \frac{b}{3}\psi^3 + \frac{c}{4}\psi^4$$



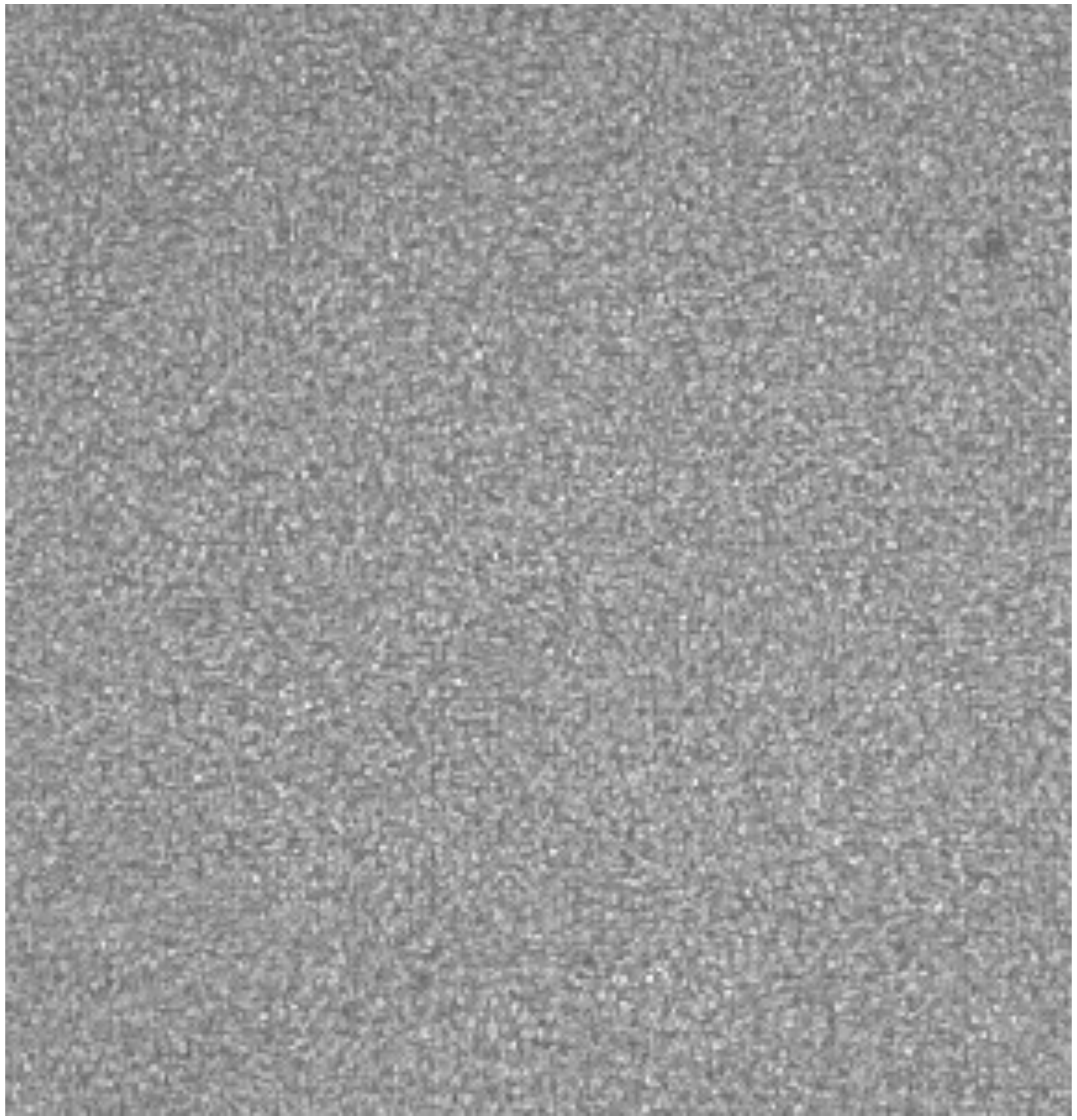


Active patterns

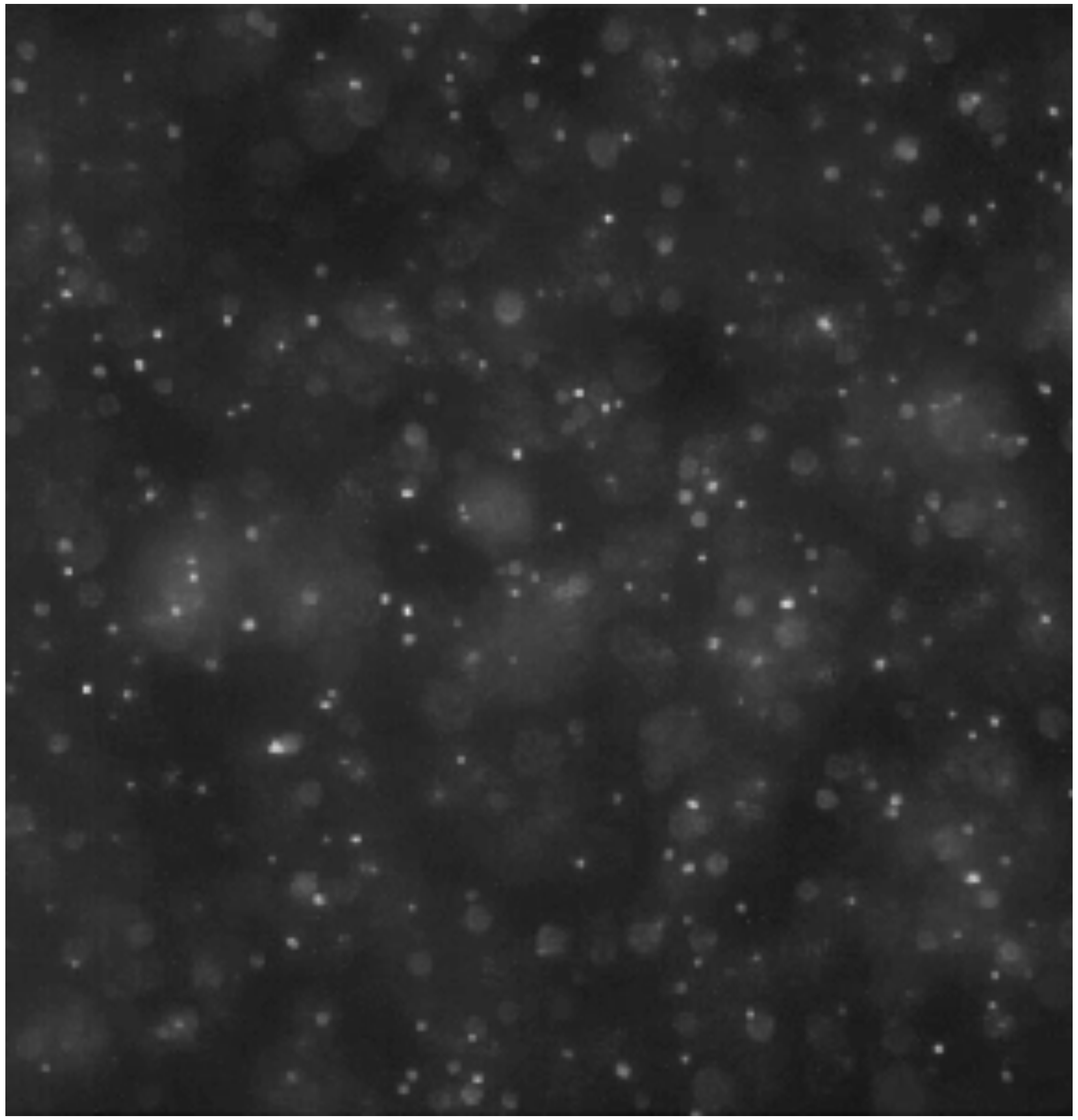
PRL (2013)

B. subtilis

tracer



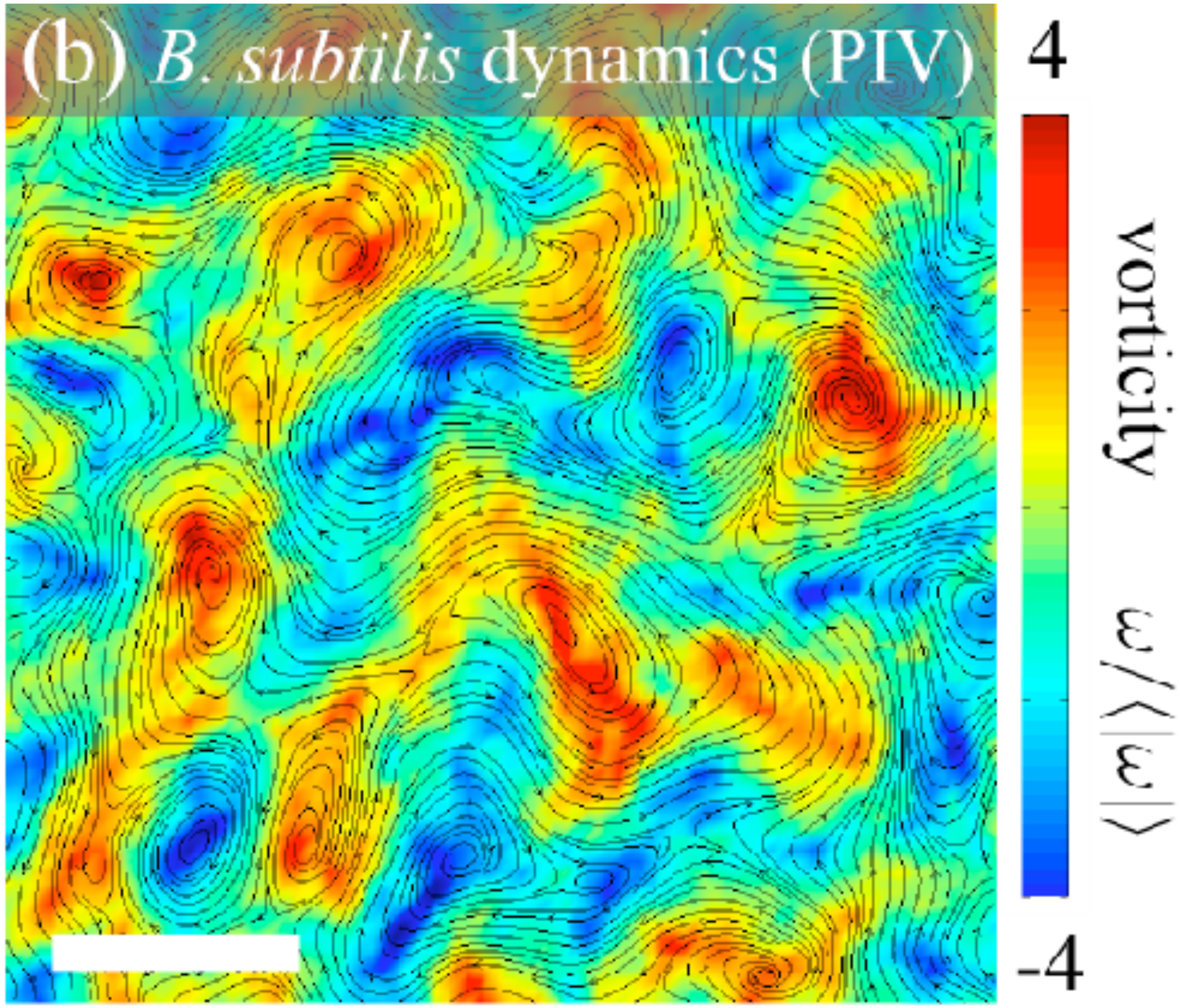
bright field



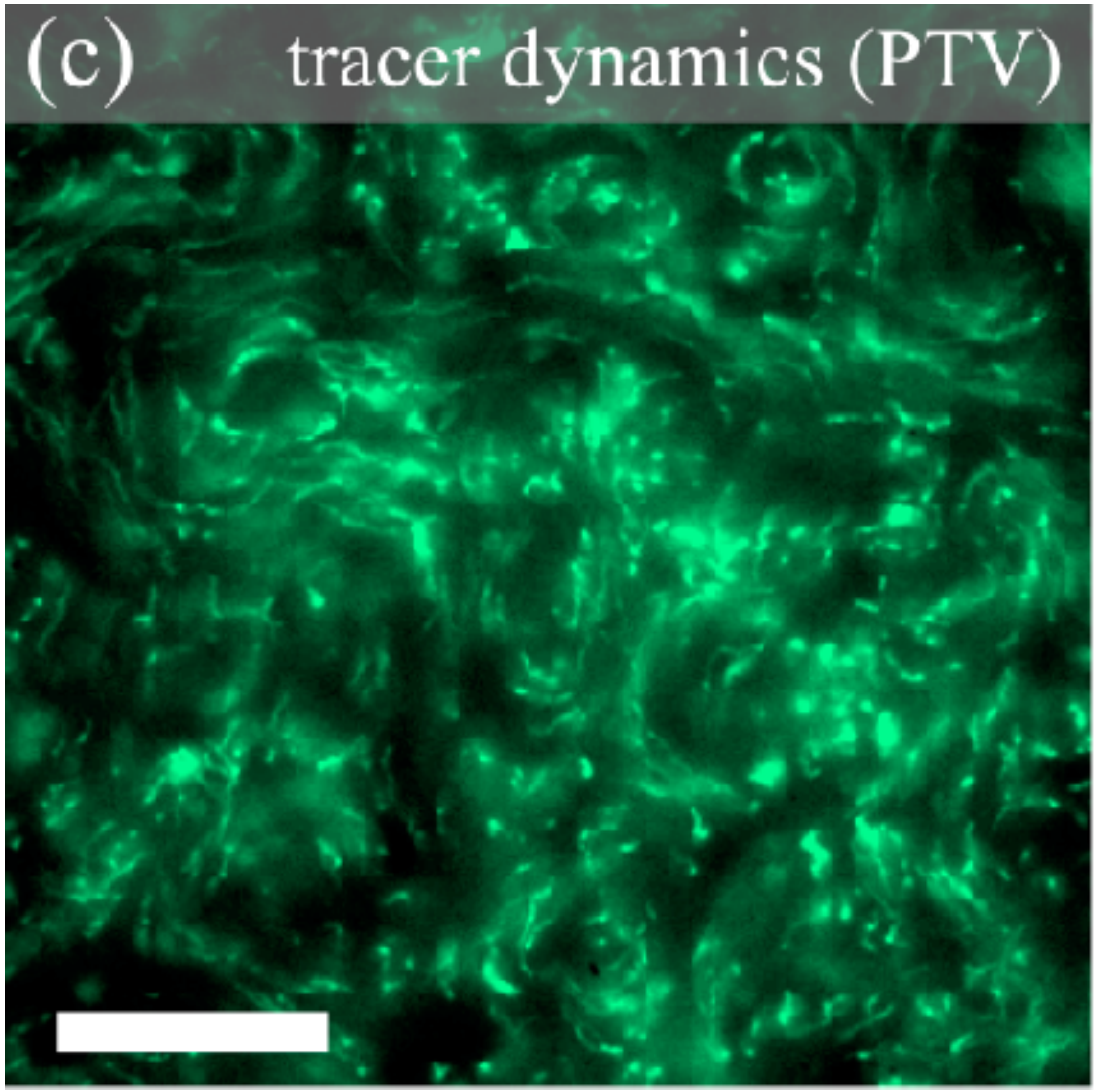
fluorescence

3D bacterial suspension

PRL (2013)



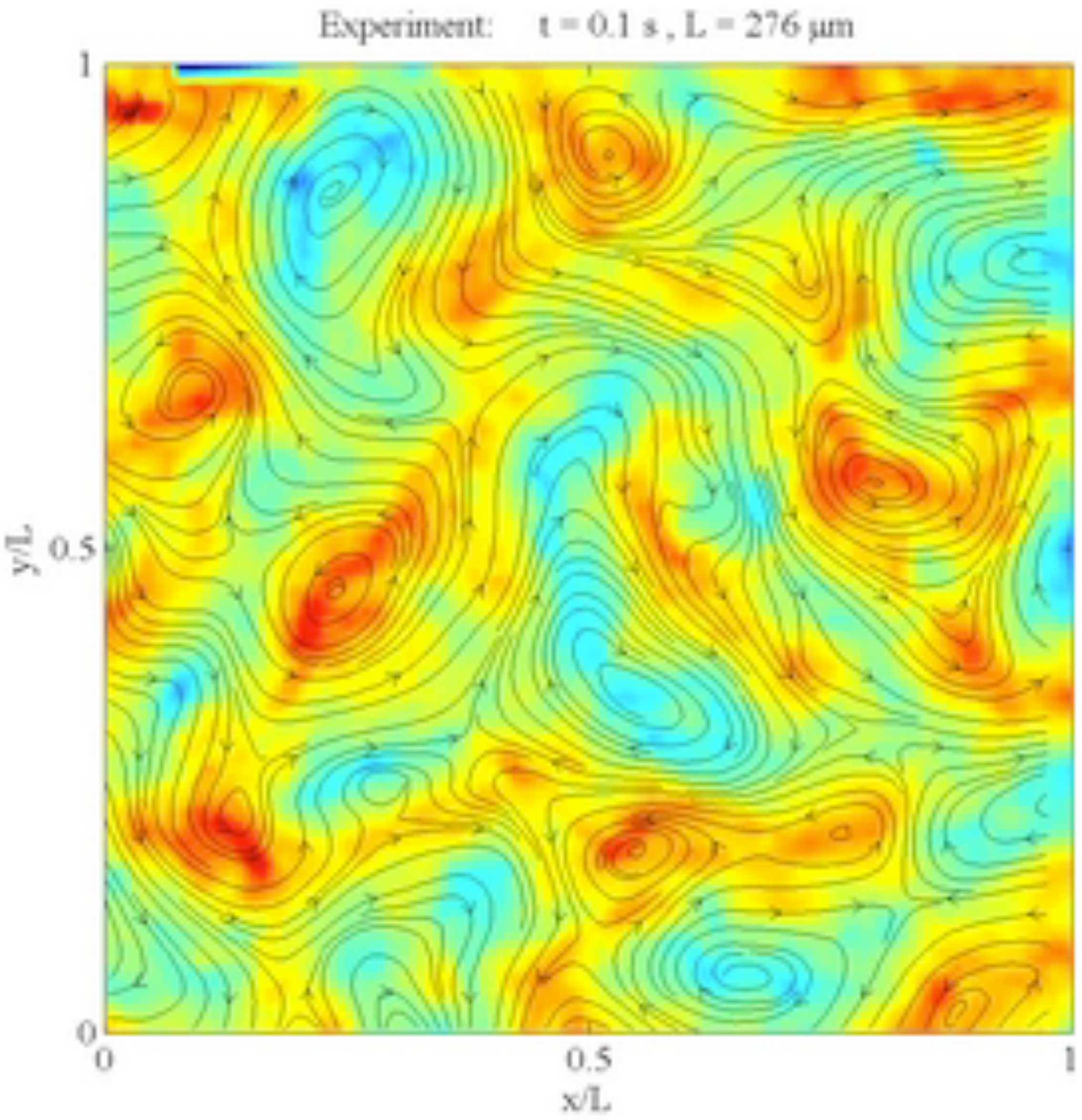
bright field



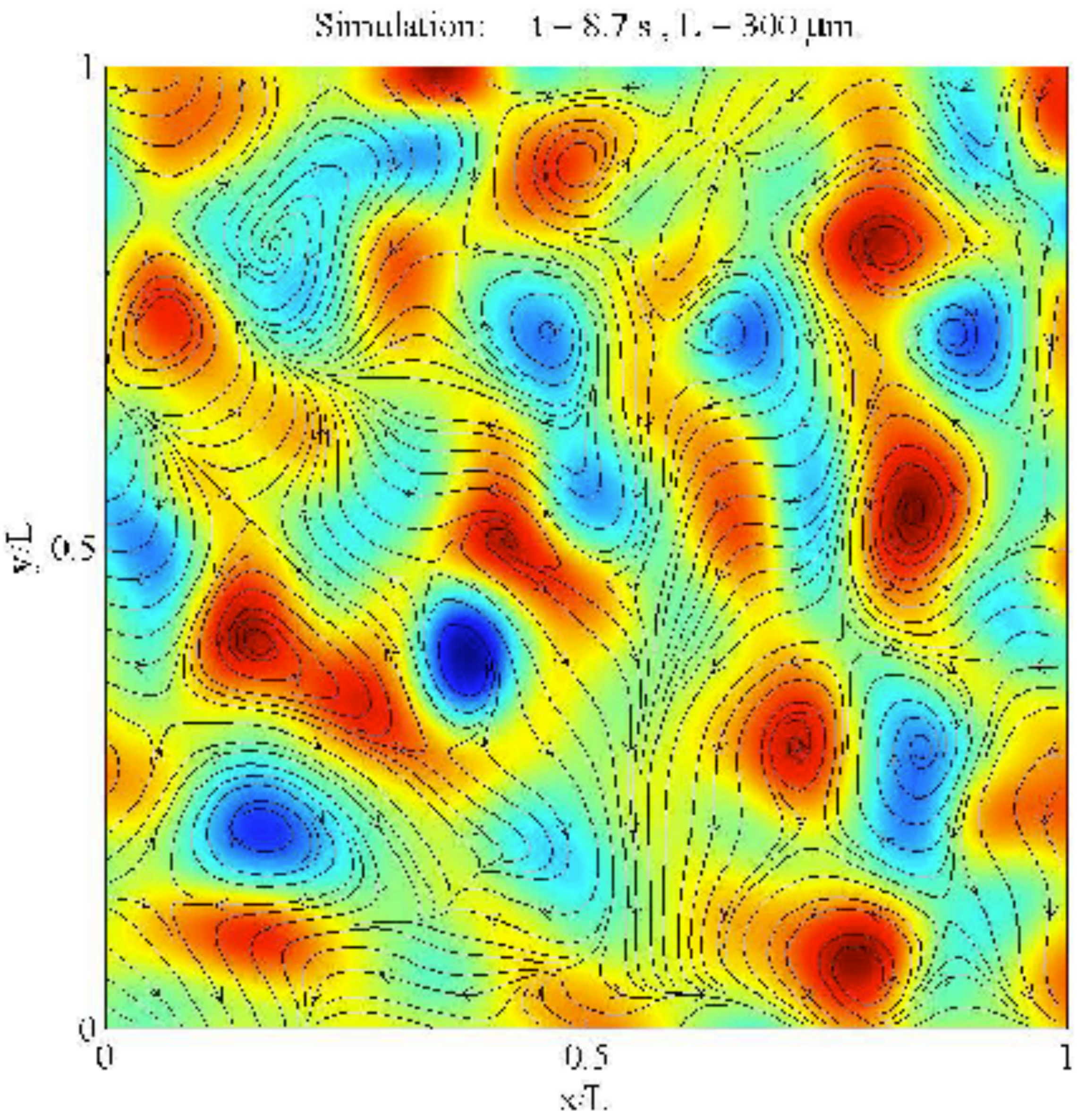
fluorescence

PRL (2013)

3D suspension



Experiment:
quasi-2D slice



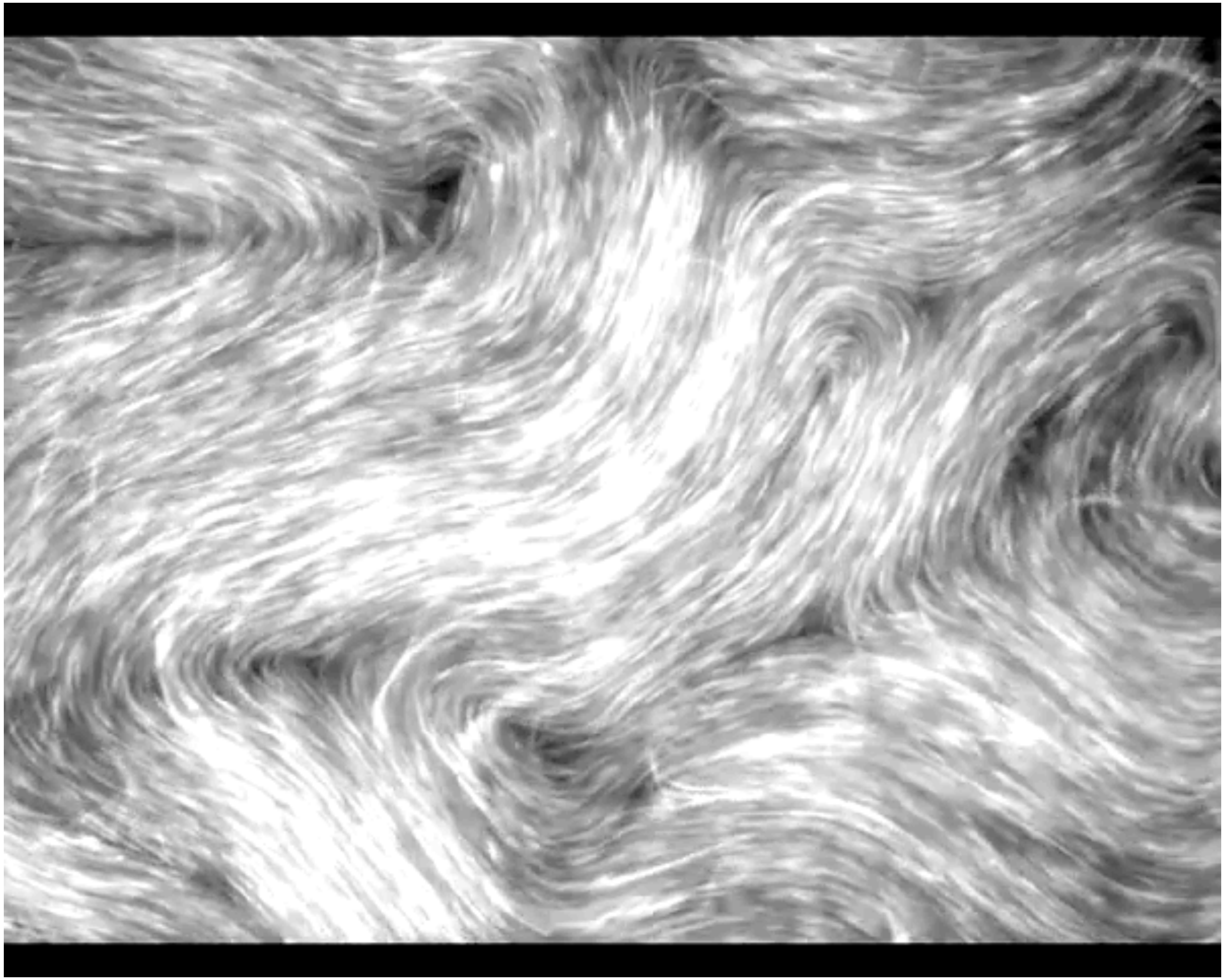
Theory:
2D slice

Vector field theory (generalized Navier-Stokes equations)

incompressibility $\nabla \cdot \boldsymbol{v} = 0$

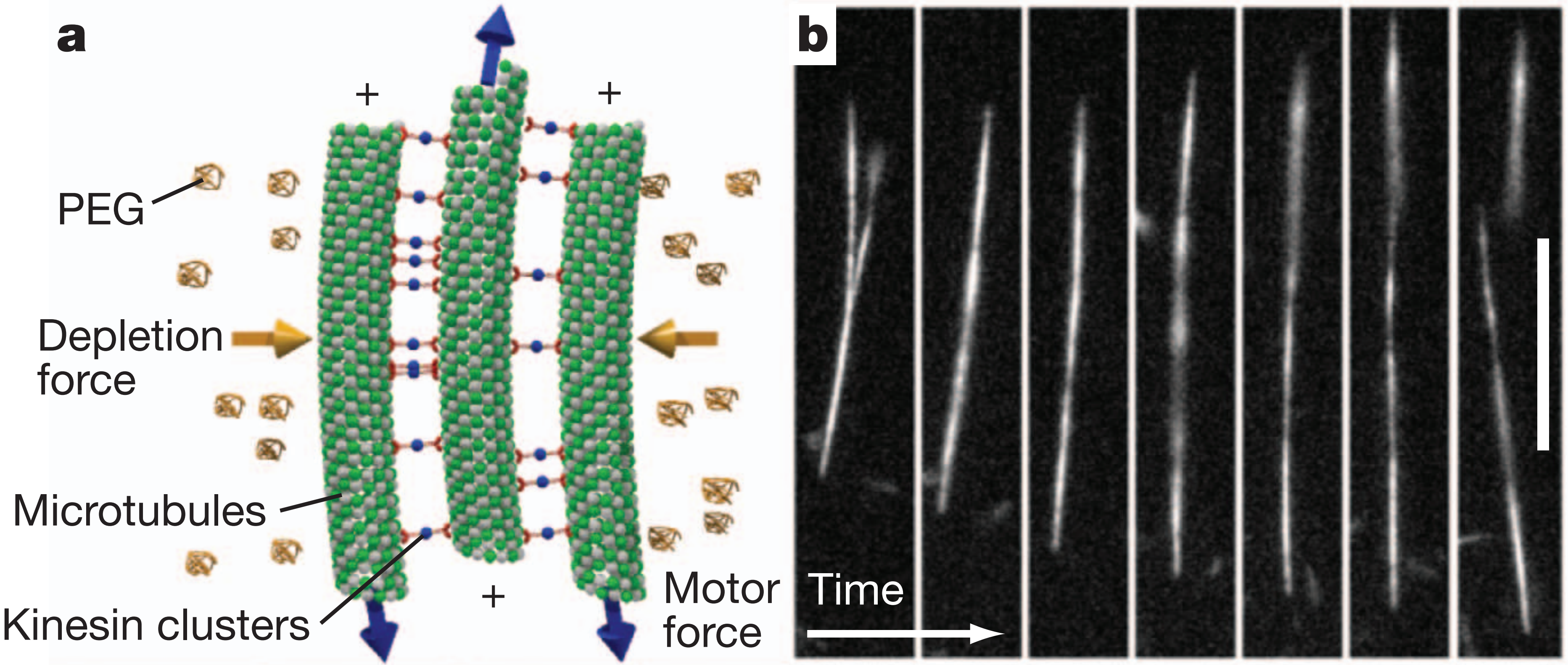
$$(\partial_t + \lambda_0 \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla(p + \lambda_1 \boldsymbol{v}^2) - (\beta \boldsymbol{v}^2 + \alpha)\boldsymbol{v} + \\ + \Gamma_0 \nabla^2 \boldsymbol{v} - \Gamma_2 (\nabla^2)^2 \boldsymbol{v}$$

Active nematics



Dogic lab (Brandeis) Nature 2012

Active nematics



Dogic lab (Brandeis) Nature 2012

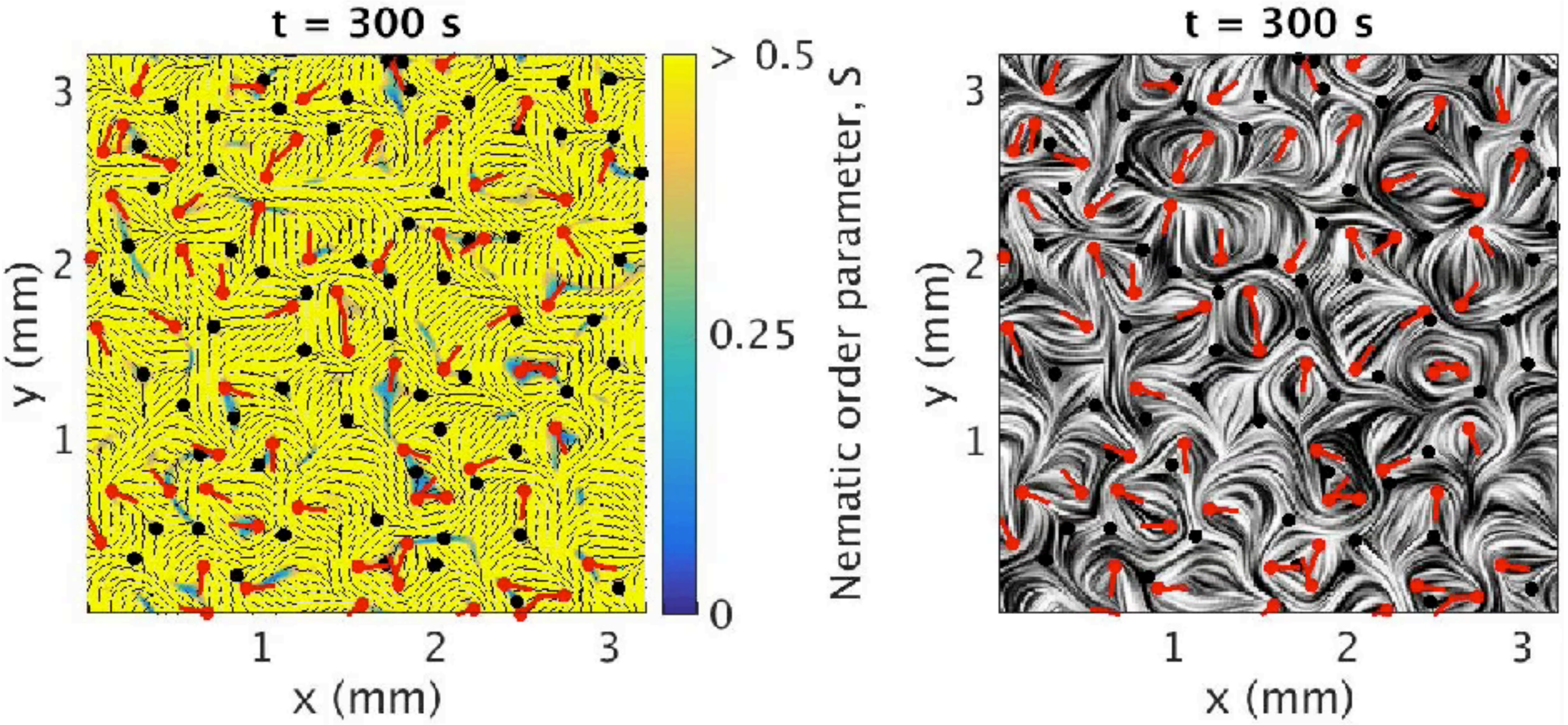
no head or tail ⇒ Q-tensor order-parameter

$$Q_{ij} = Q_{ji}\ , \qquad \mathrm{Tr}\, Q = 0 \qquad Q = \begin{pmatrix} \lambda & \mu \\ \mu & -\lambda \end{pmatrix} .$$

$$\Delta = \sqrt{\lambda^2 + \mu^2}, \qquad \Lambda^{\pm} = \pm\Delta$$

$$\partial_t Q_{ij} + \partial_k (v_k Q_{ij}) = -\frac{\delta \mathcal{F}}{\delta Q_{ij}}$$

$$v_k \;=\; D\,\partial_n Q_{nk}$$



biological networks

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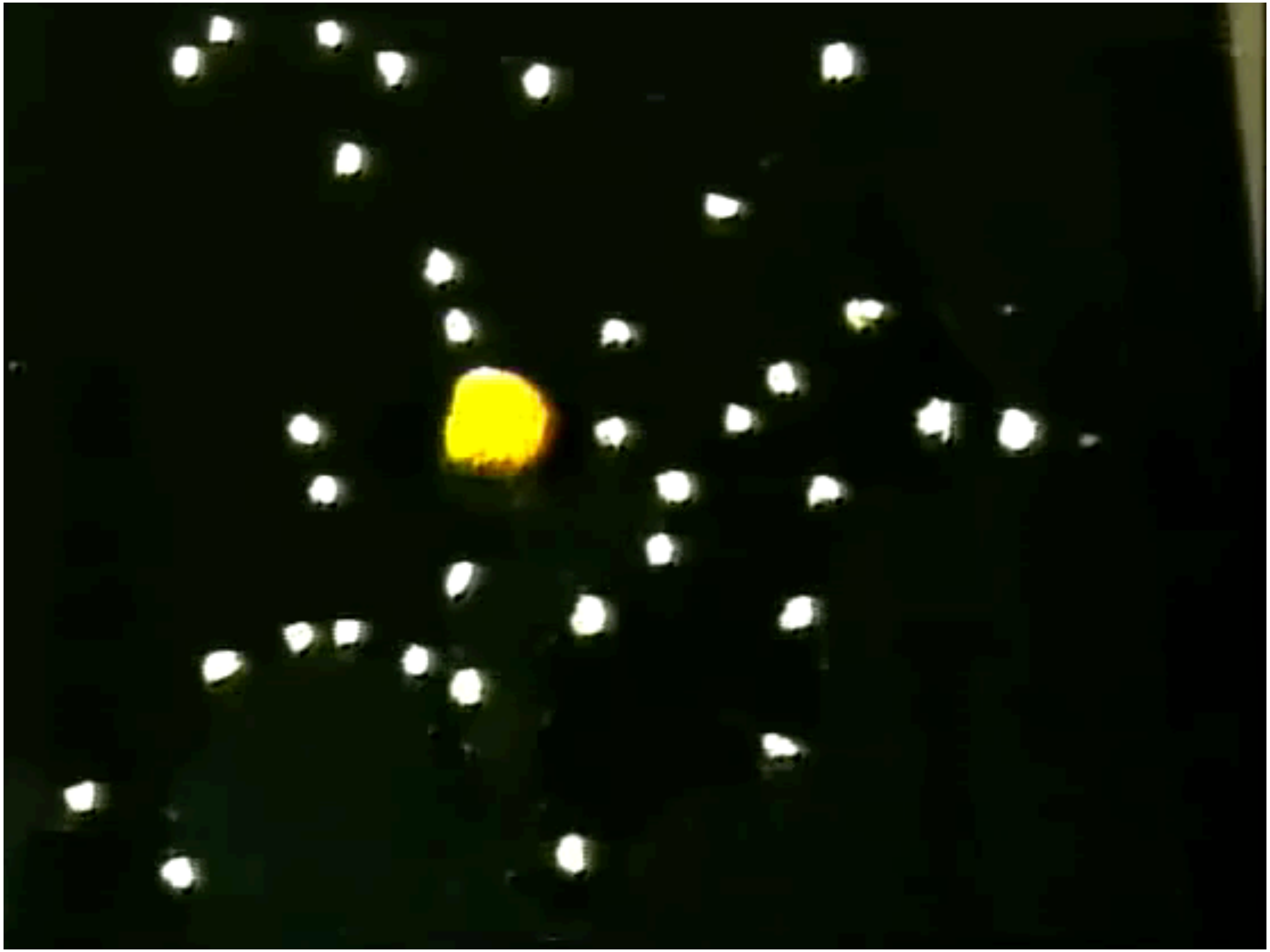
Tokyo rail network by *Physarum plasmodium*



Tero et al (2010) Science

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Compressible AFN model

$$\dot{\varrho}_v = \sum_e \nabla_{ve}\phi_e$$

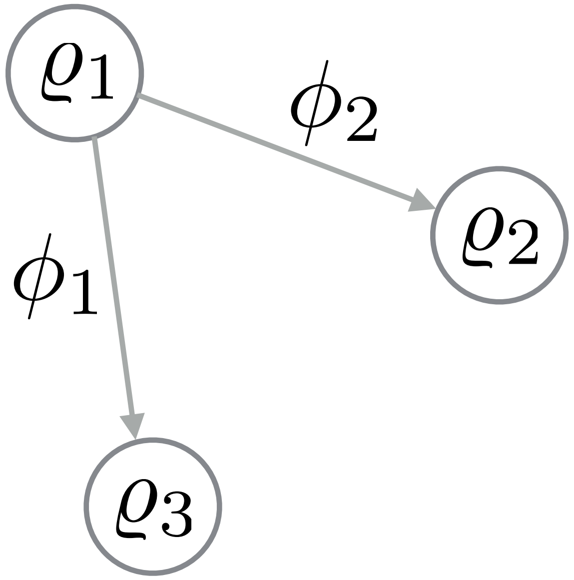
$$\dot{\phi}_e = -\sum_v \nabla^\top_{ev}\varrho_v + \epsilon\frac{\mu-\phi_e^2}{1+\phi_e^2}\phi_e + \sqrt{2D}\xi_e(t)$$

noise

mass conservation

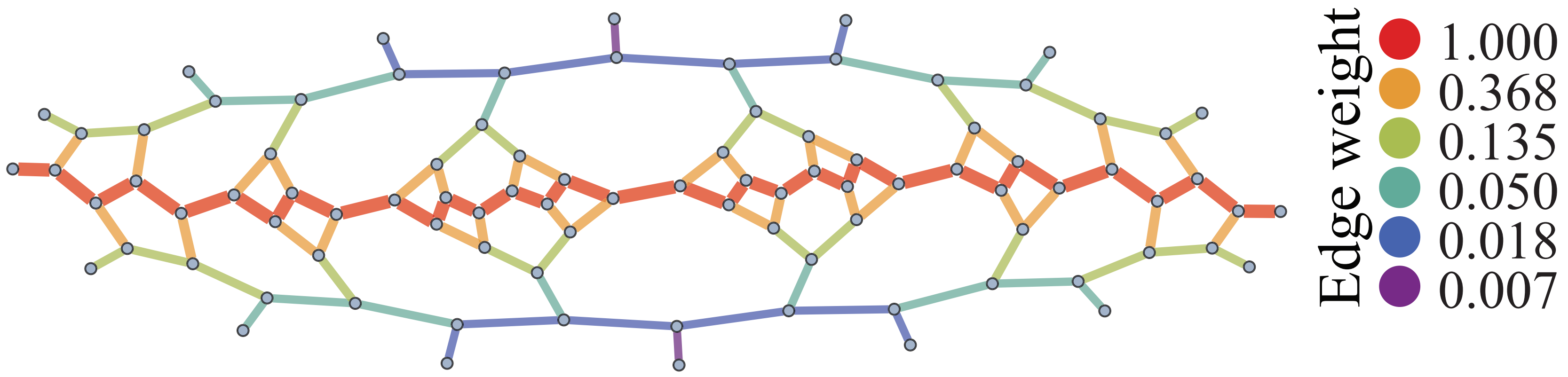
vertex pressure gradient

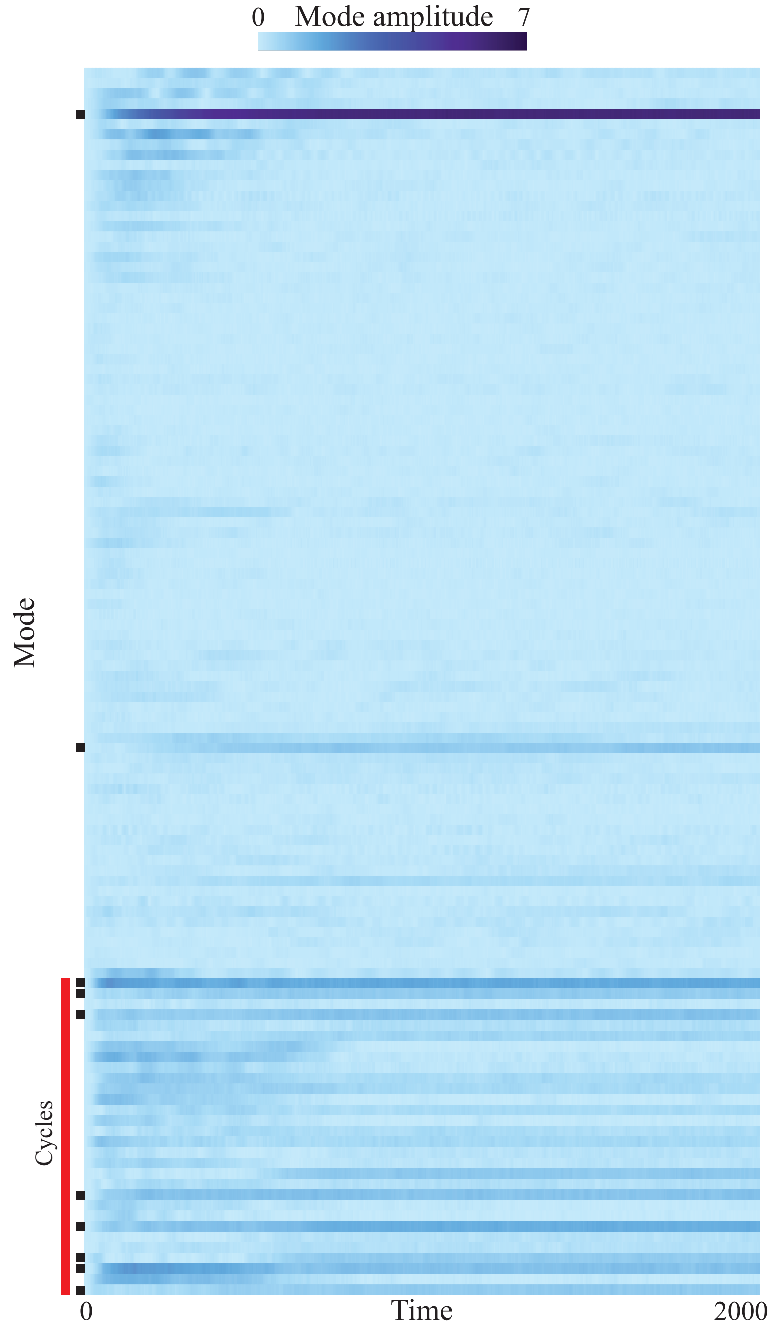
active friction (depot model)



Schweitzer, Ebeling, Tilch (1998) PRL

Forrow et al, PRL 2017





Mode selection in compressible active flow networks

Aden Forrow, Francis G. Woodhouse, and Jörn Dunkel

Single mode selection

$\epsilon = 0.1,\ \mu = 1,\ D = 10^{-4}$

1 second is $\Delta t = 6$

Forrow et al, PRL 2017