(Some) Numbers and Maths in Biology

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http://bionumbers.hms.harvard.edu/

B10NUMB3R5

THE DATABASE OF USEFUL BIOLOGICAL NUMBERS

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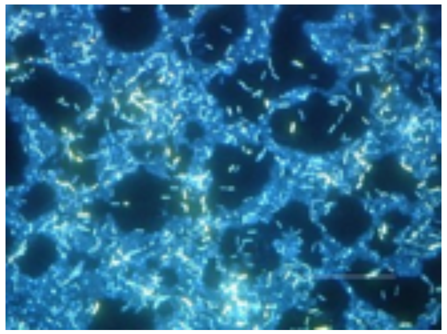
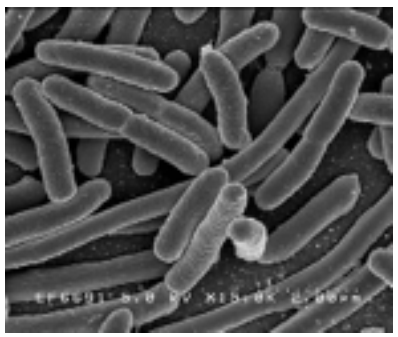
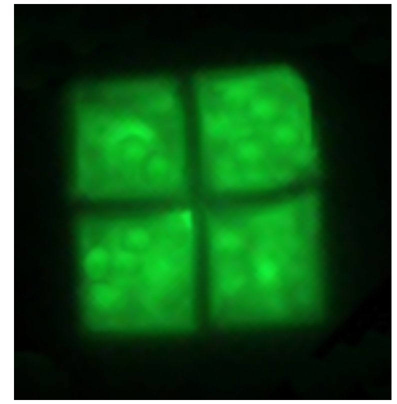
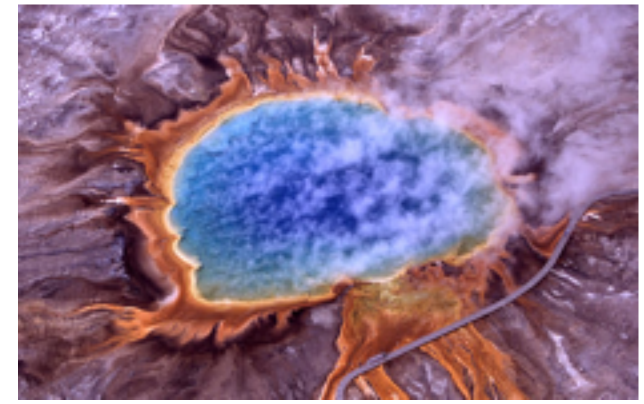
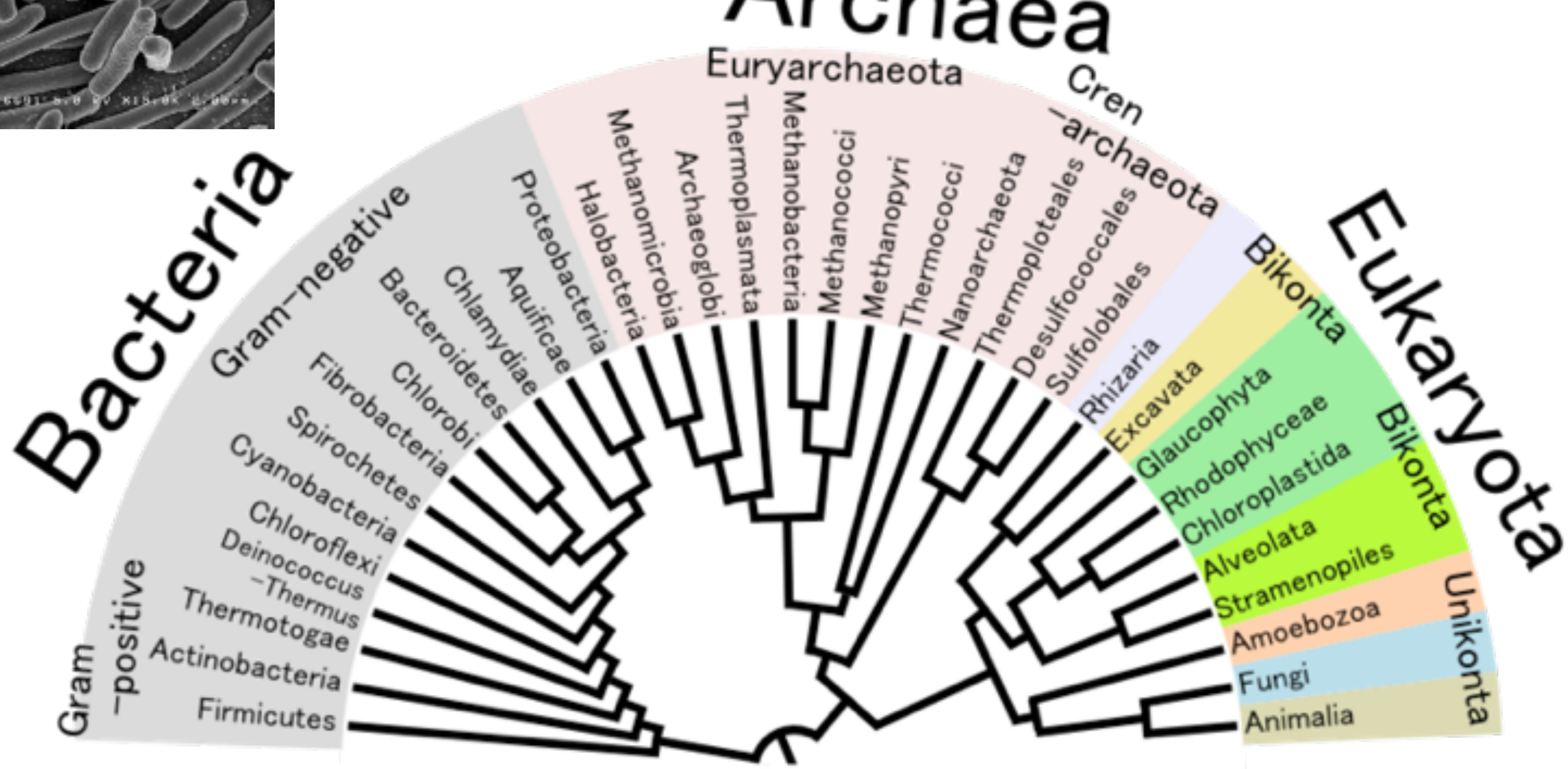
Phylogenetic tree











source: wiki

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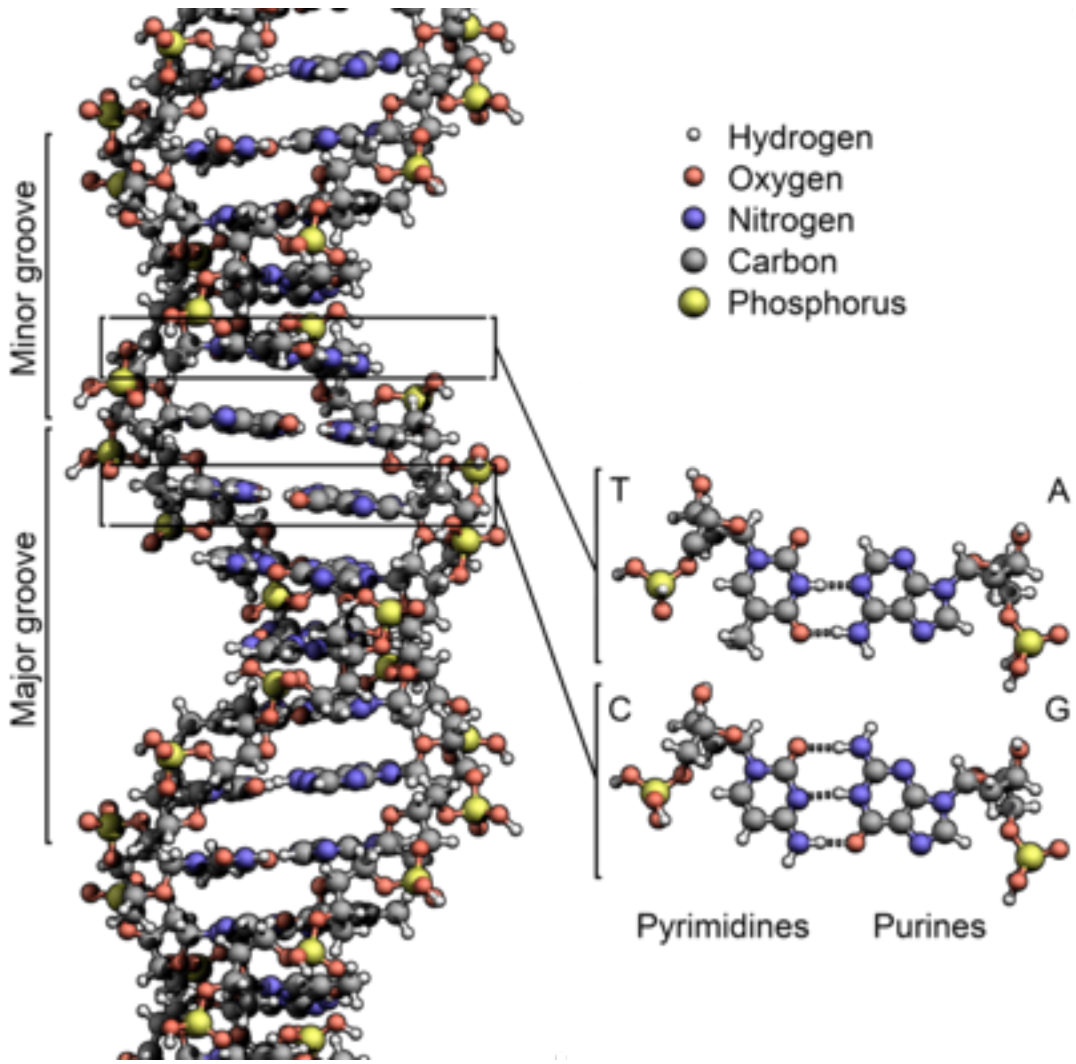
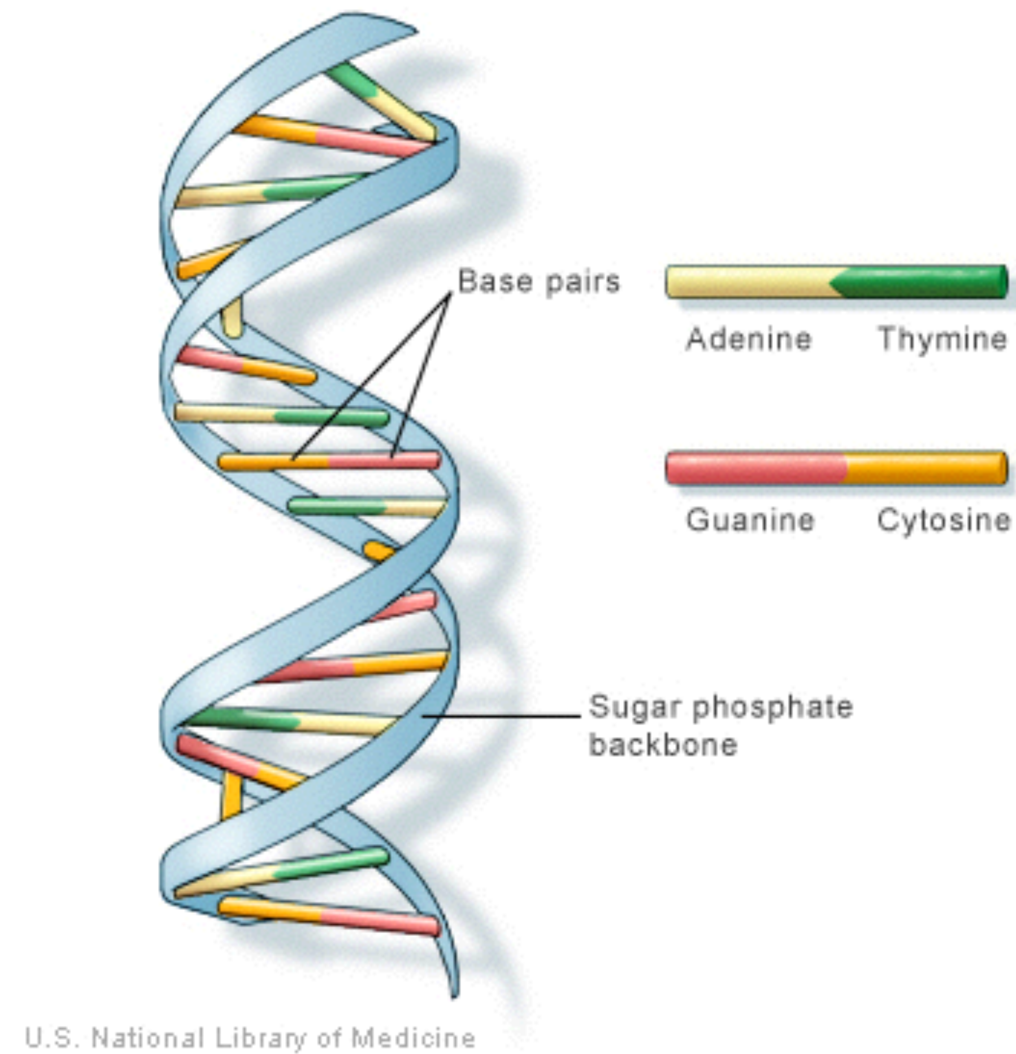
DNA



source: wiki



http://ghr.nlm.nih.gov/handbook/basics/dna

- DNA contour length in bacteria: ~1.5mm
- Length of DNA in nucleus of mammals: ~ 2m

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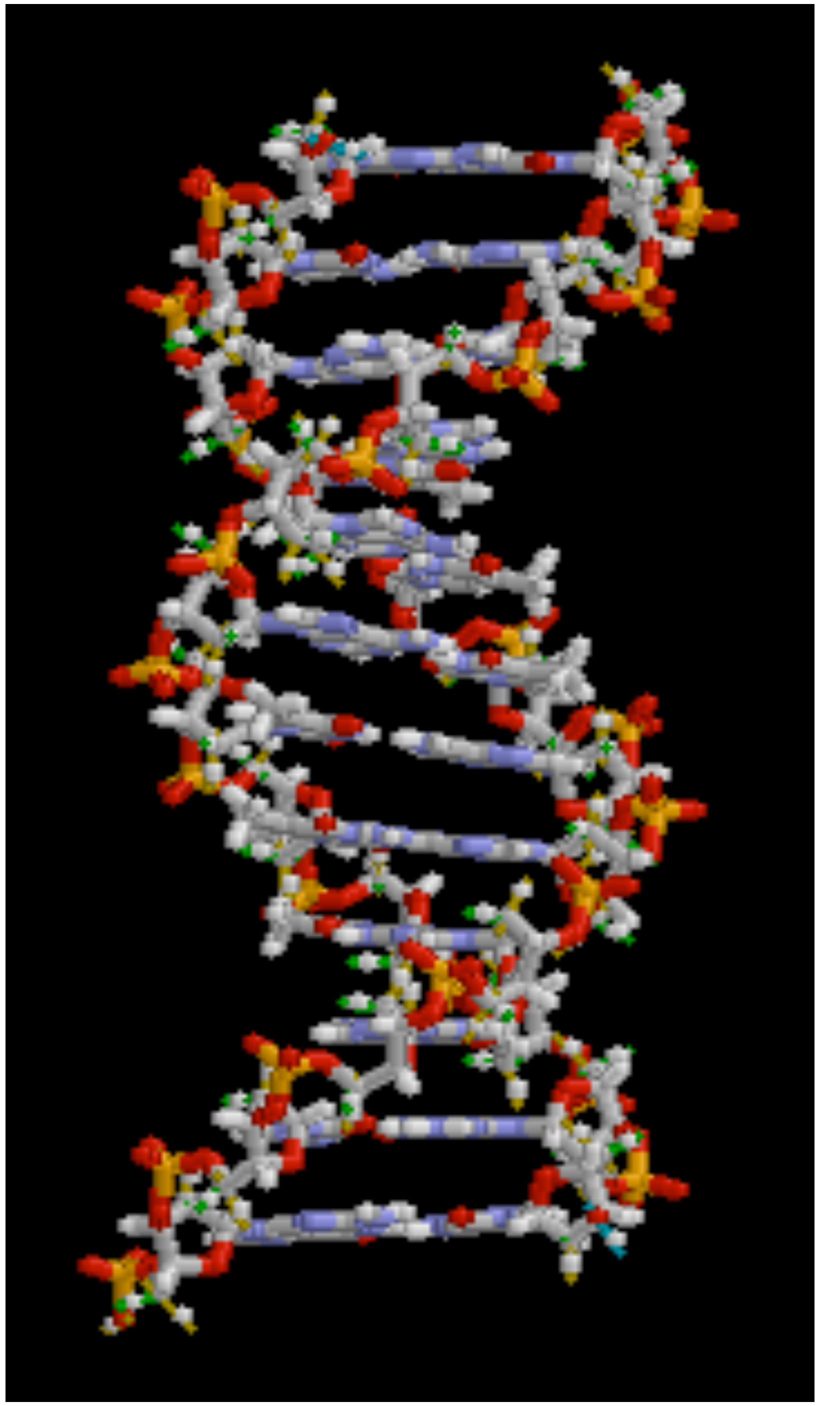
DNA = biopolymer pair



~ 3m per cell

~ 10^14 cells/human

> max. distance between Earth and Pluto (~50 AU = 7.5 x 10^12 m)

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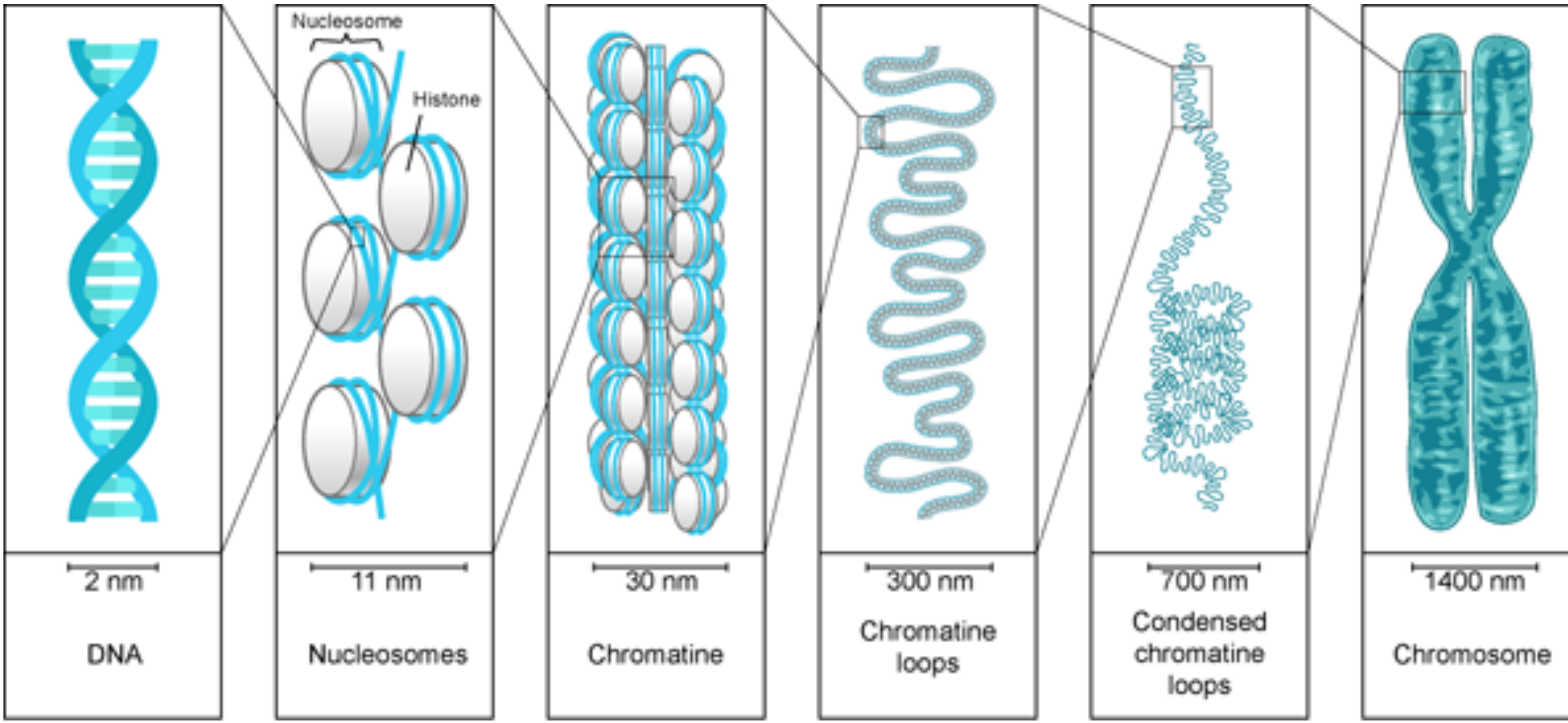
DNA packaging in eukaryotes



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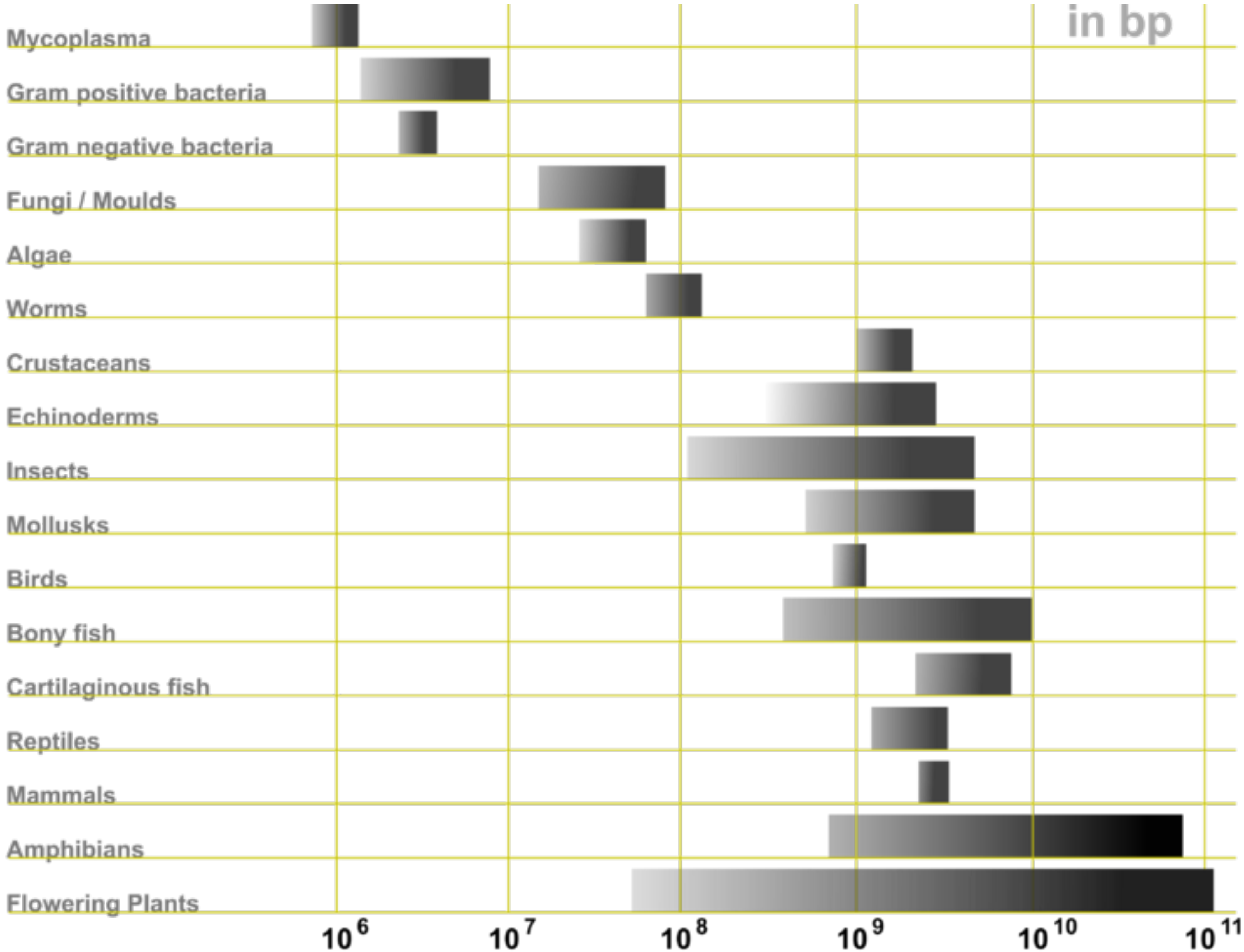

source: wiki

mass $1\text{pg} = 978\text{Mb}$

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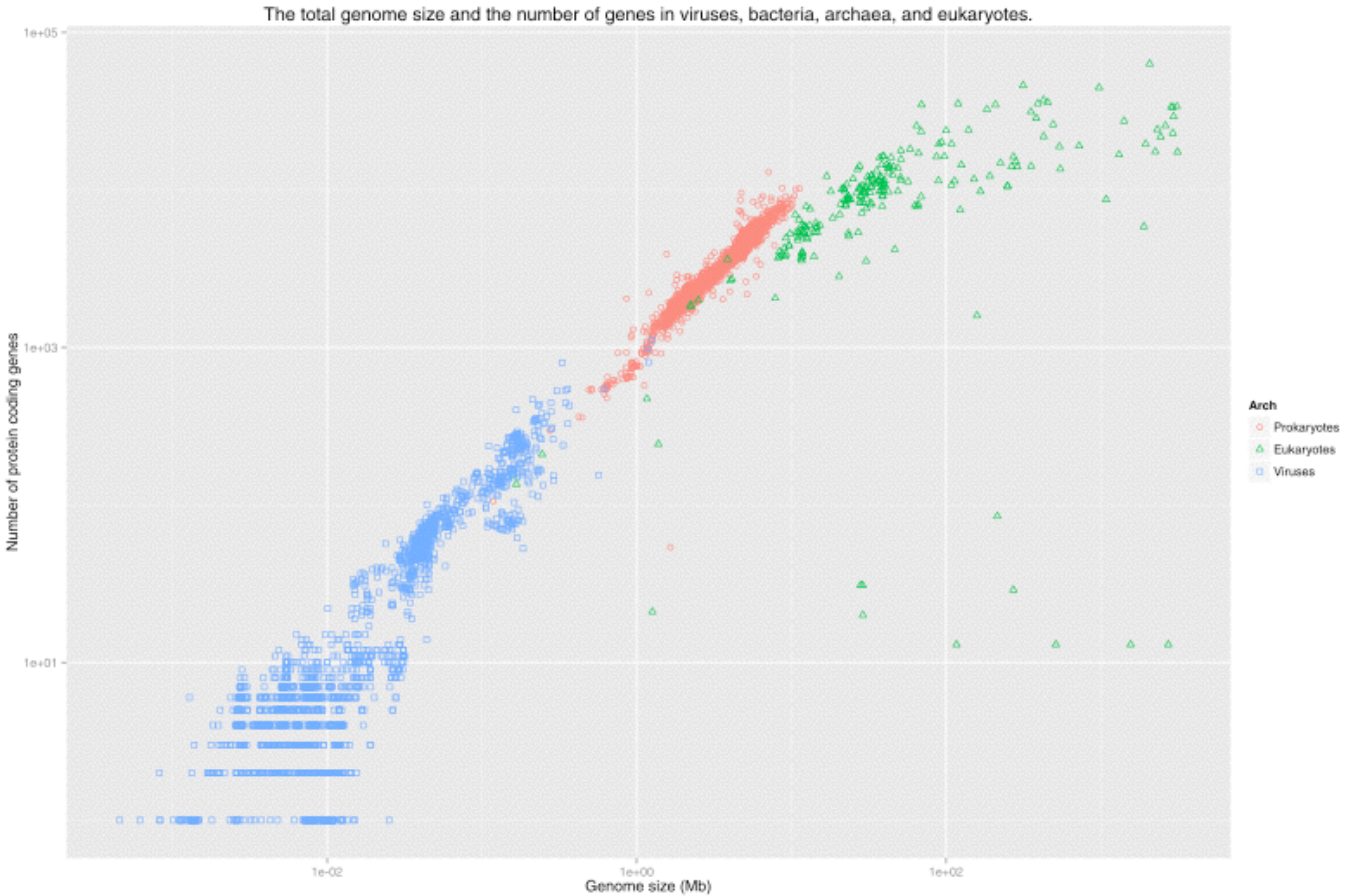

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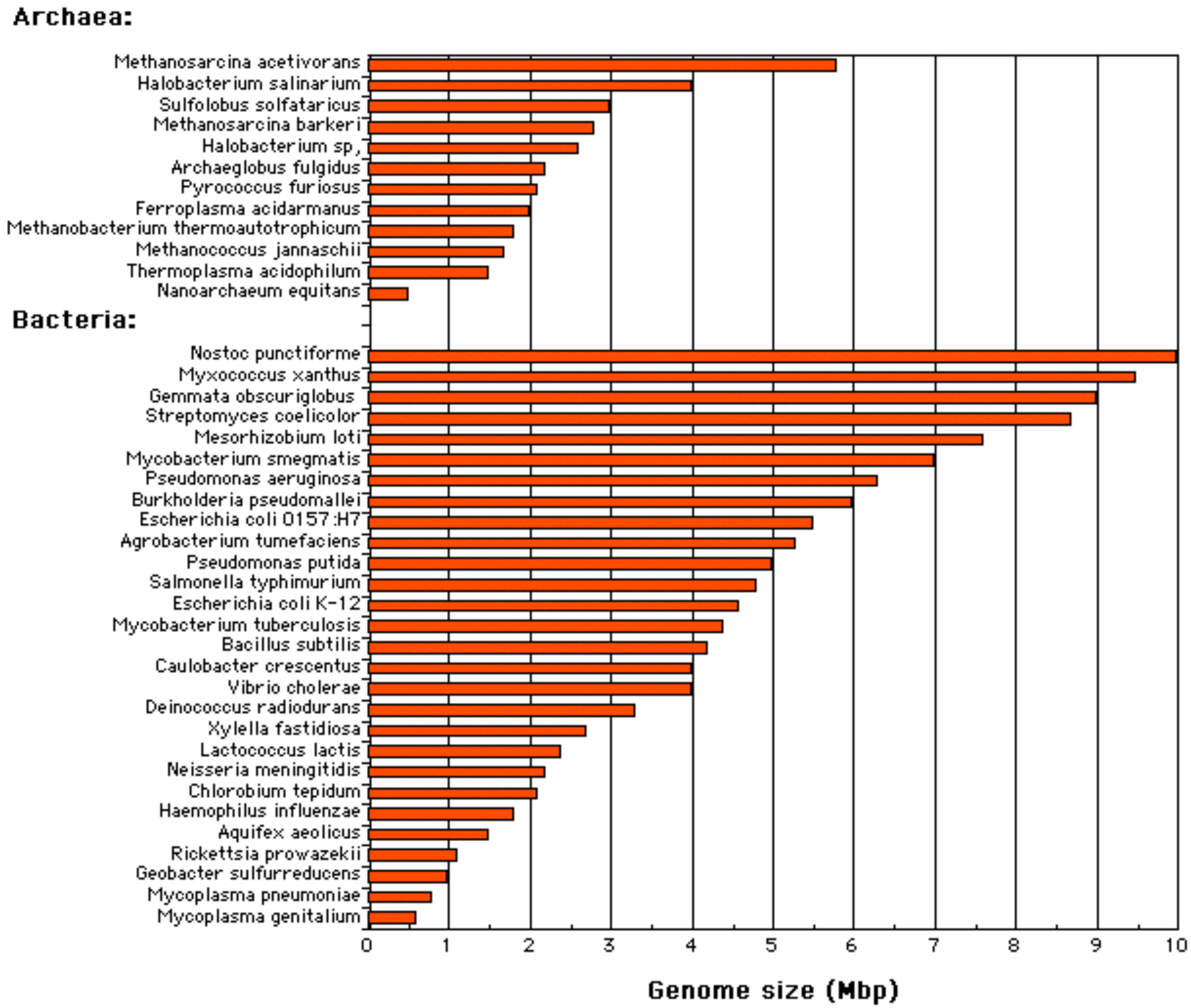
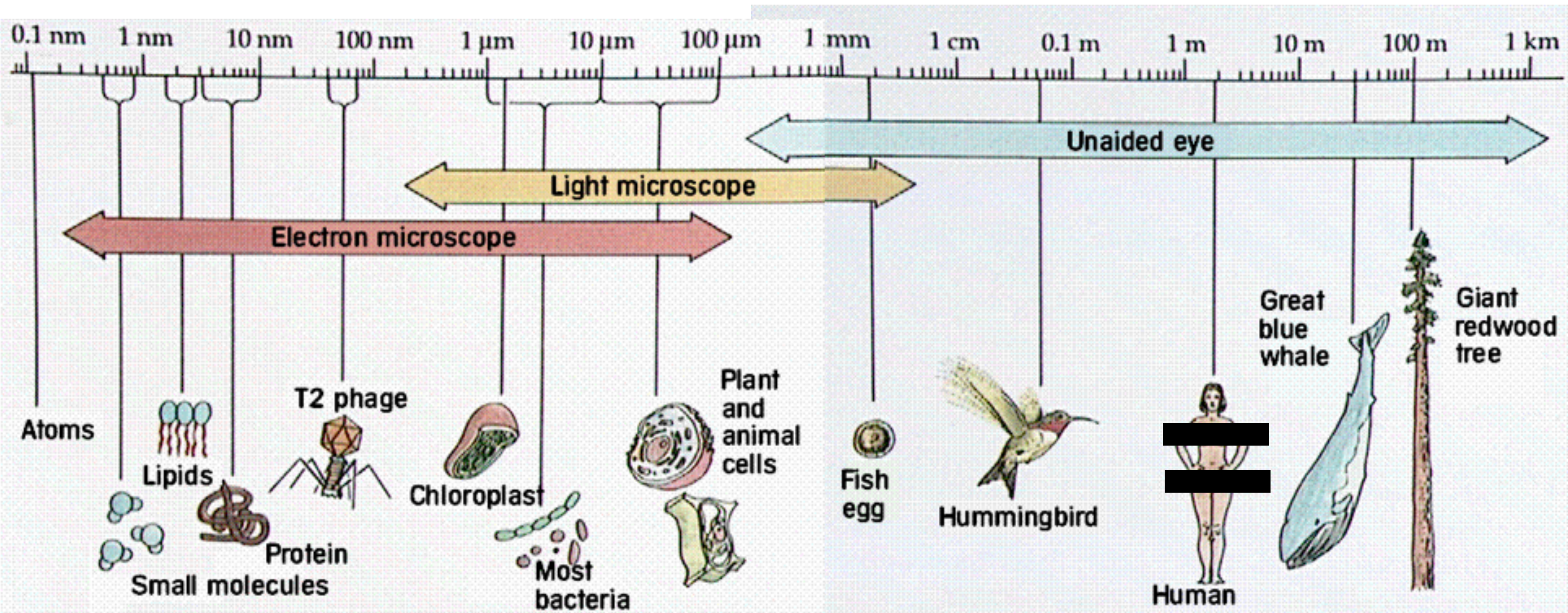
Prokaryotes



http://www.sci.sdsu.edu/~smaloy/MicrobialGenetics/topics/chroms-genes-prots/genomes.html

Typical length scales



http://www2.estrellamountain.edu/faculty/farabee/BIOBK/biobookcell2.html

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Species estimates

- estimated number of eukaryotic species on Earth: 8.7 million (Nature, 2011)
- undiscovered: 86% land spec. & 91%marine spec
- ~ 300,000 plant species
- prokaryotic biomass ~ eukaryotic biomass
- oldest known fossilized prokaryotes from 3.5 billion years ago

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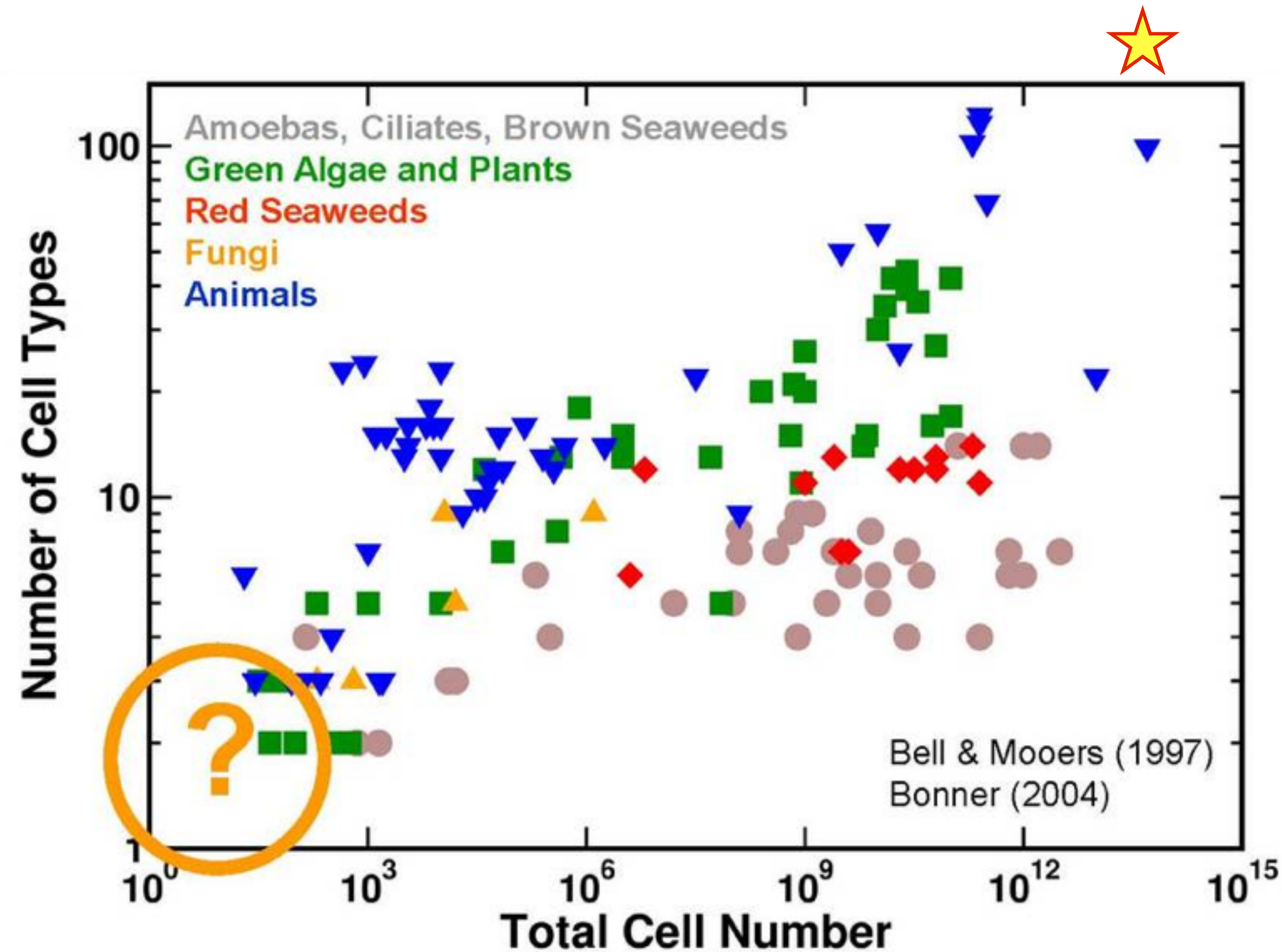
Size-Complexity relation



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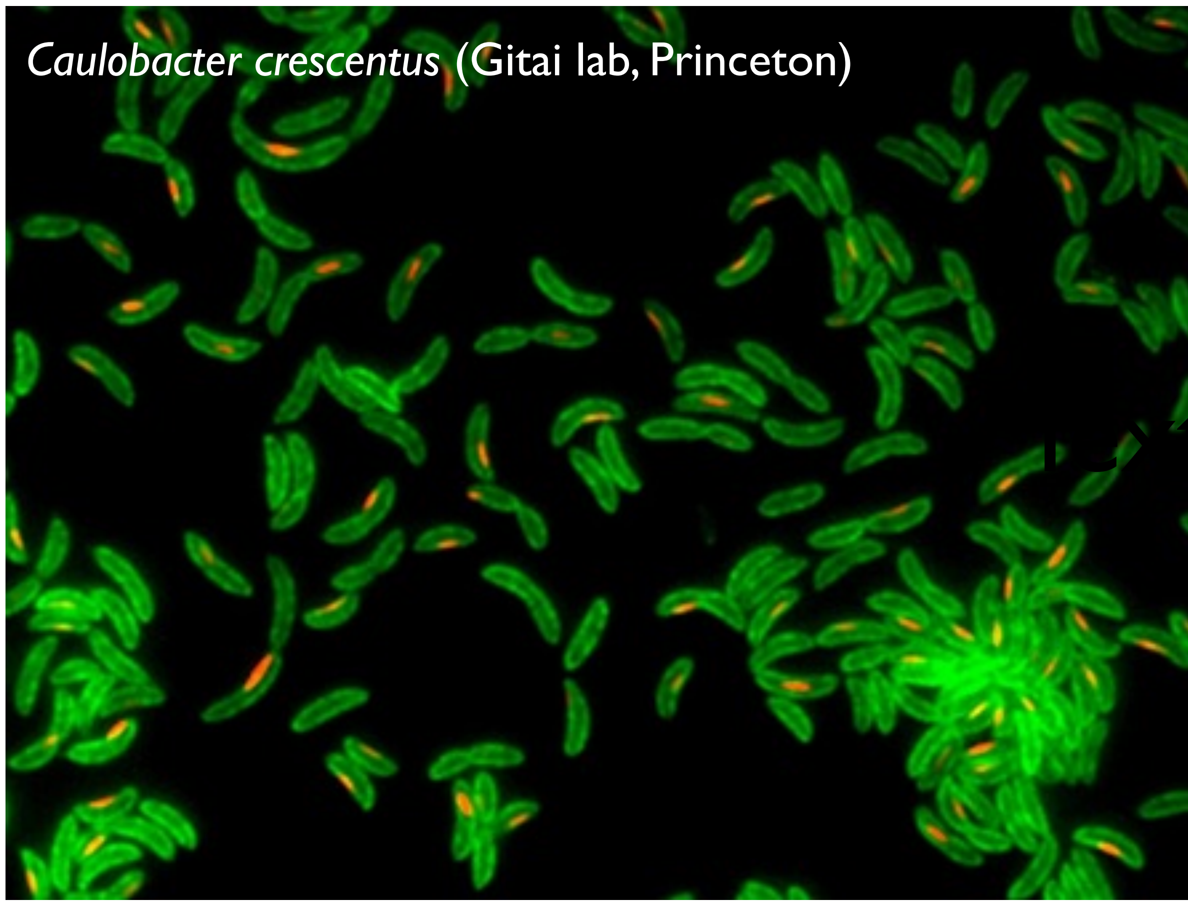
Unicellular organisms

Bacteria



size ~ 1µm
doubling time ~ 2h

Algae

size ~ 10µm
doubling time ~ 5-8h

Amoeba

size ~ 1mm
doubling time ~ 1d

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evolution from

unicellular to multicellular ?

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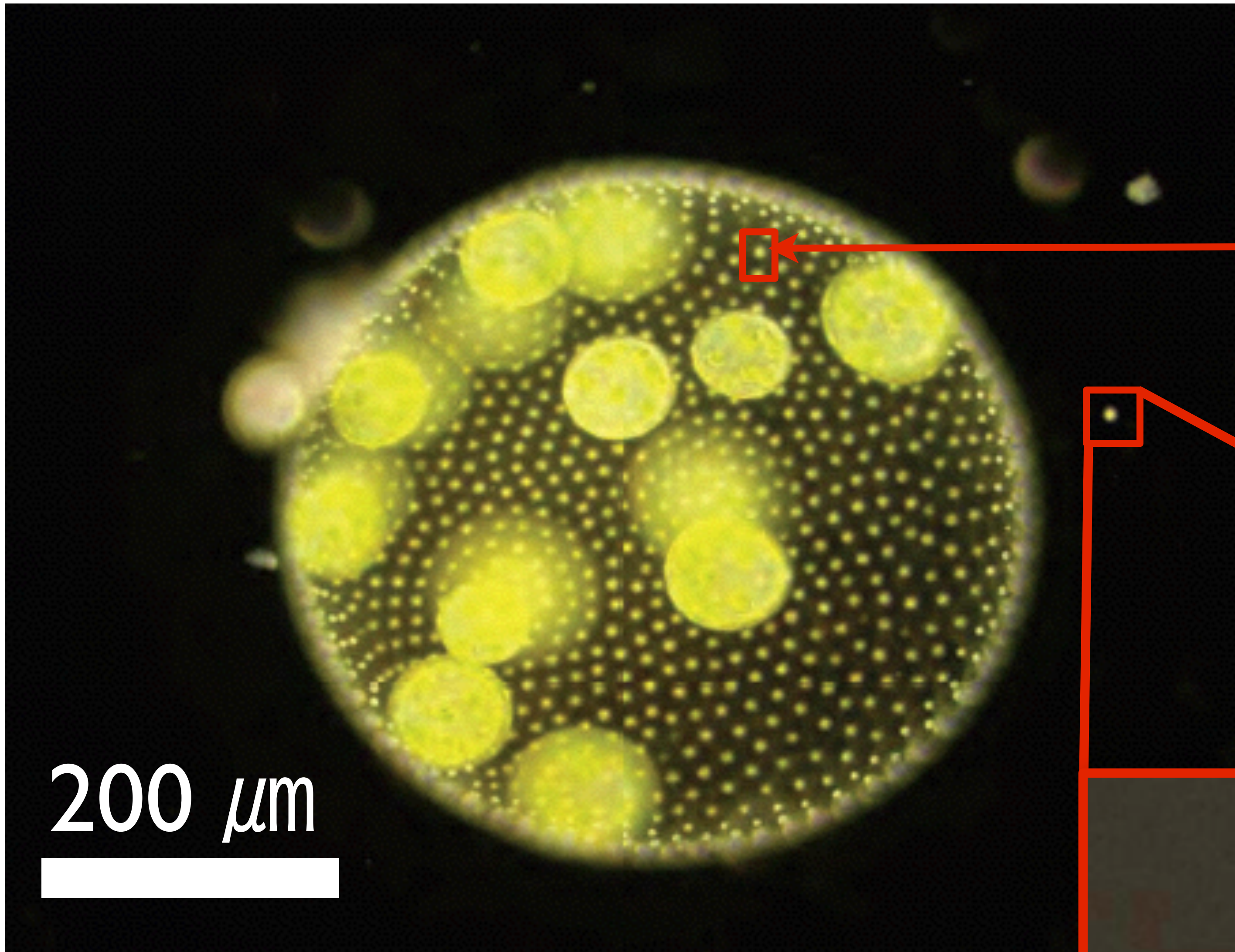
Volvox carteri



Chlamydomonas reinhardtii

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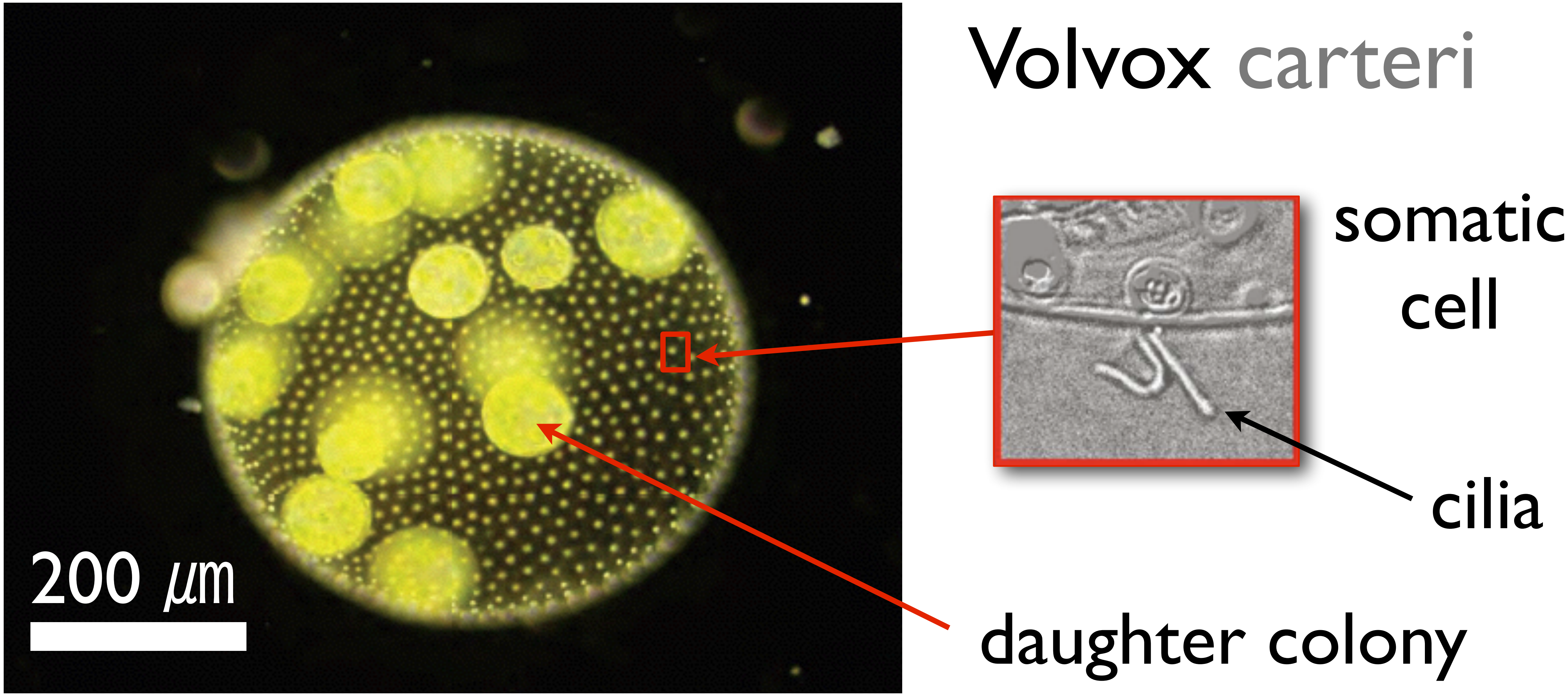
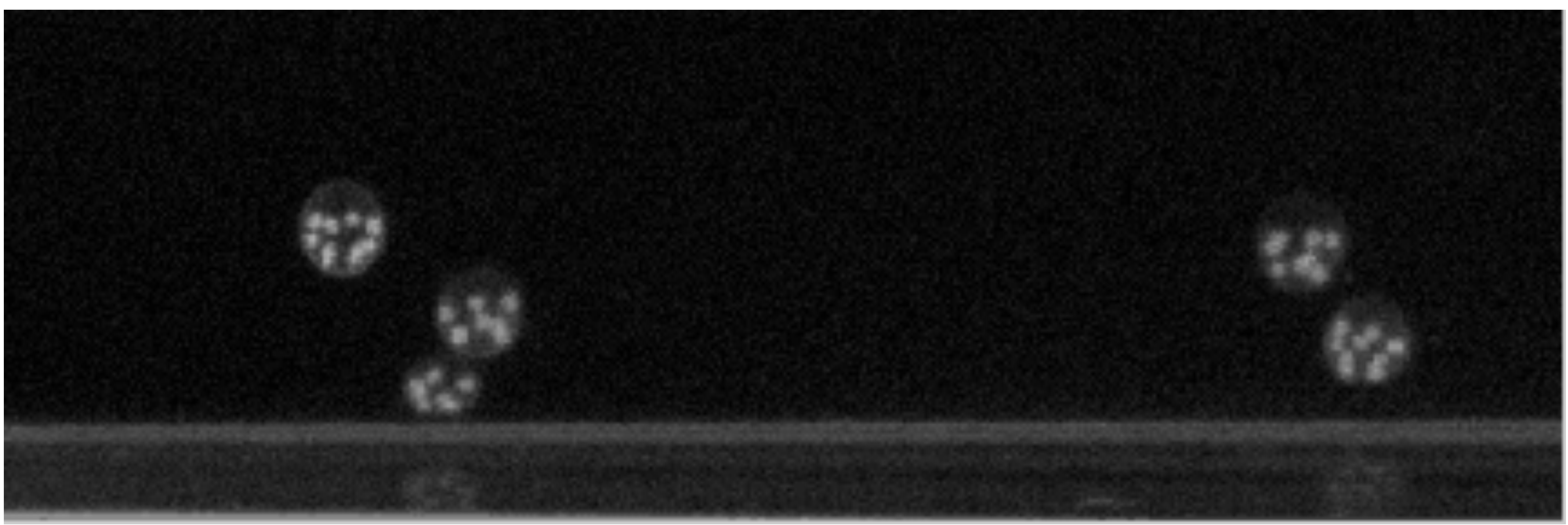
Volvox carteri





Drescher et al (2010) PRL

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how do organisms
achieve locomotion ?

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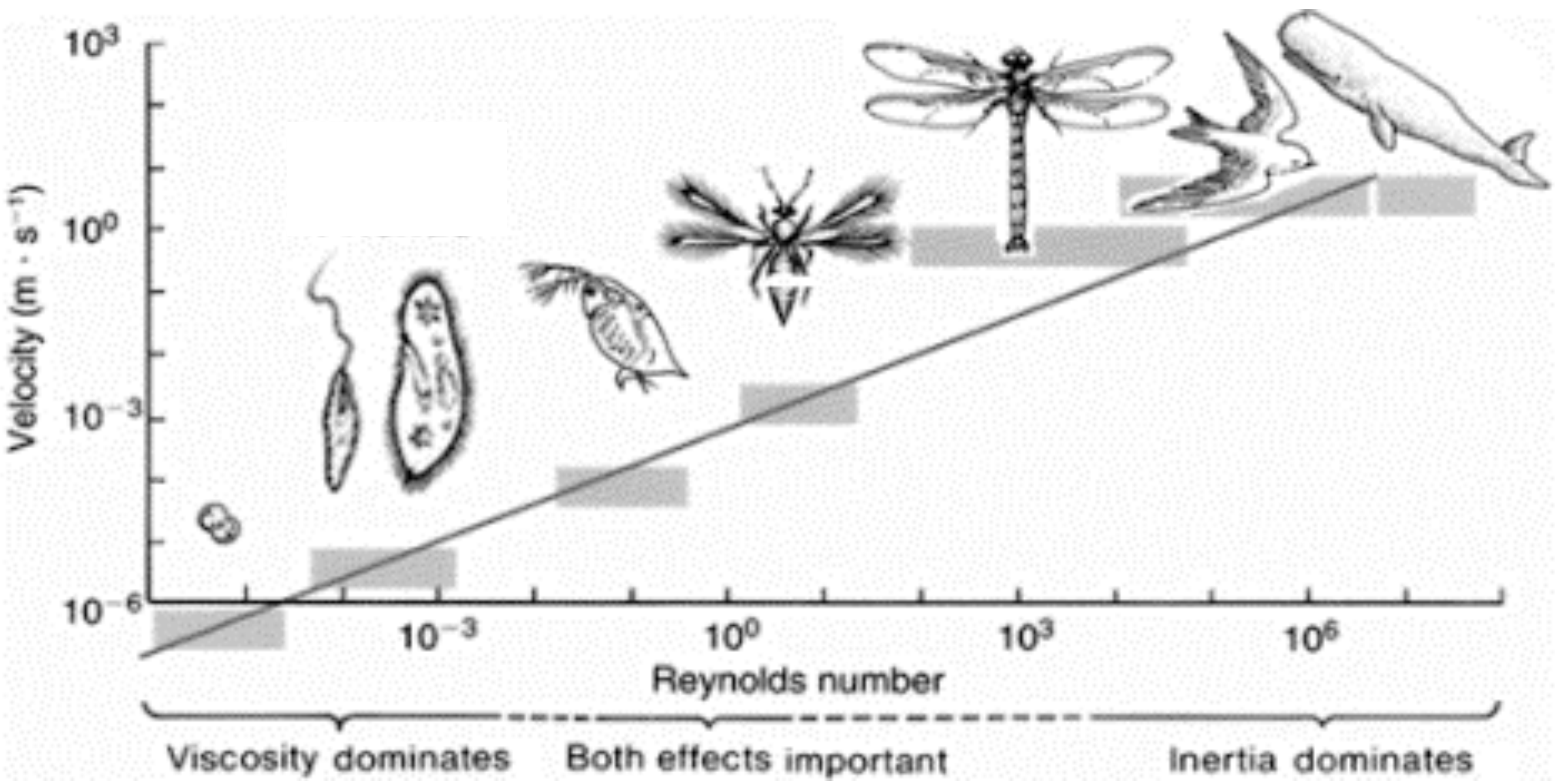
Reynolds numbers

$$Re = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$



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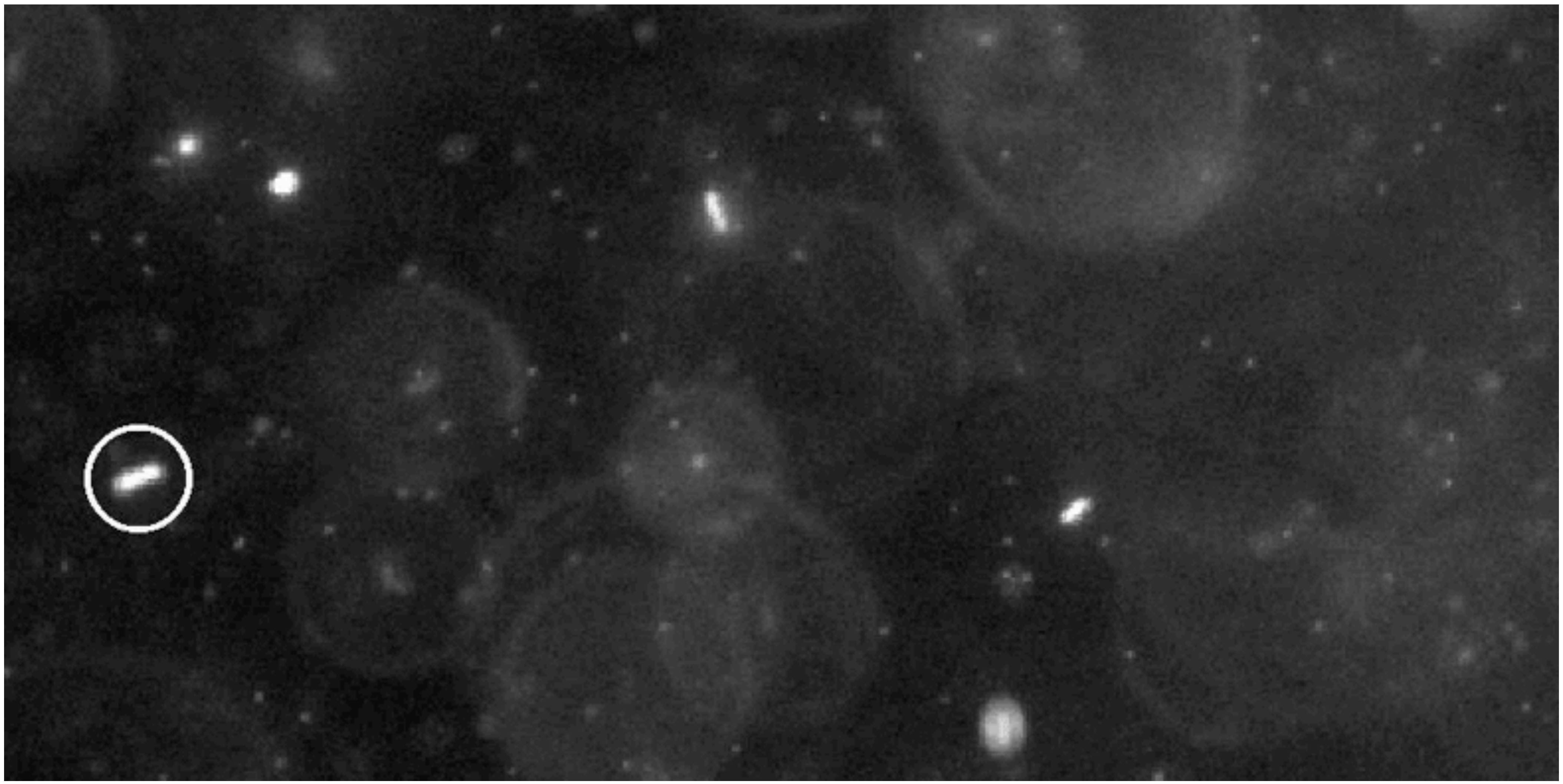
E.coli (non-tumbling HCB 437)



Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

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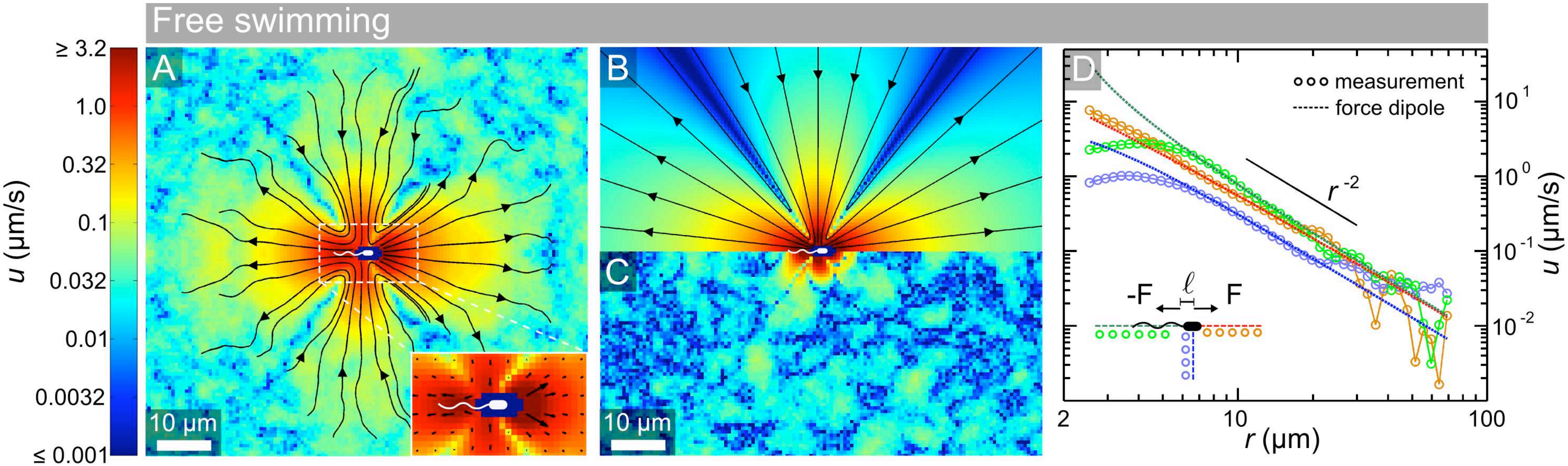
E.coli (non-tumbling HCB 437)



$$\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$V_0 = 22 \pm 5 \ \mu\text{m/s}$$

$$\ell = 1.9 \ \mu\text{m}$$

$$F = 0.42 \ \text{pN}$$

Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

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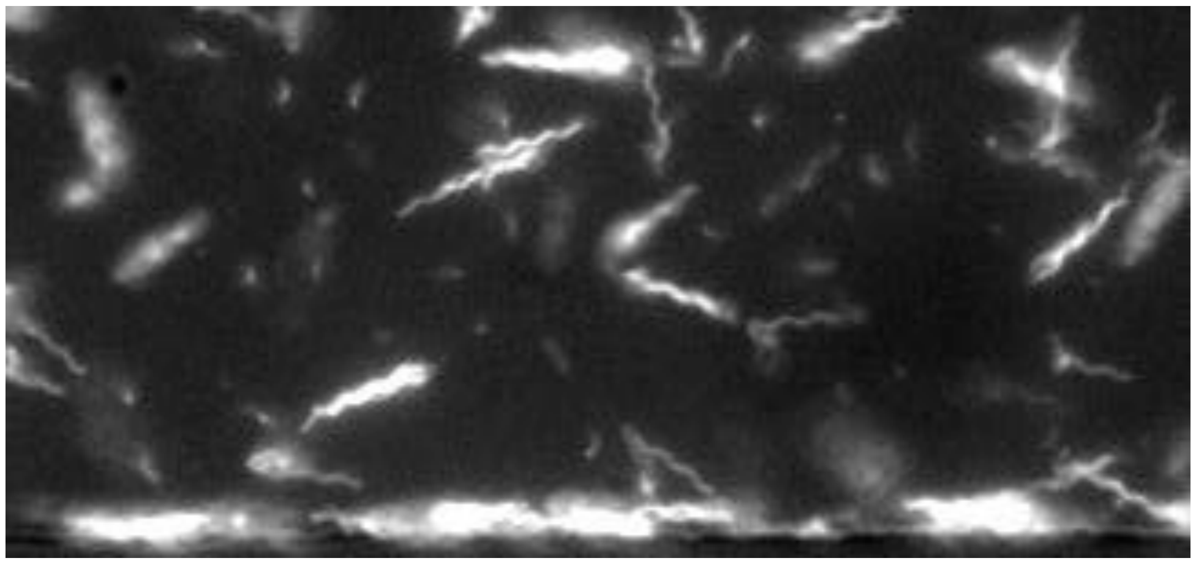
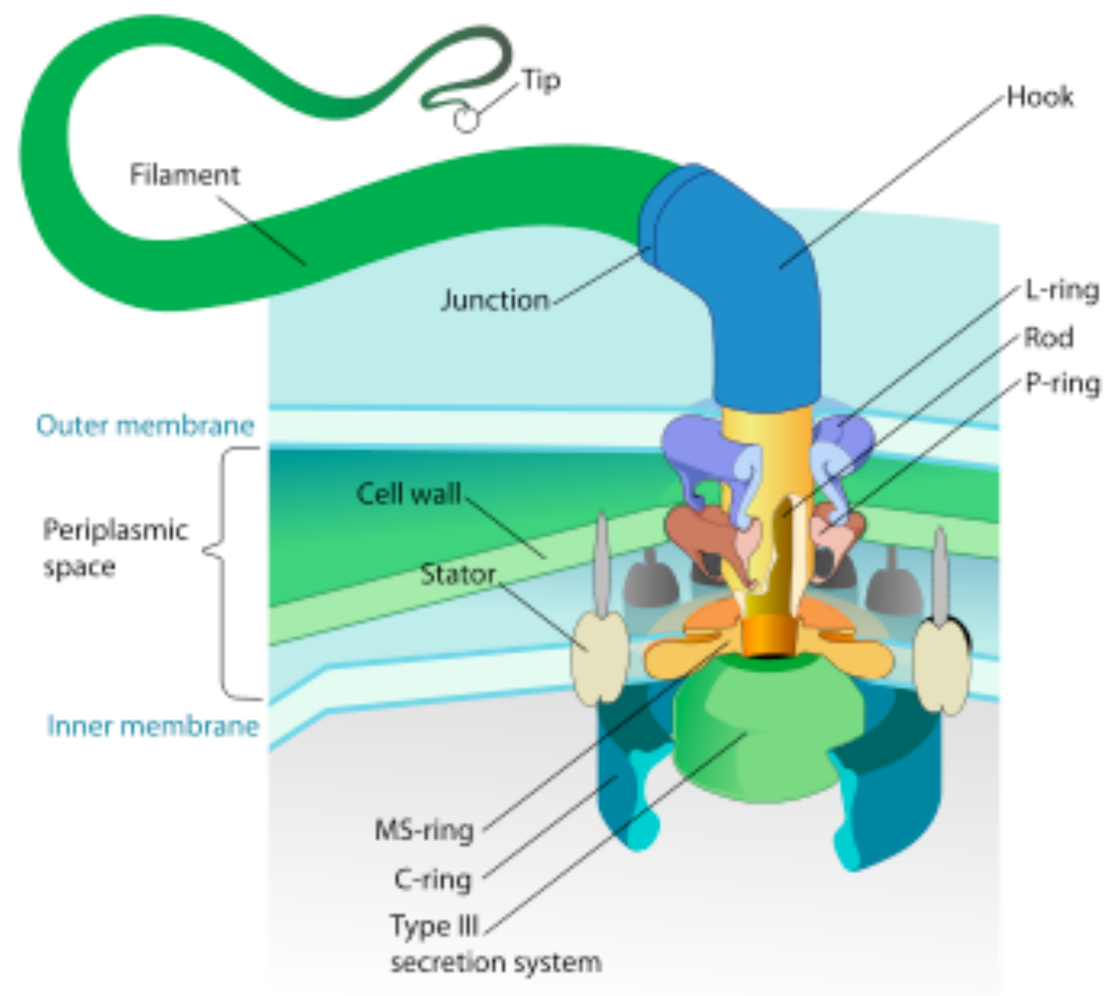
Bacterial motors

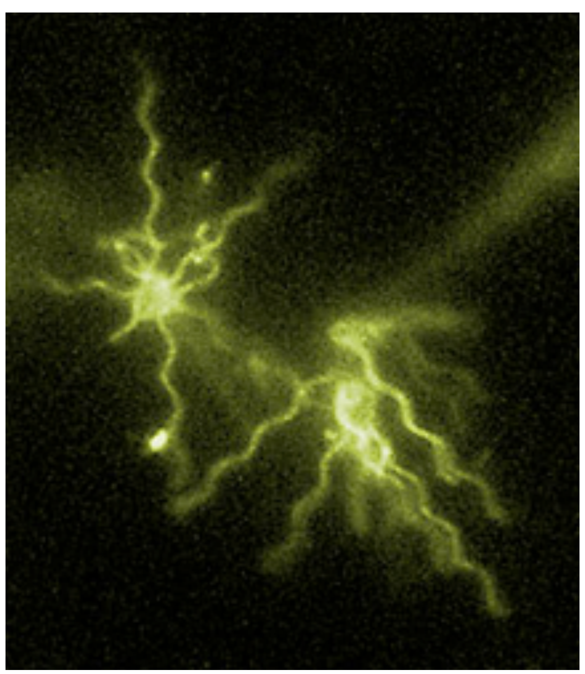
movie: V. Kantsler



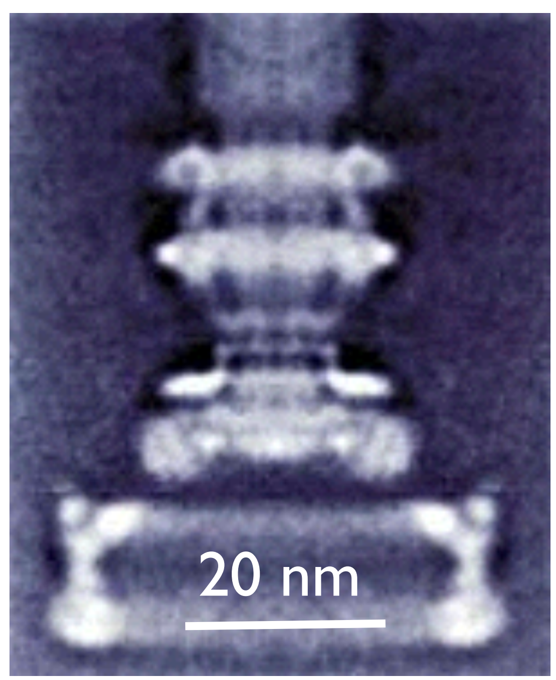
~20 parts



source: wiki



Berg (1999) Physics Today



Chen et al (2011) EMBO Journal

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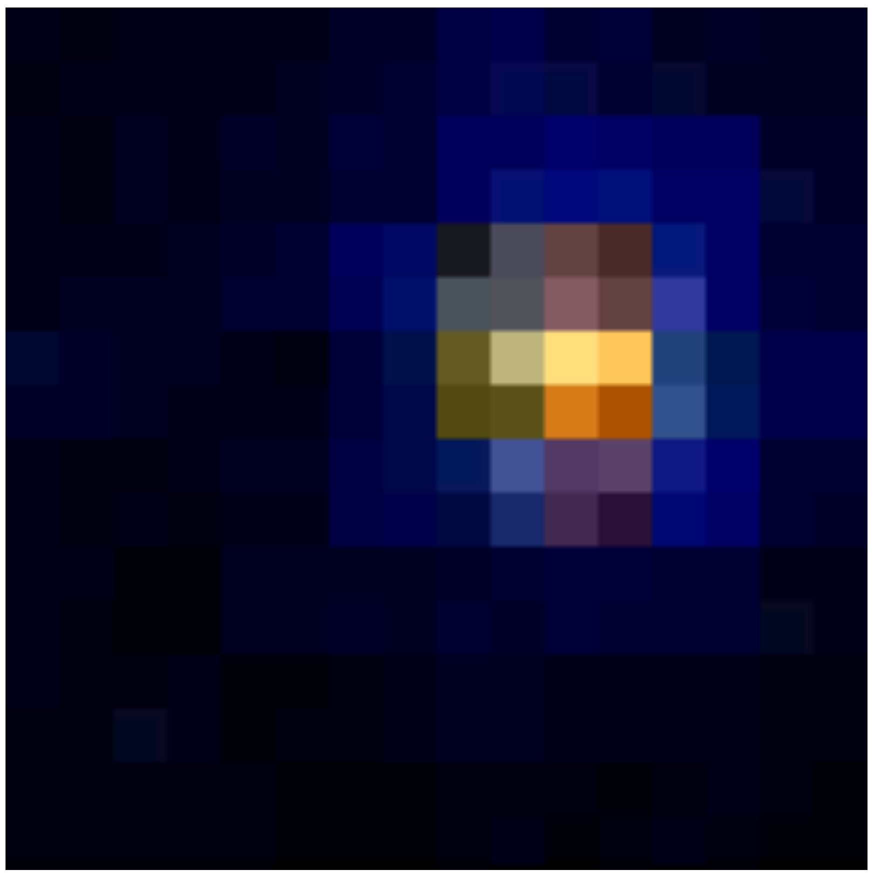
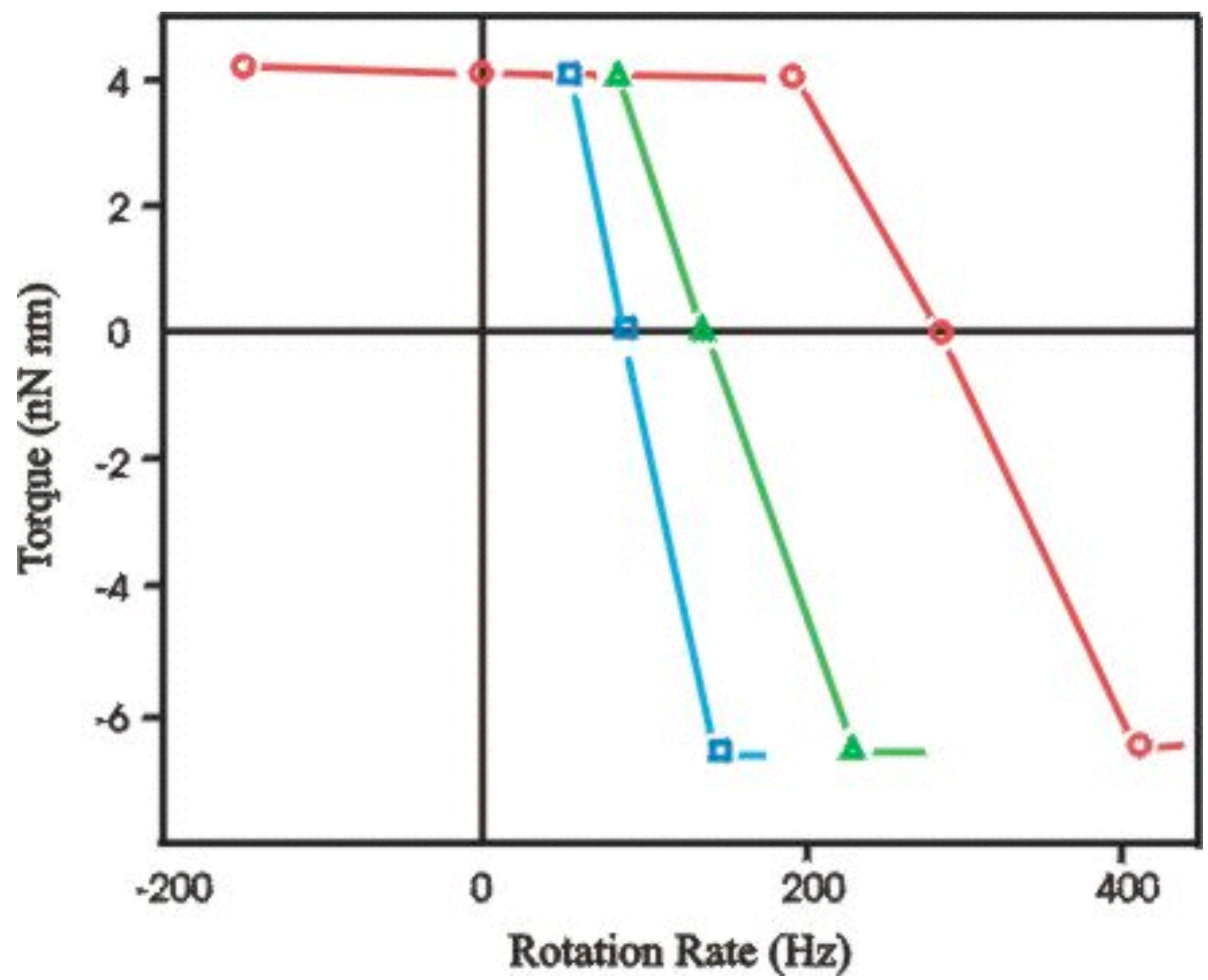
Torque-speed relation





200 nm fluorescent bead attached to a flagellar motor
26 steps per revolution
30x slower than real time
2400 frames per second
position resolution ~5 nm

Berry group, Oxford

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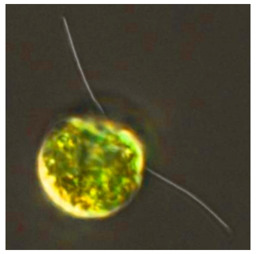
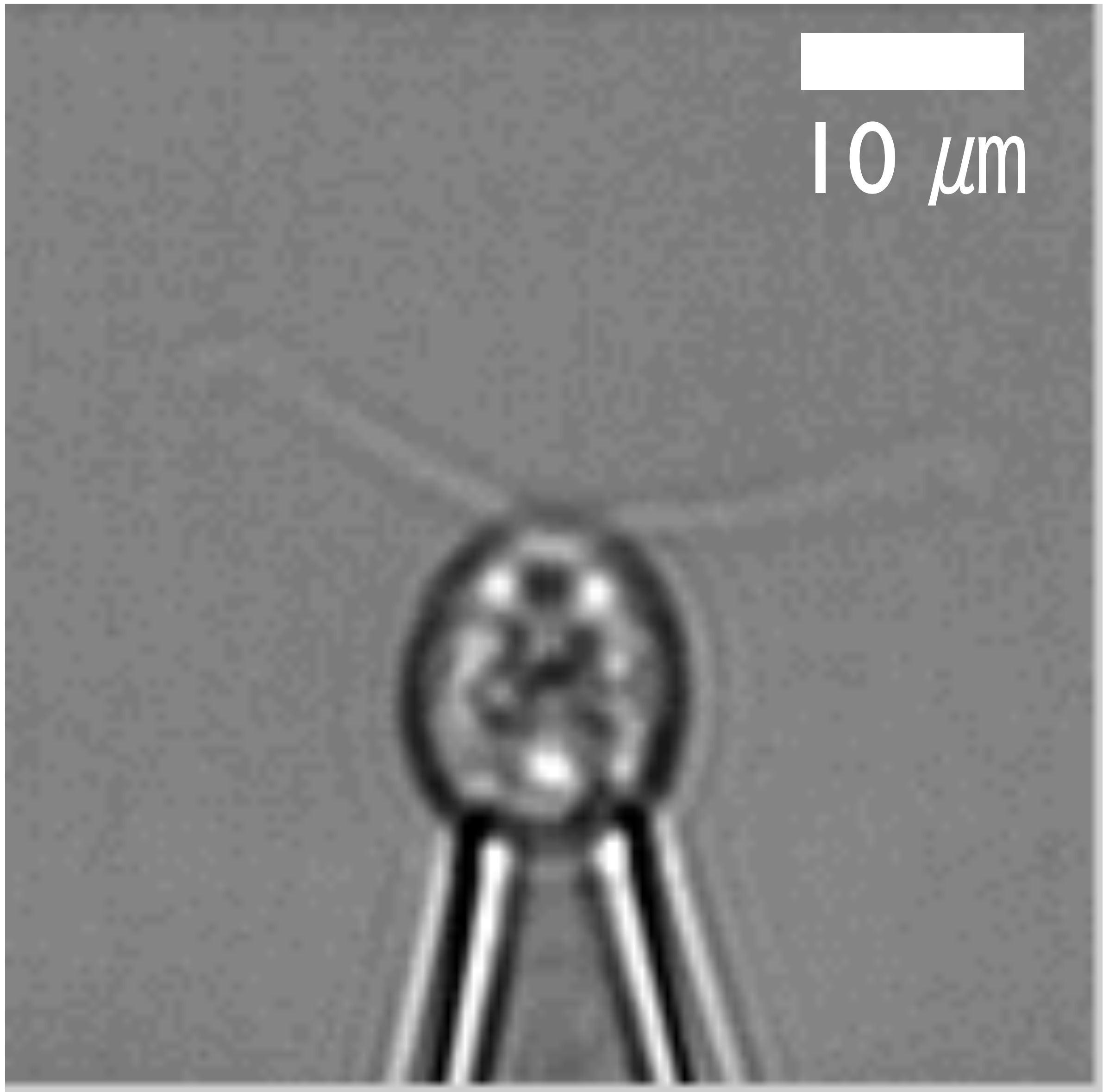
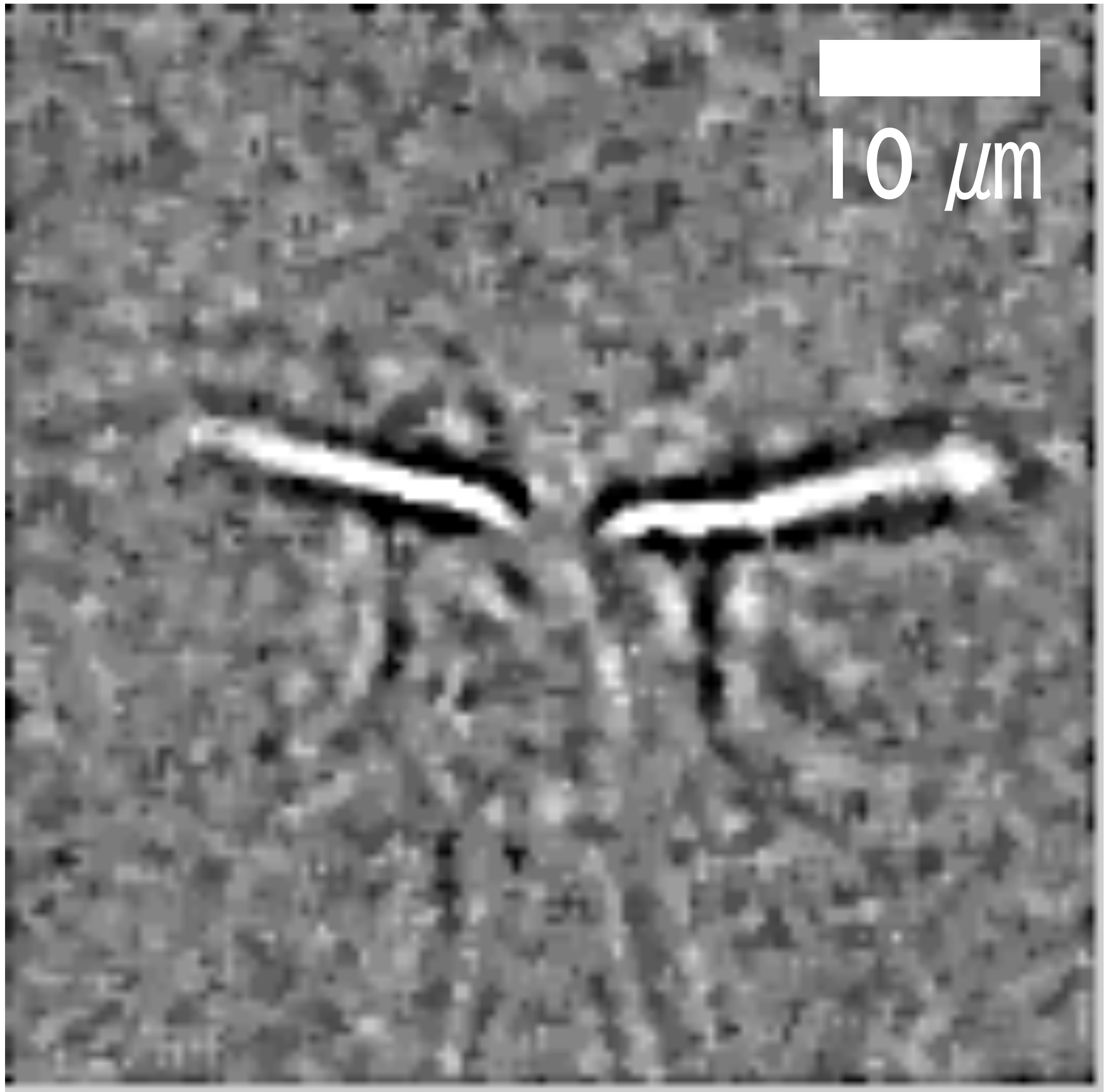
Chlamydomonas alga





~ 50 beats / sec



speed ~100 μm/s

Goldstein et al (2011) PRL

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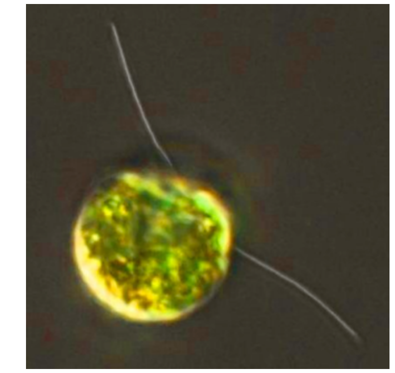
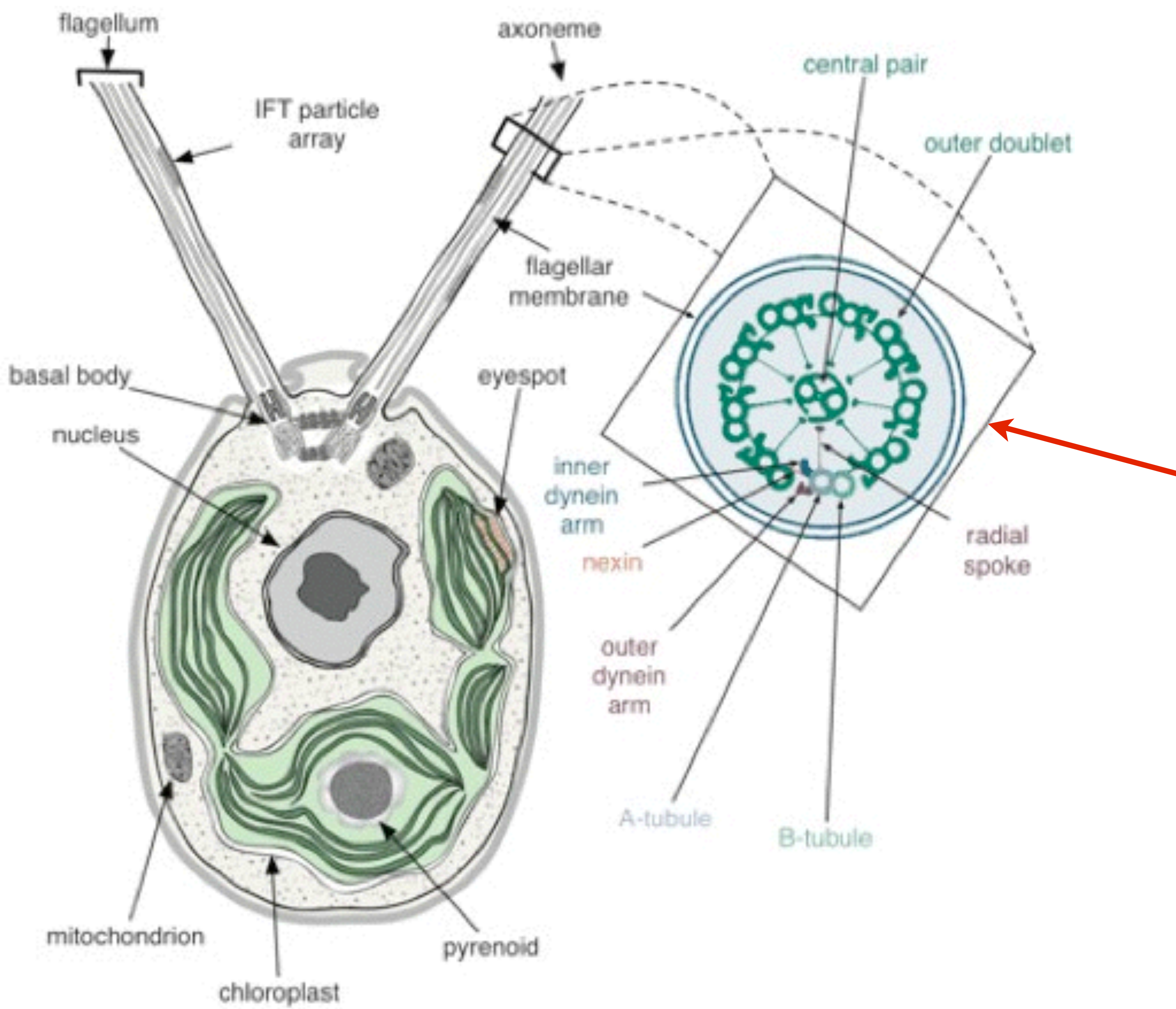
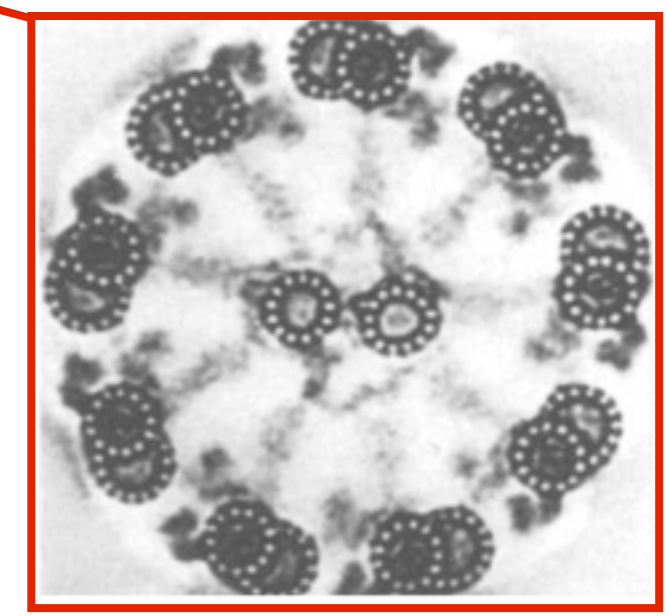
Chlamy







Merchant et al (2007) Science

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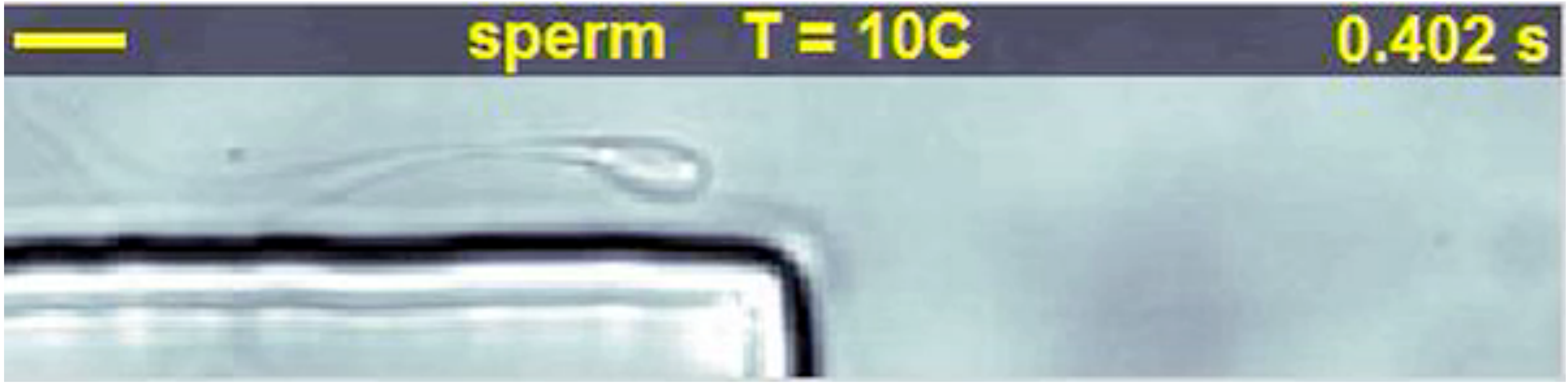
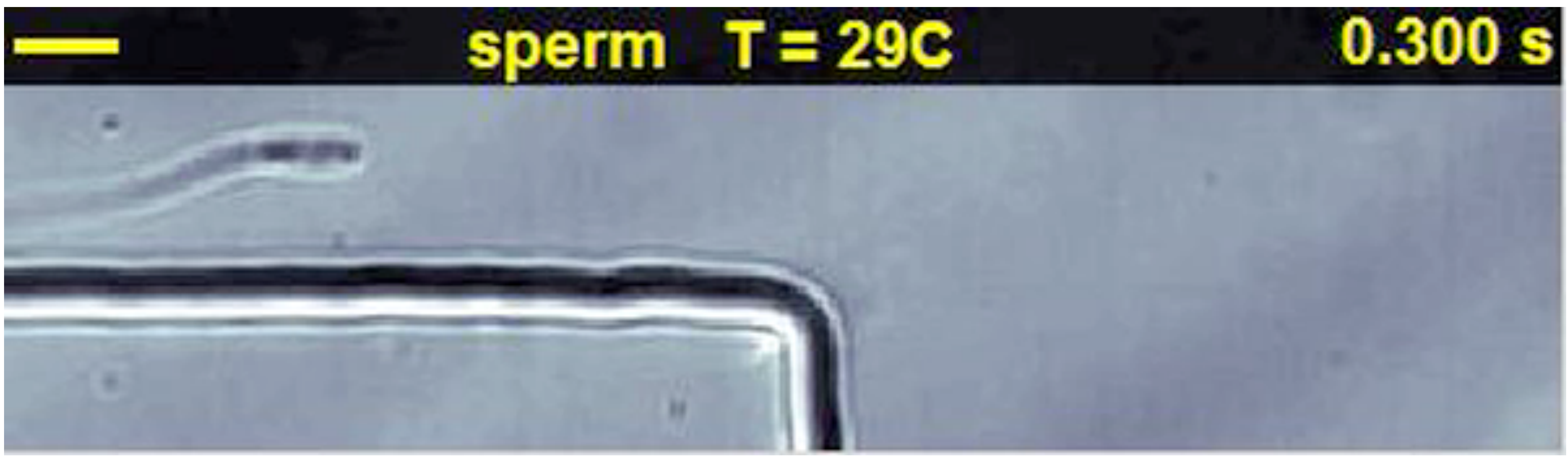
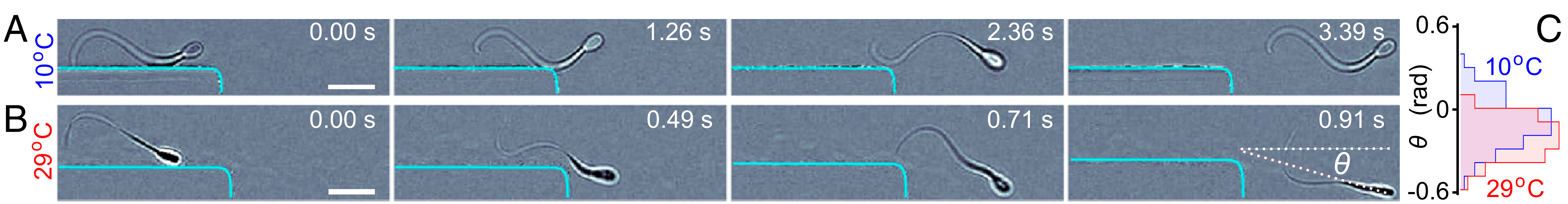
Sperm near surfaces

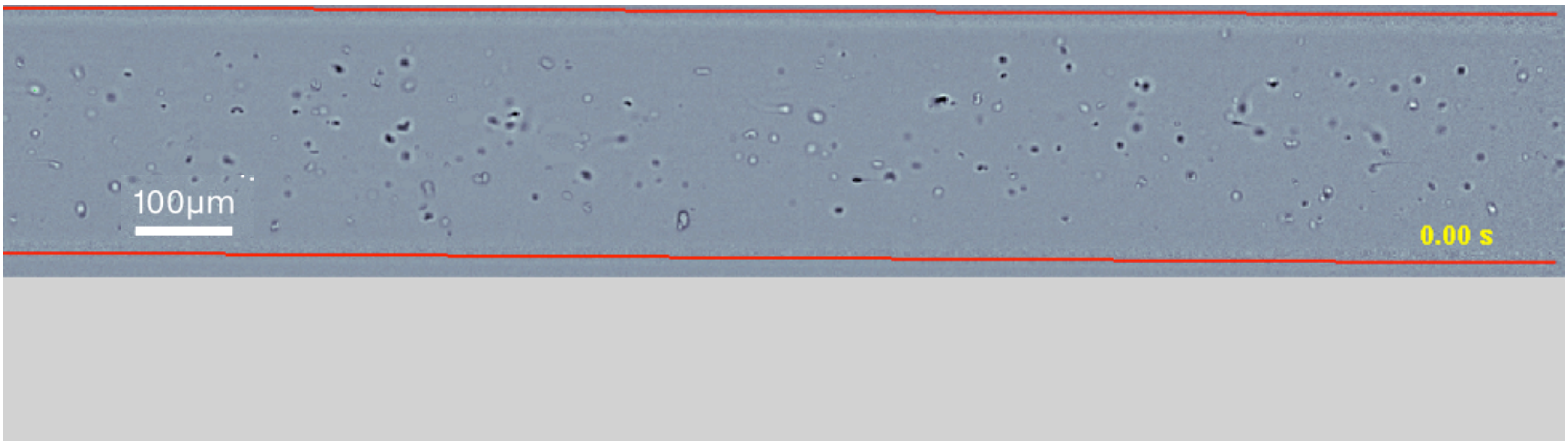






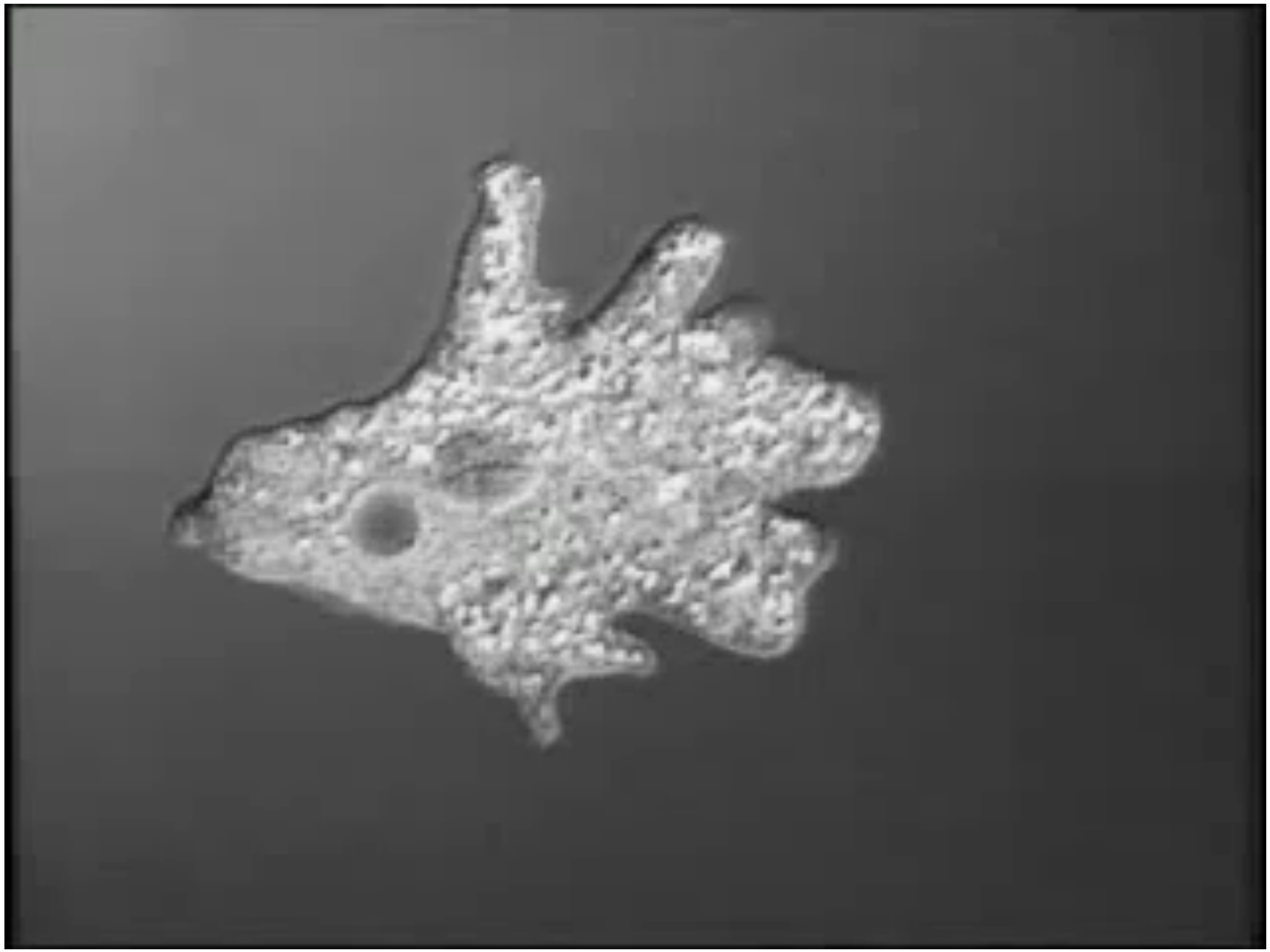
Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

Surface + shear flow

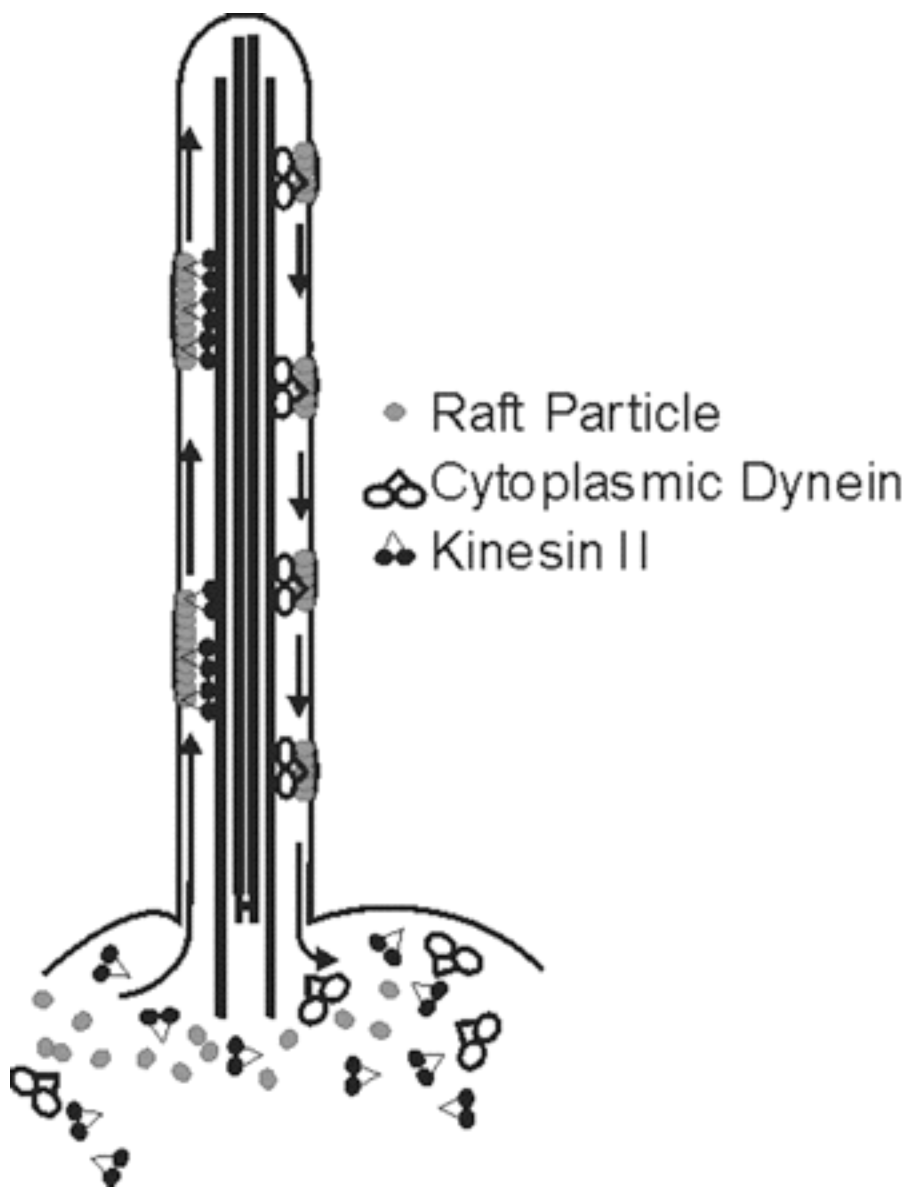


Kantsler et al 2014 (submitted)

Amoeba

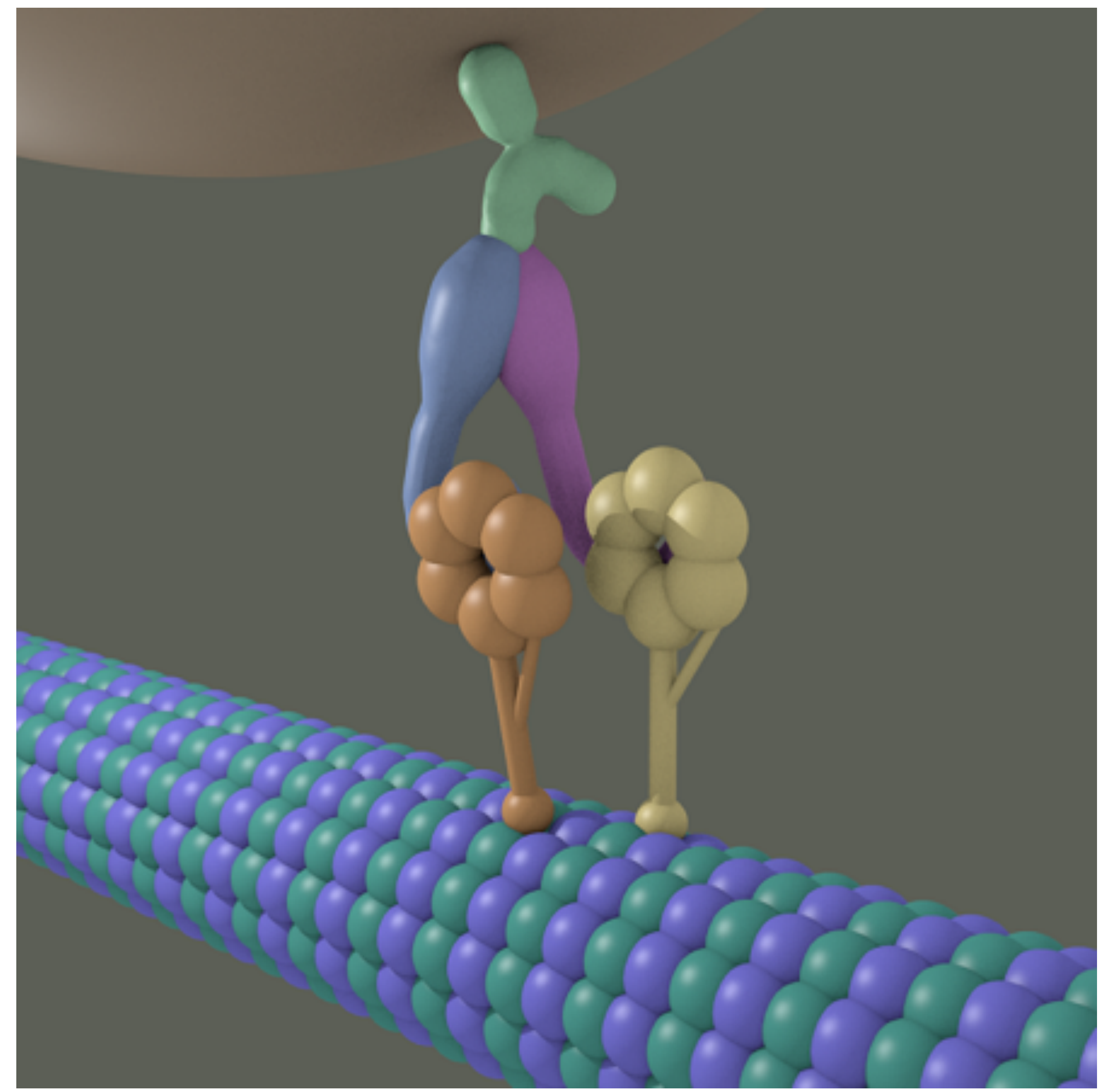


Eukaryotic motors



http://www.plantphysiol.org/content/127/4/1500/F4.expansion.html

Sketch: dynein molecule carrying cargo down a microtubule



Yildiz lab, Berkeley

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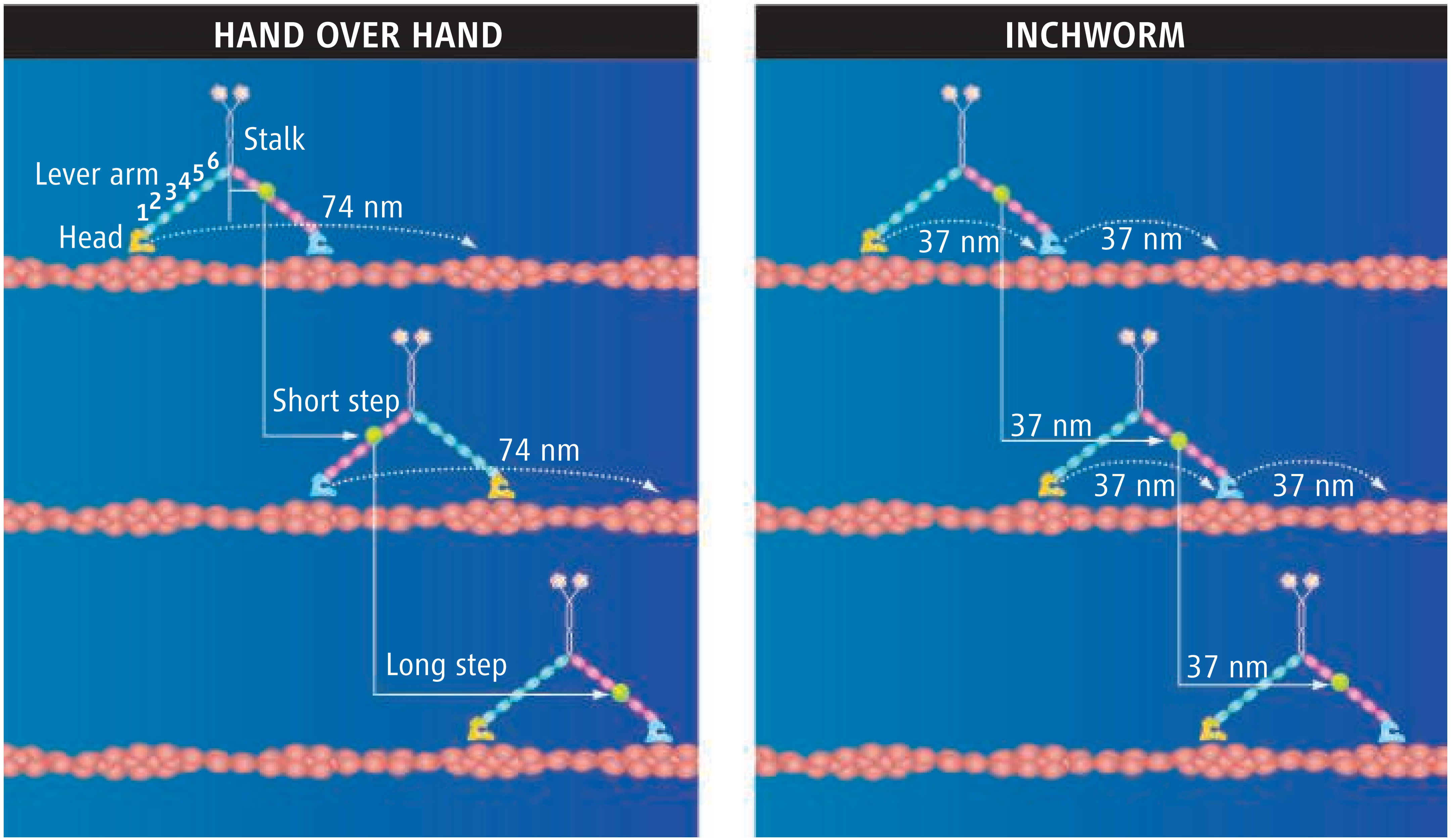
Walking modes



Myosin V: Walking or inchworming? Predicted movement for the heads and a dye molecule label (green dot) on the lever arm in the hand-over-hand model **(left)** and the inchworm model (**right**). The FIONA assay has revealed that myosin V, along with kinesin and myosin VI, walks hand-over-hand.

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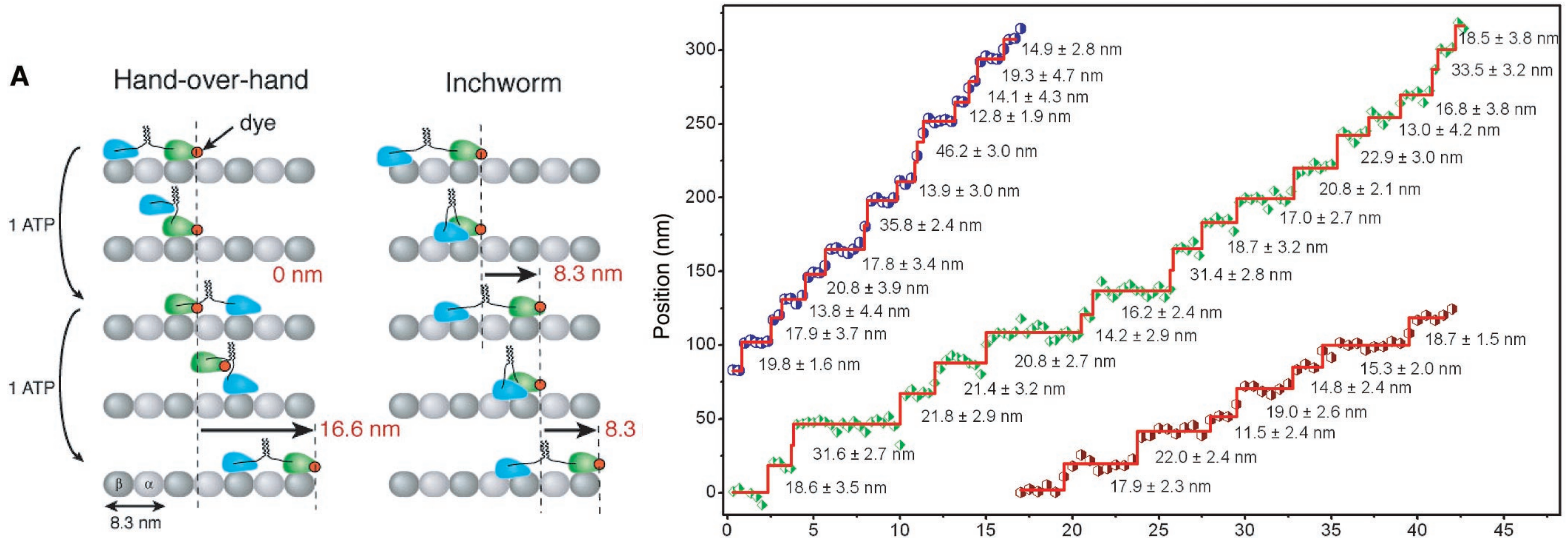
Kinesin walks hand-over-hand



Yildiz et al (2005) Science

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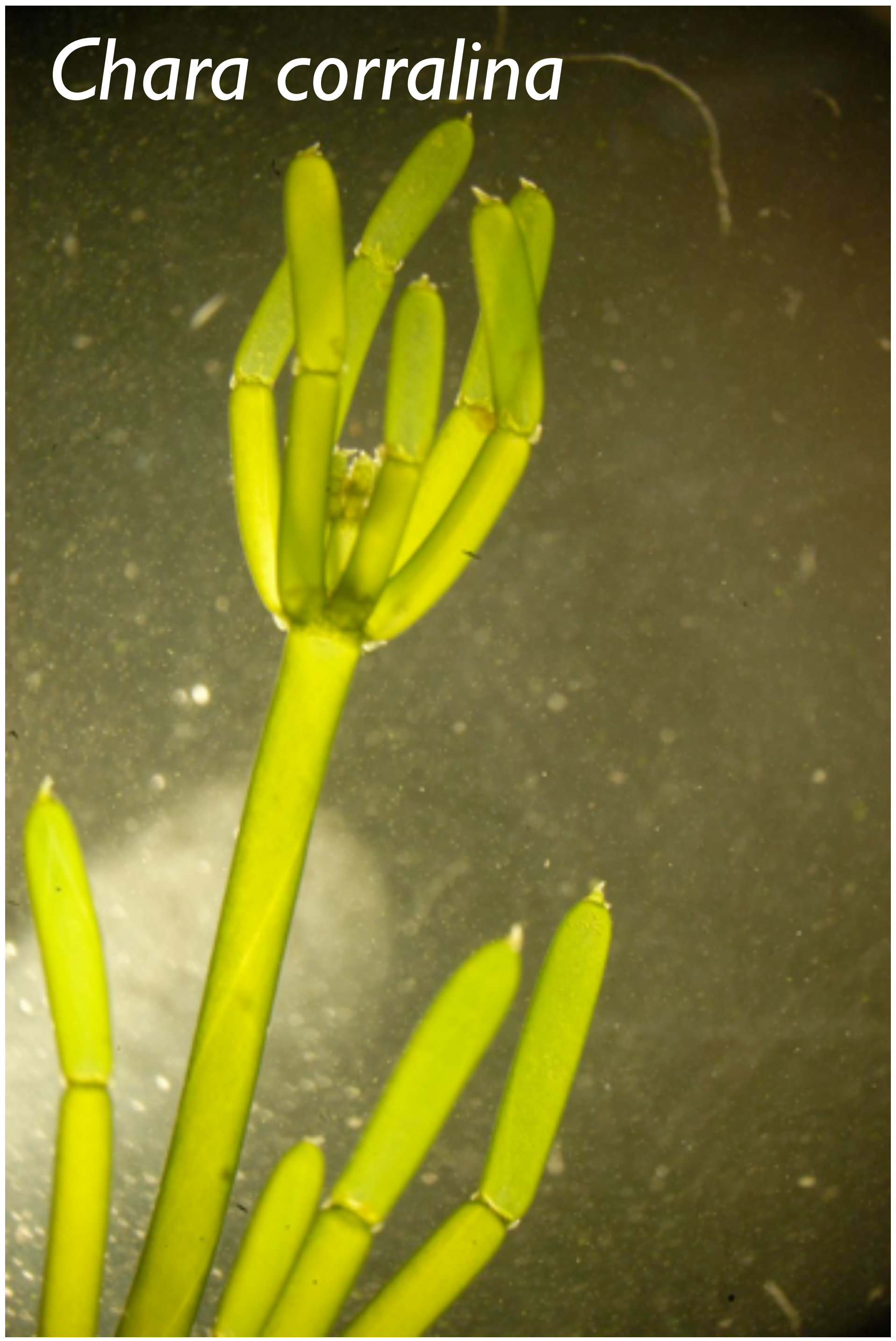
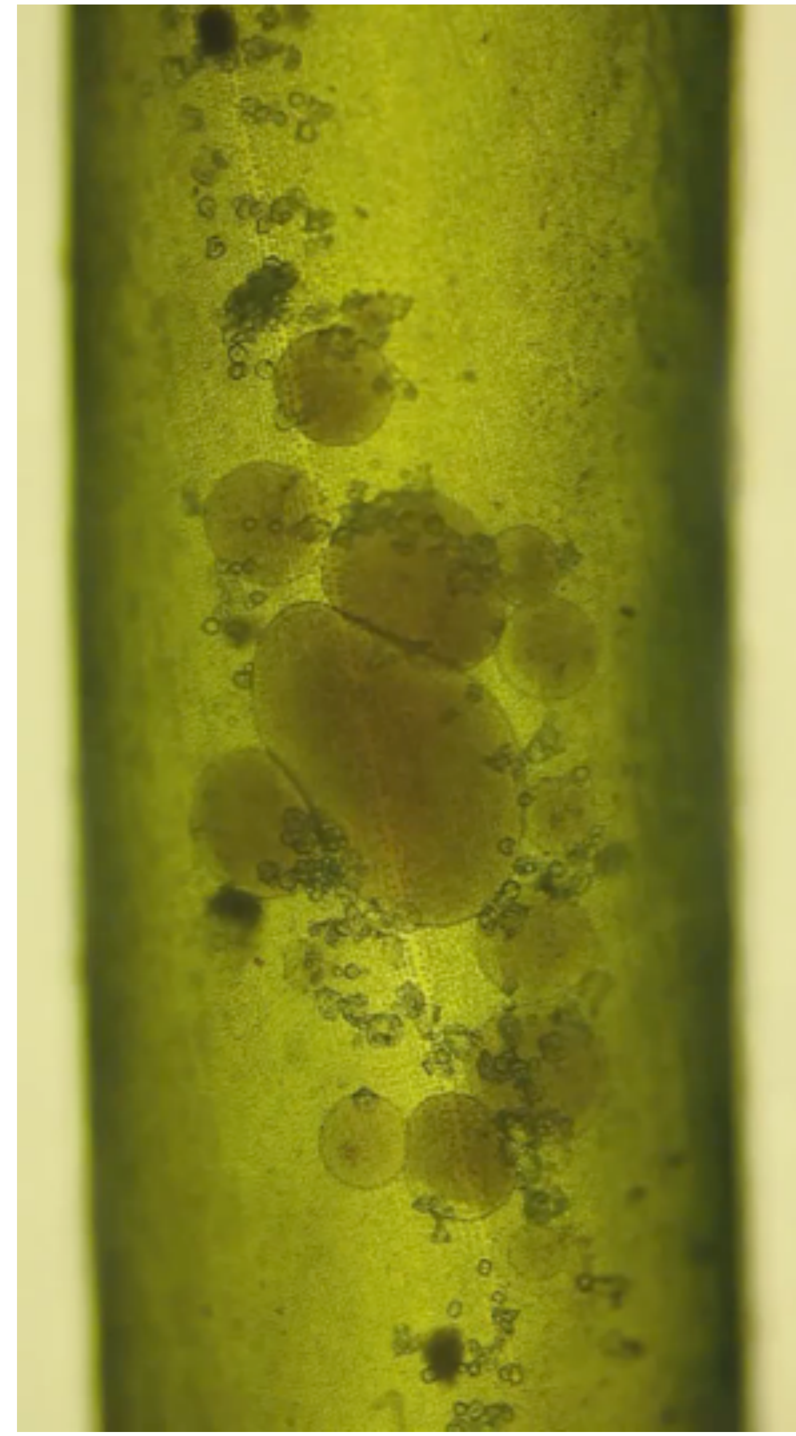
Intracellular transport





http://damtp.cam.ac.uk/user/gold/movies.html

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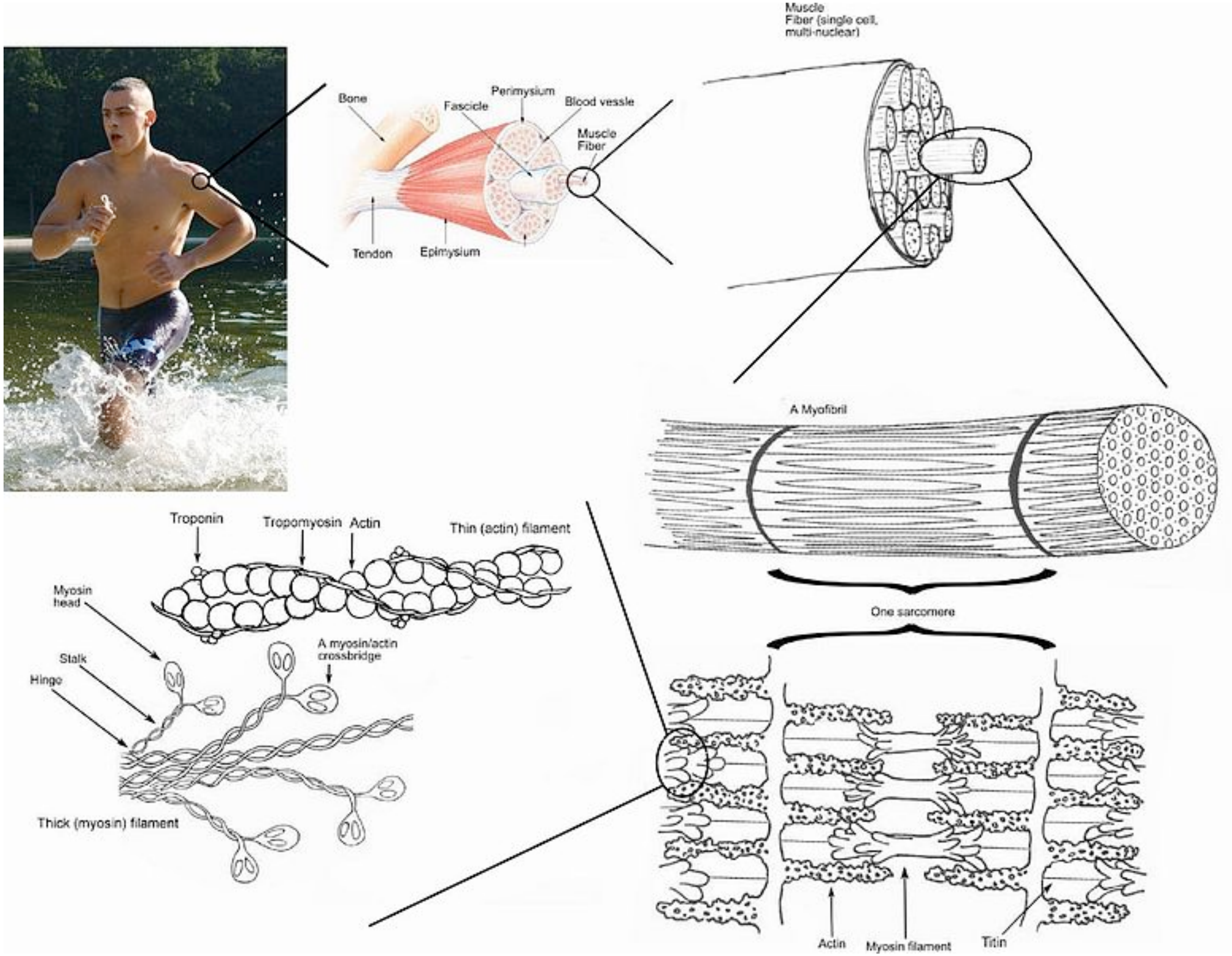

wiki

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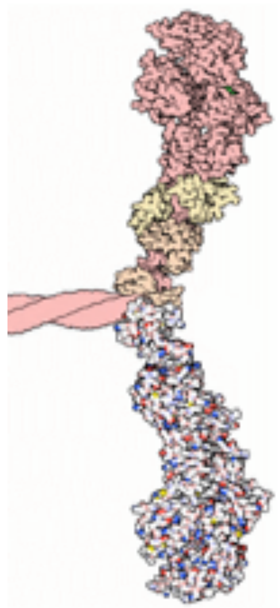
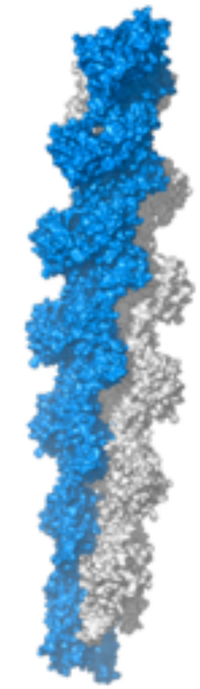
Actin-Myosin

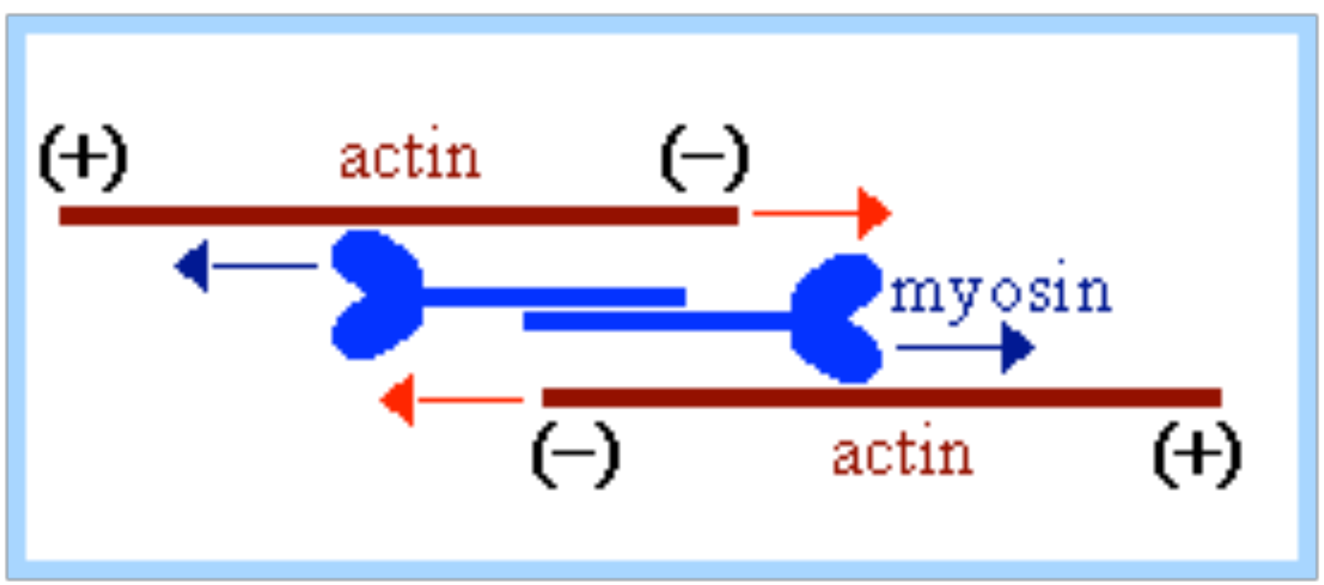
Myosin



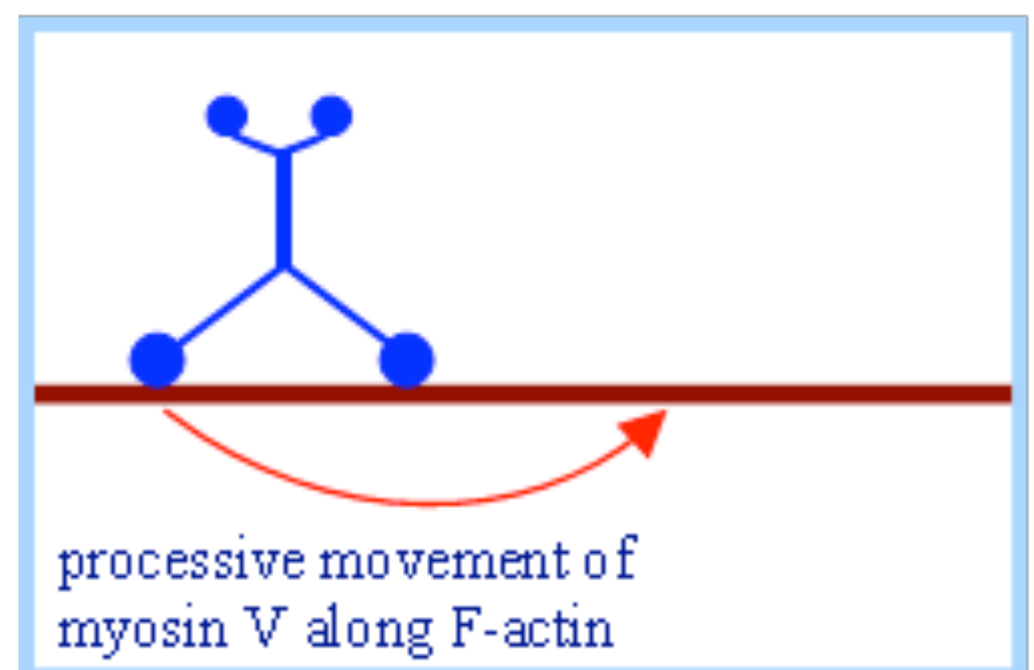


F-Actin

helical filament



myosin-II



myosin-V

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our lecture course:

generic models of micro-motors

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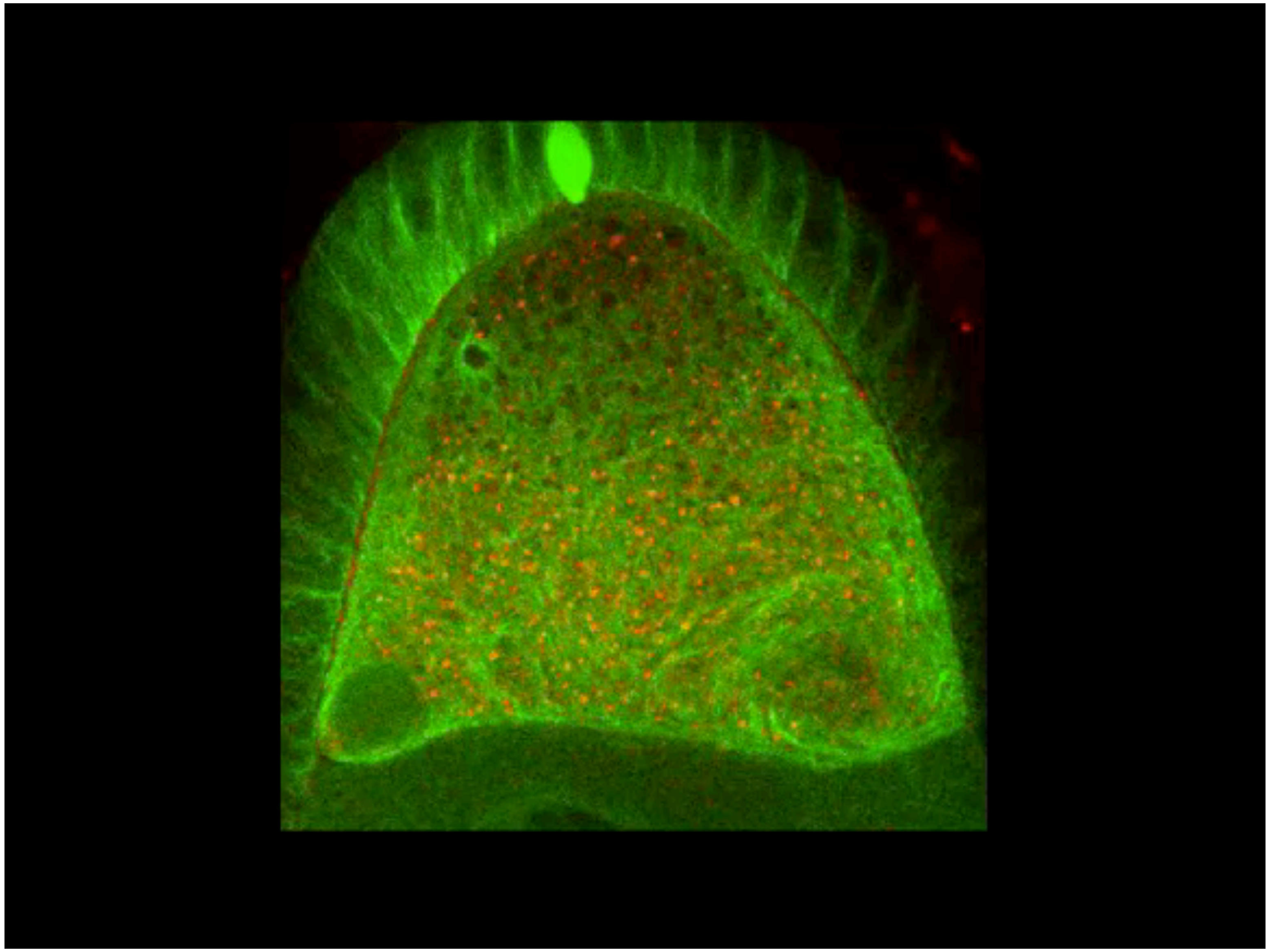
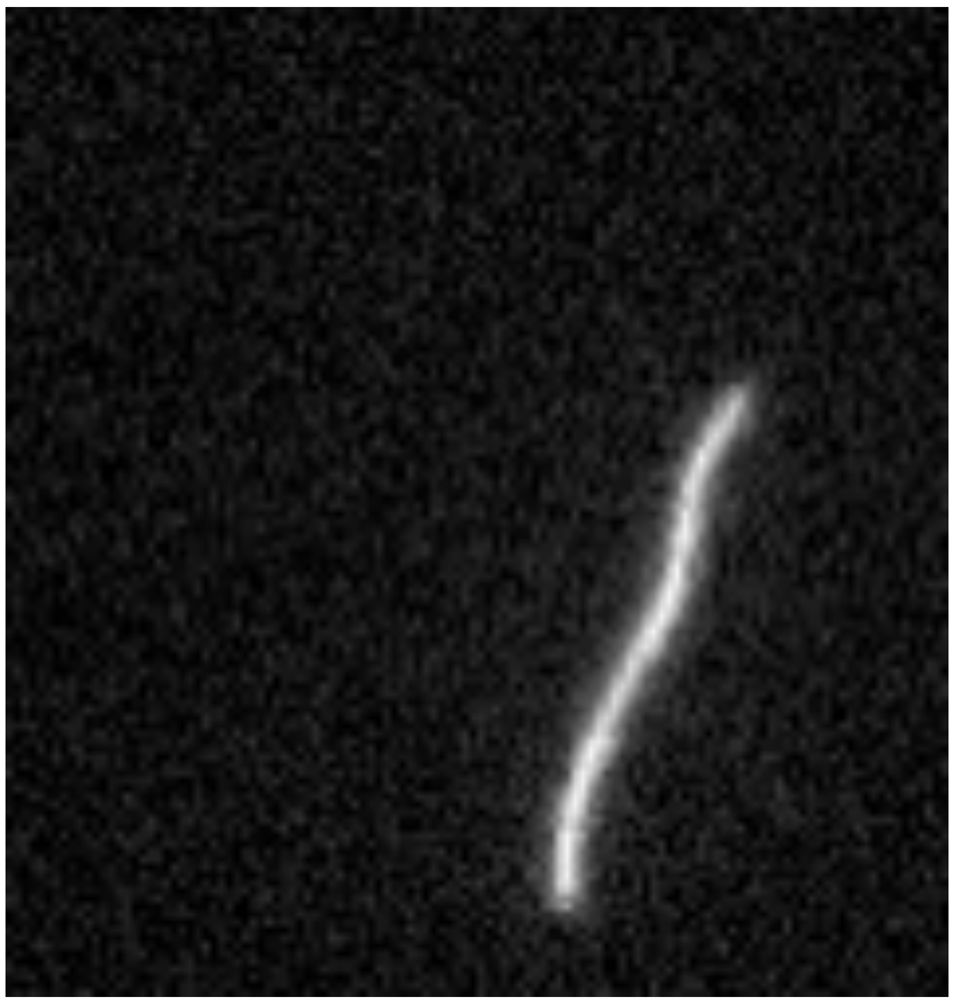
Polymers & filaments (D=1)



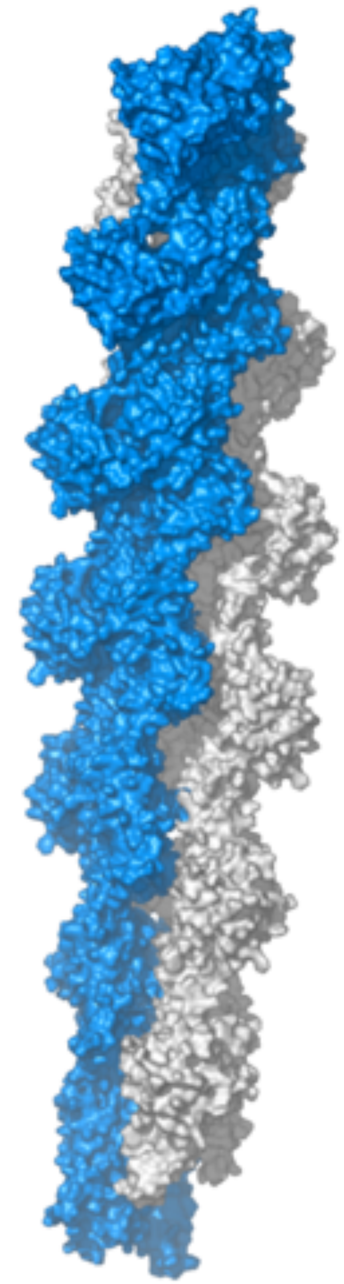
Drosophila oocyte

Physical parameters (e.g. bending rigidity) from fluctuation analysis

Actin in 2D



Dogic Lab (Brandeis)



F-Actin

helical filament

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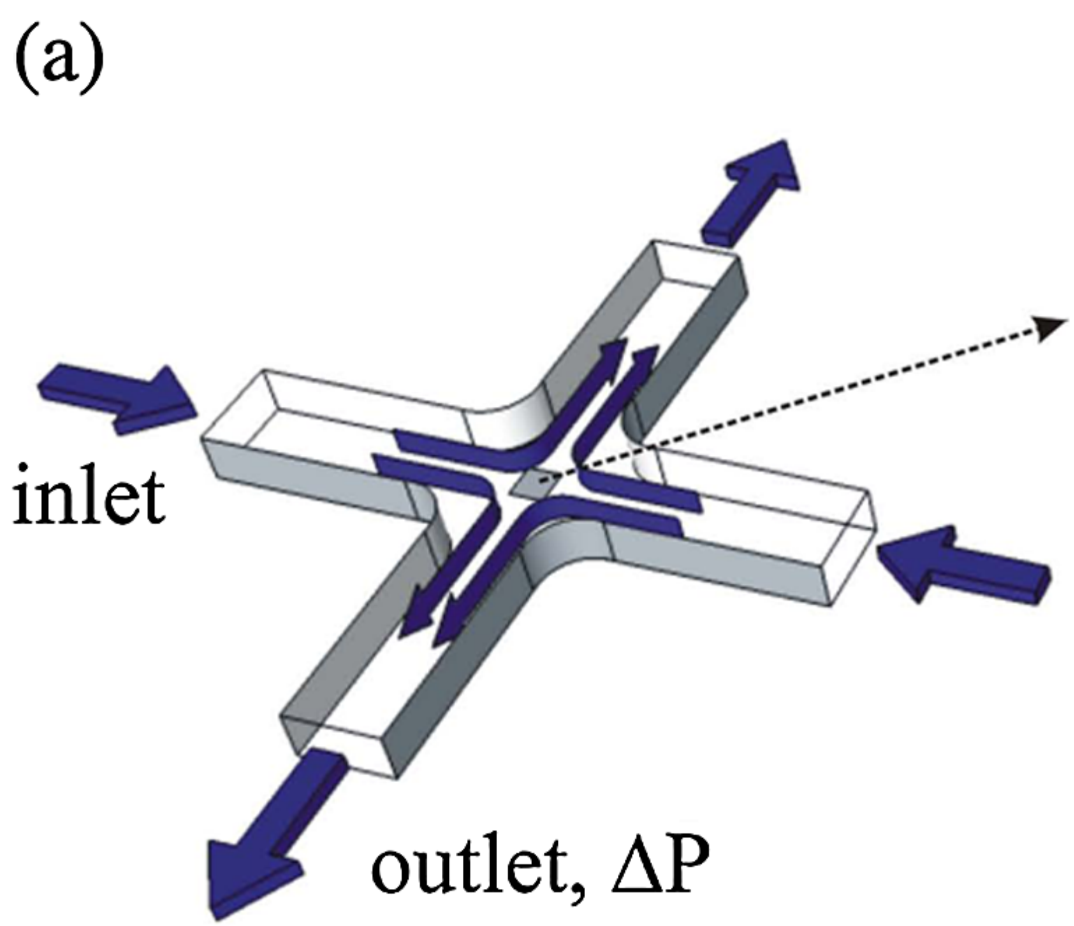
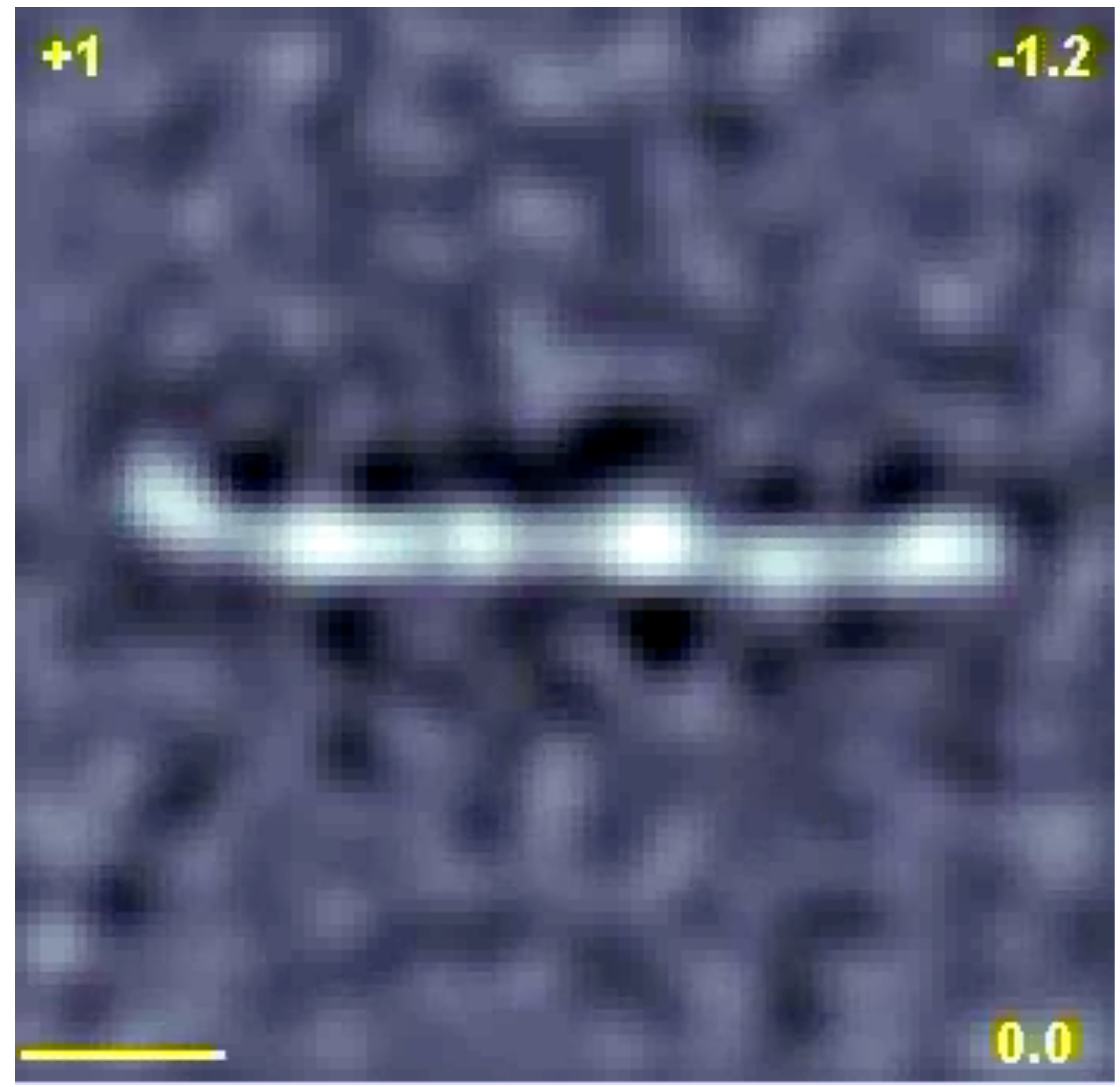
Actin in flow





Kantsler & Goldstein (2012) PRL

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our lecture course:

- polymer models
- how to relate fluctuations to mechanical properties

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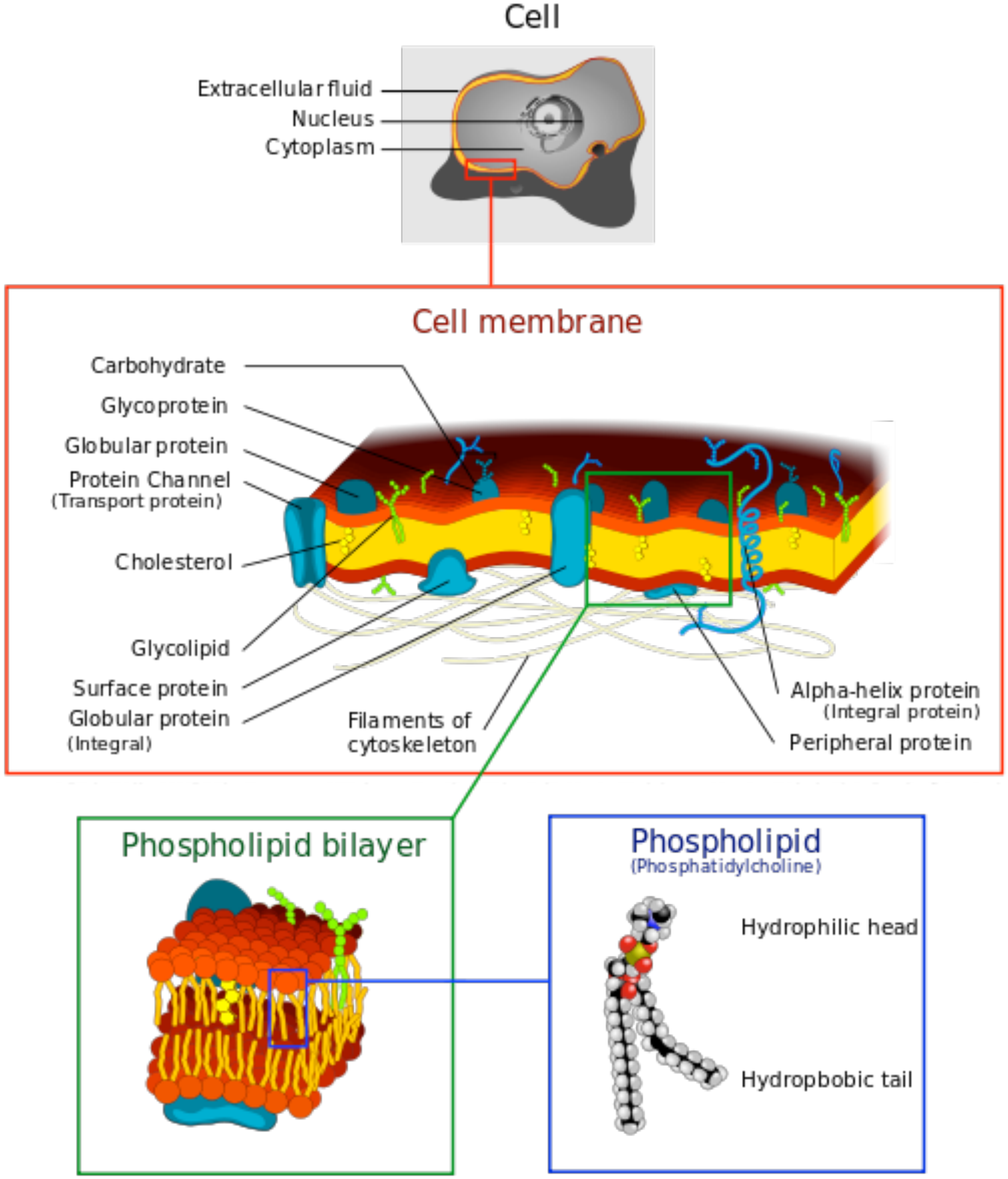
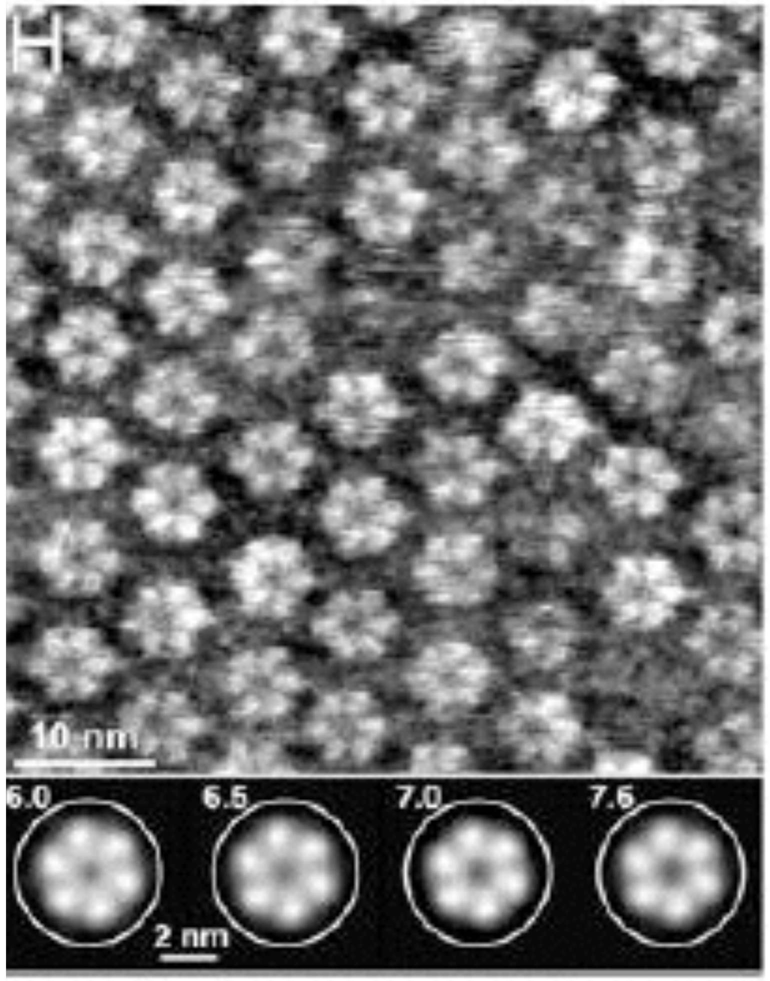
Cell membranes (D=2)





http://www.sbmp-itn.eu/sbmps/research_method/

source: wiki

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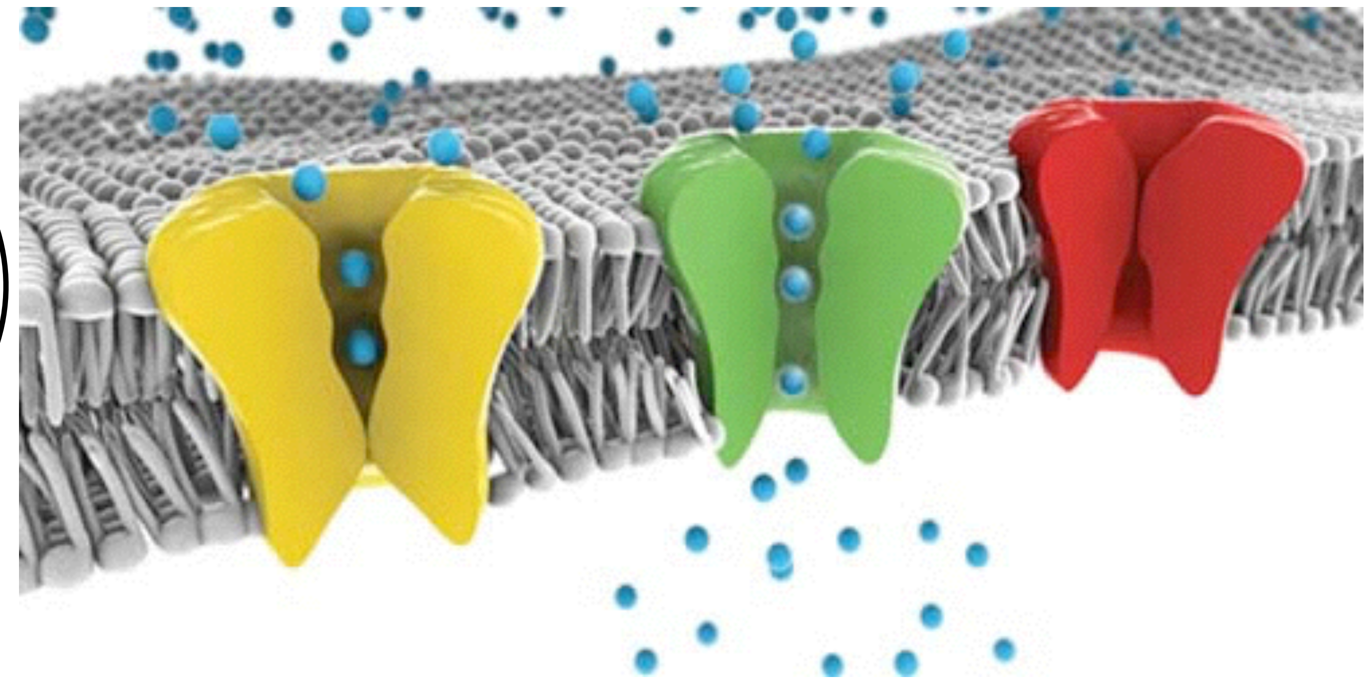
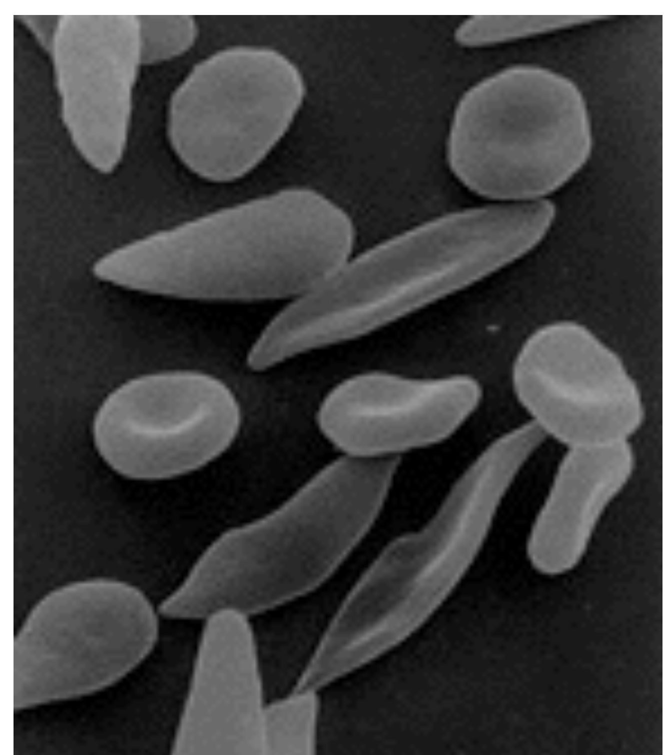
Cell membranes (D=2)

transport:
stochastic
escape problems



shape:
differential
geometry



red blood cells
affected by
sickle-cell disease

source: wiki

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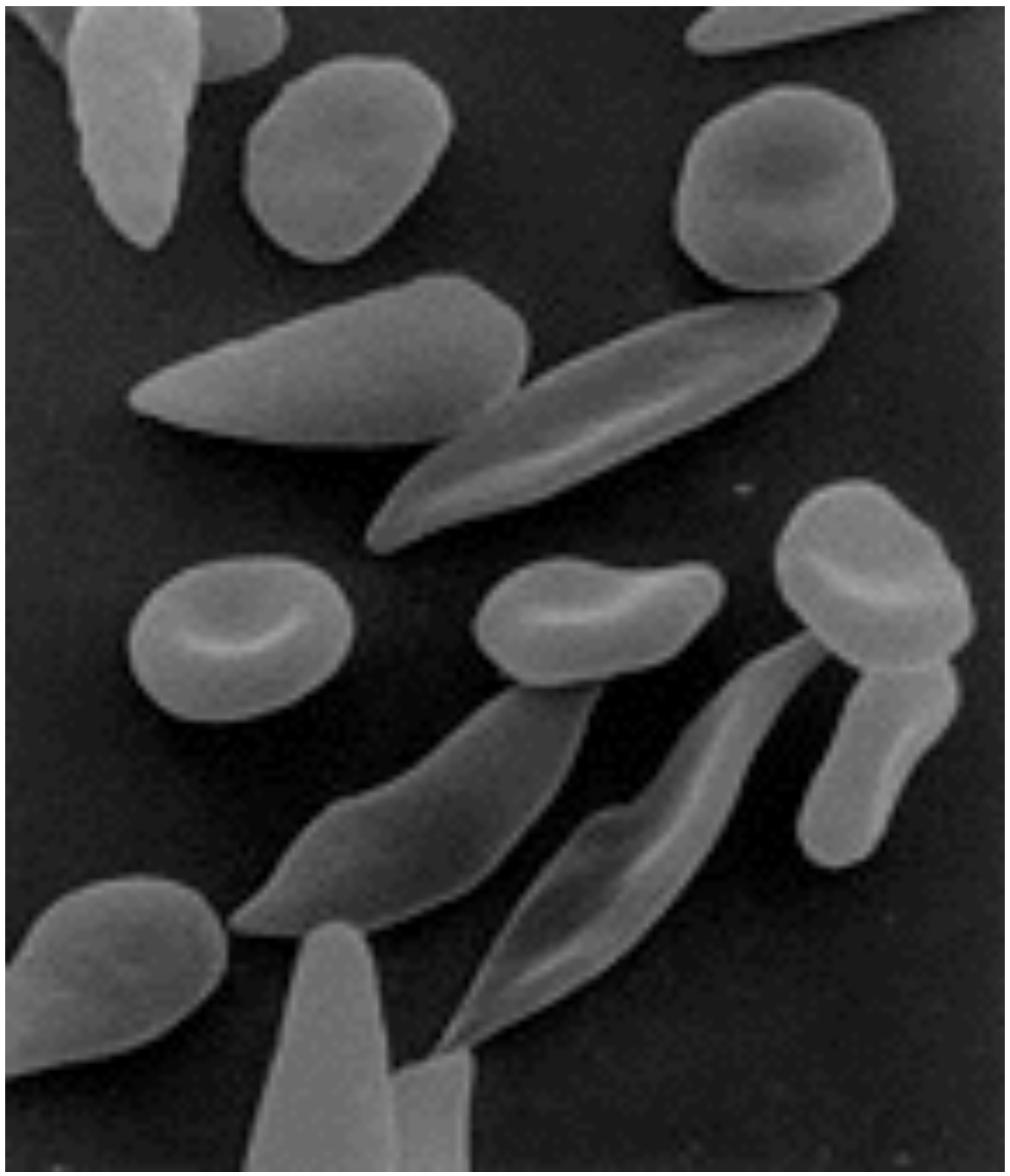
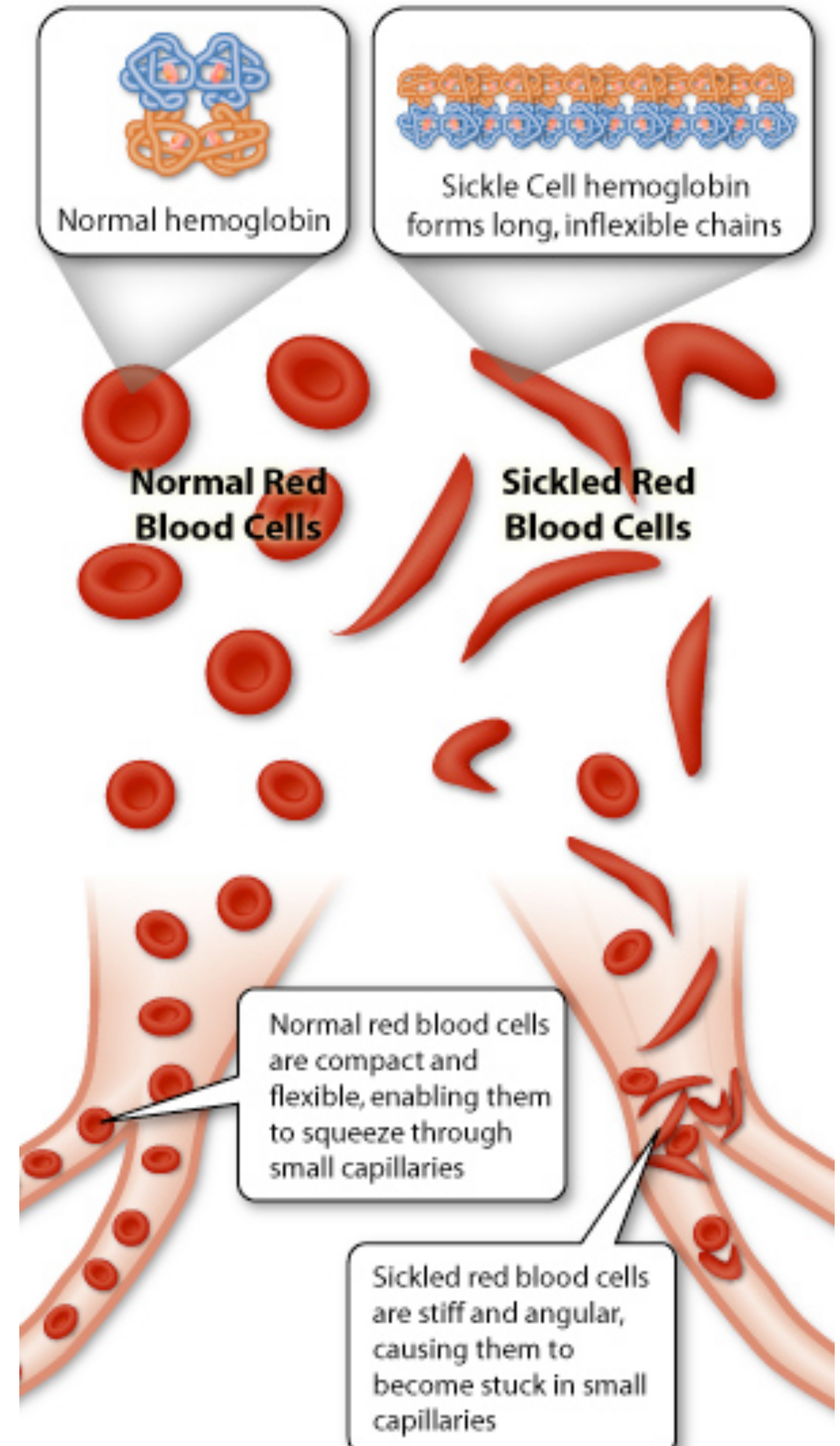
Blood cells: shape & function

source: wiki



red blood cells affected by sickle-cell disease



http://learn.genetics.utah.edu/

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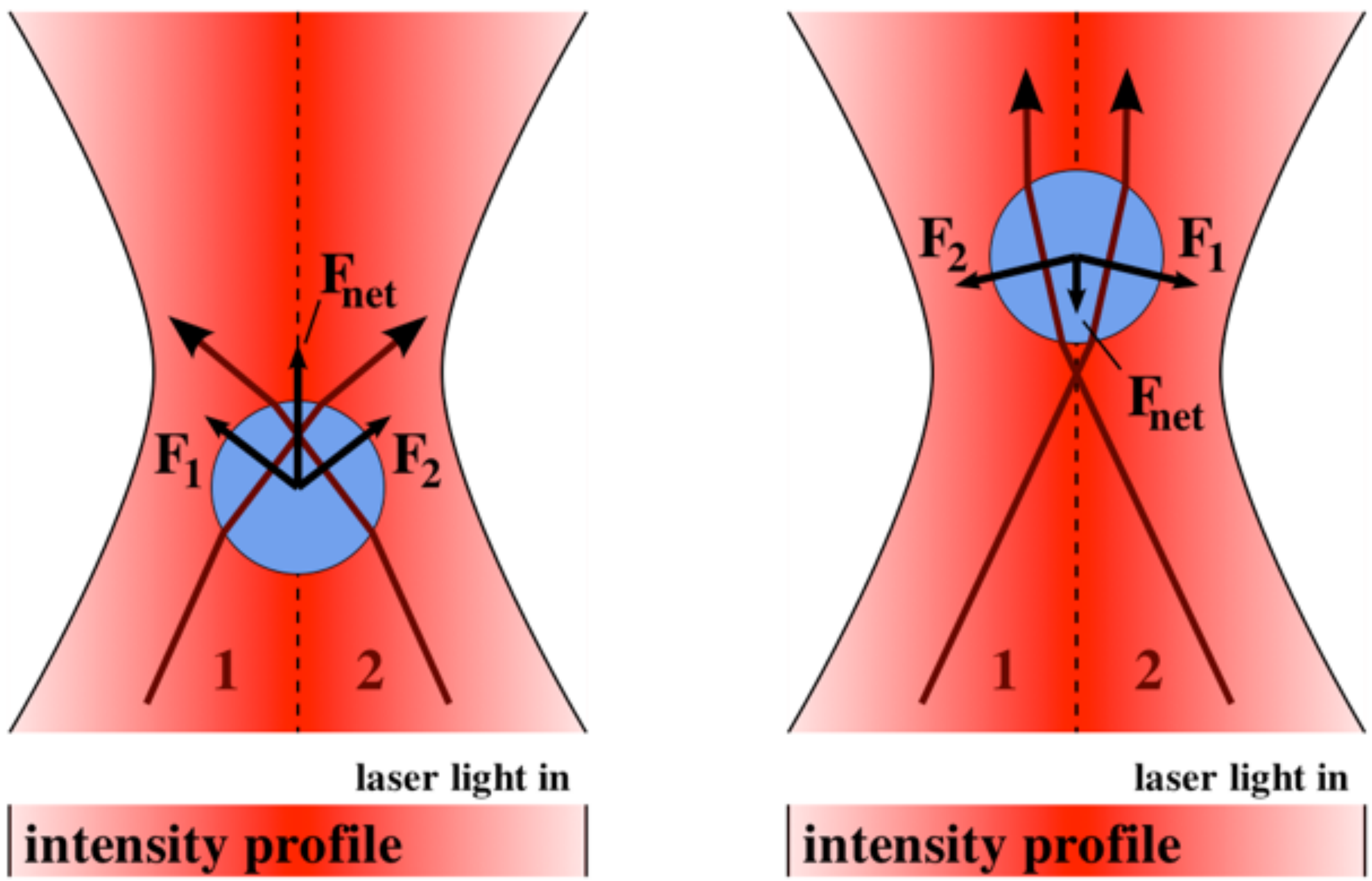
Optical tweezer

source: wiki



http://www.nature.com/ncomms/journal/v4/n4/extref/ncomms2786-s1.swf

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Dynamics of a vesicle in general flow

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11444–11447 | PNAS | **July 14, 2009** | vol. 106 | no. 28

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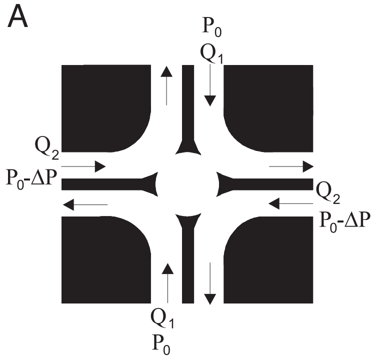

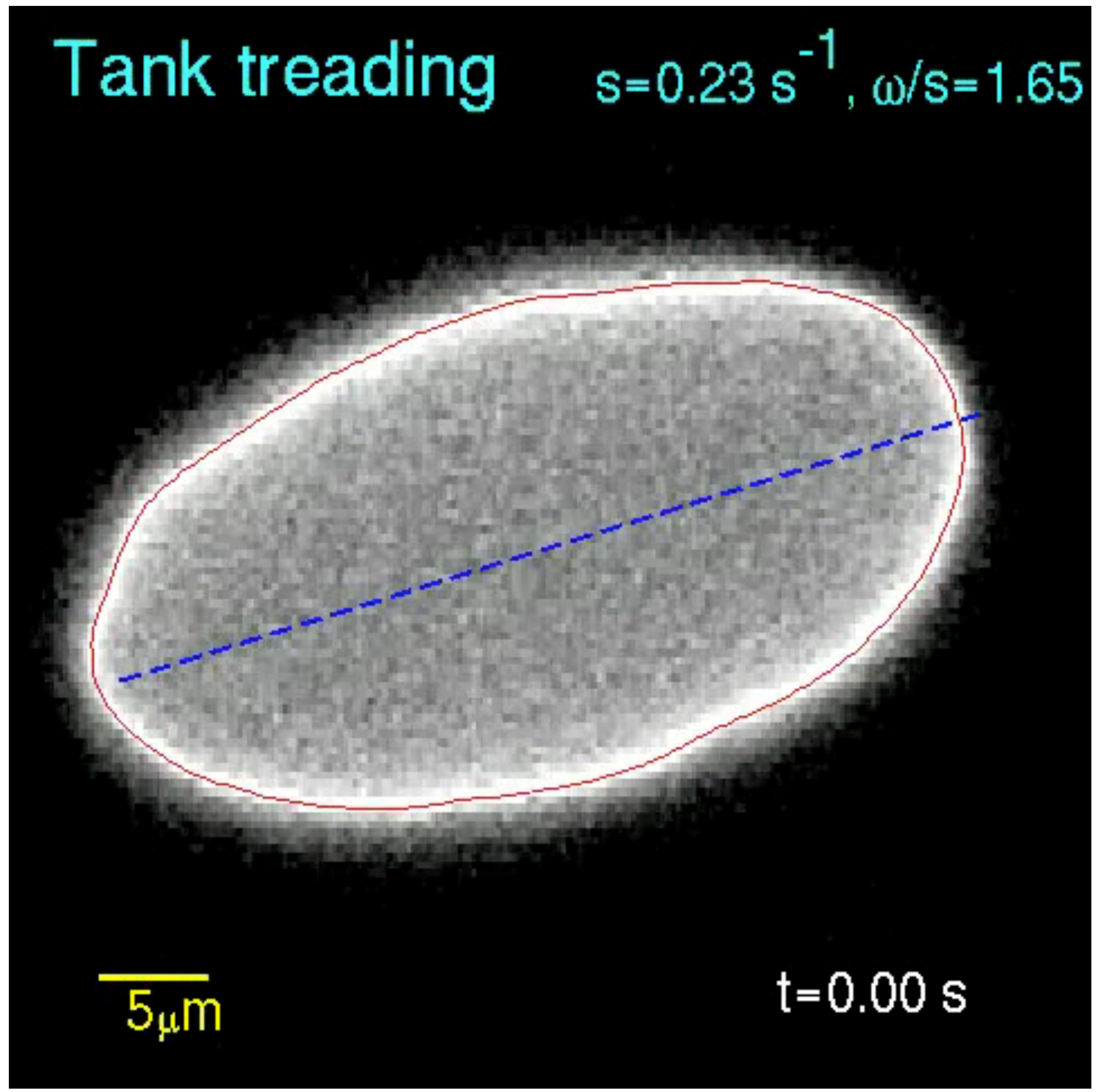

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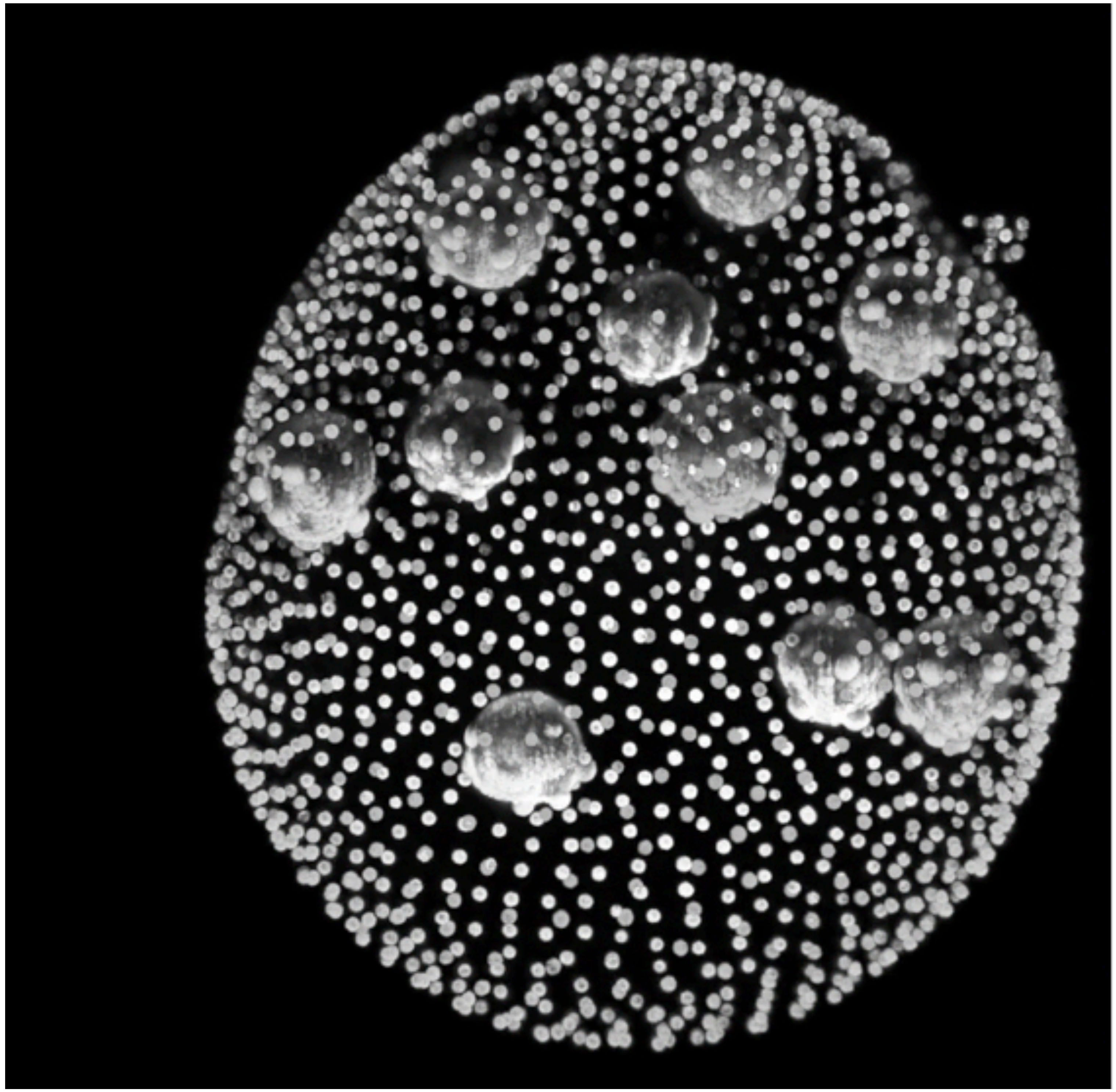
Volvox inversion



http://www.damtp.cam.ac.uk/user/gold/movies.html

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our lecture course:

- ‘differential geometry’ of membranes

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Stationary patterns

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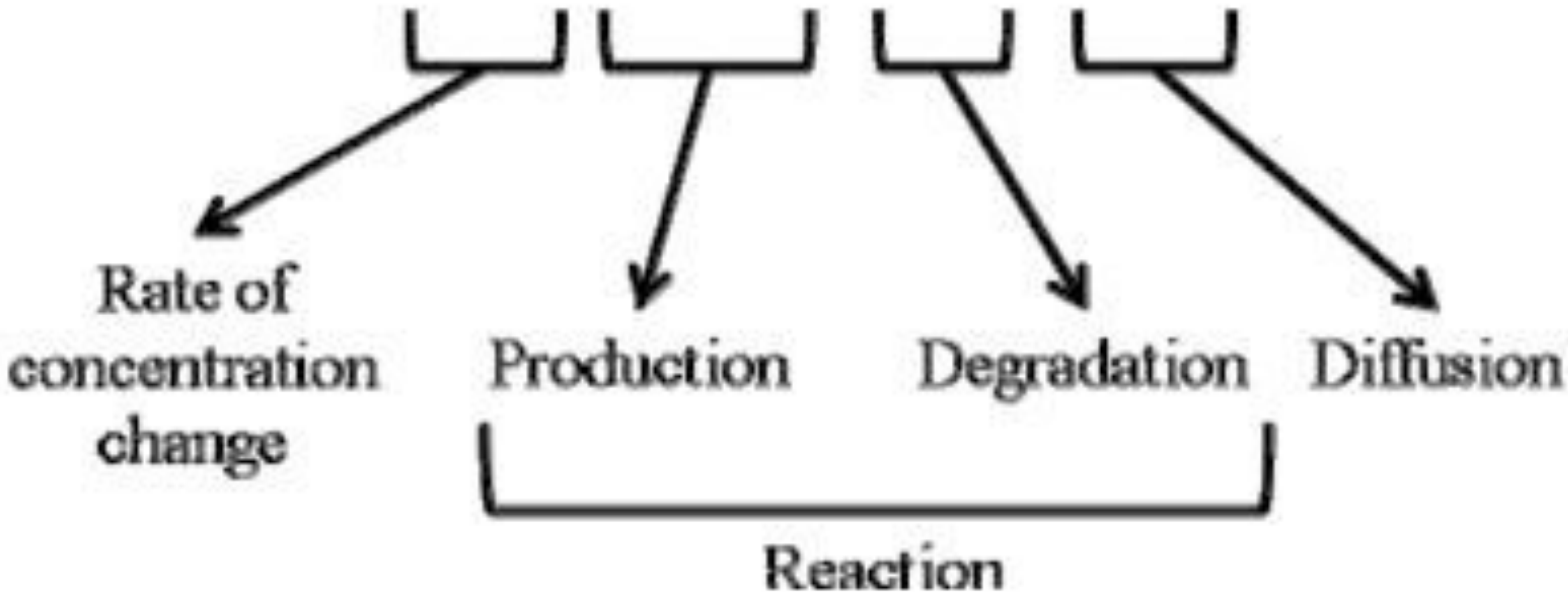
Turing model

$$\frac{\partial u}{\partial t} = F(u,v) - d_u v + D_u \Delta u$$

$$\frac{\partial v}{\partial t} = G(u,v) - d_v v + D_v \Delta v$$



A. M. Turing. **The chemical basis of morphogenesis.** *Phil. Trans. Royal Soc. London. B* **327**, 37–72 (1952)

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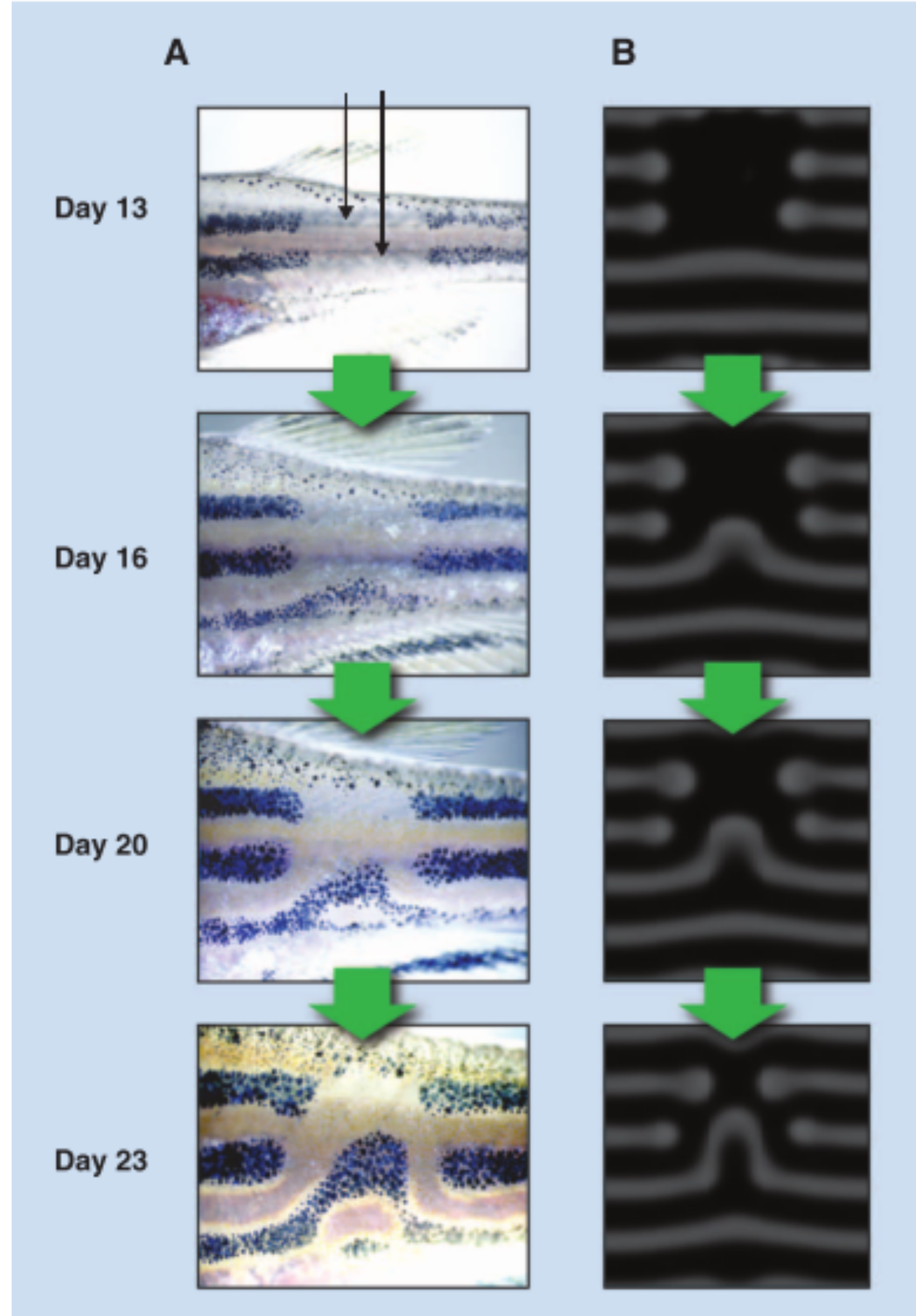
wiki



The matching of zebrafish stripe formation and a Turing model

Kondo S, & Miura T (2010). Reaction-diffusion model as a framework for understanding biological pattern formation. *Science, 329* (5999), 1616-20

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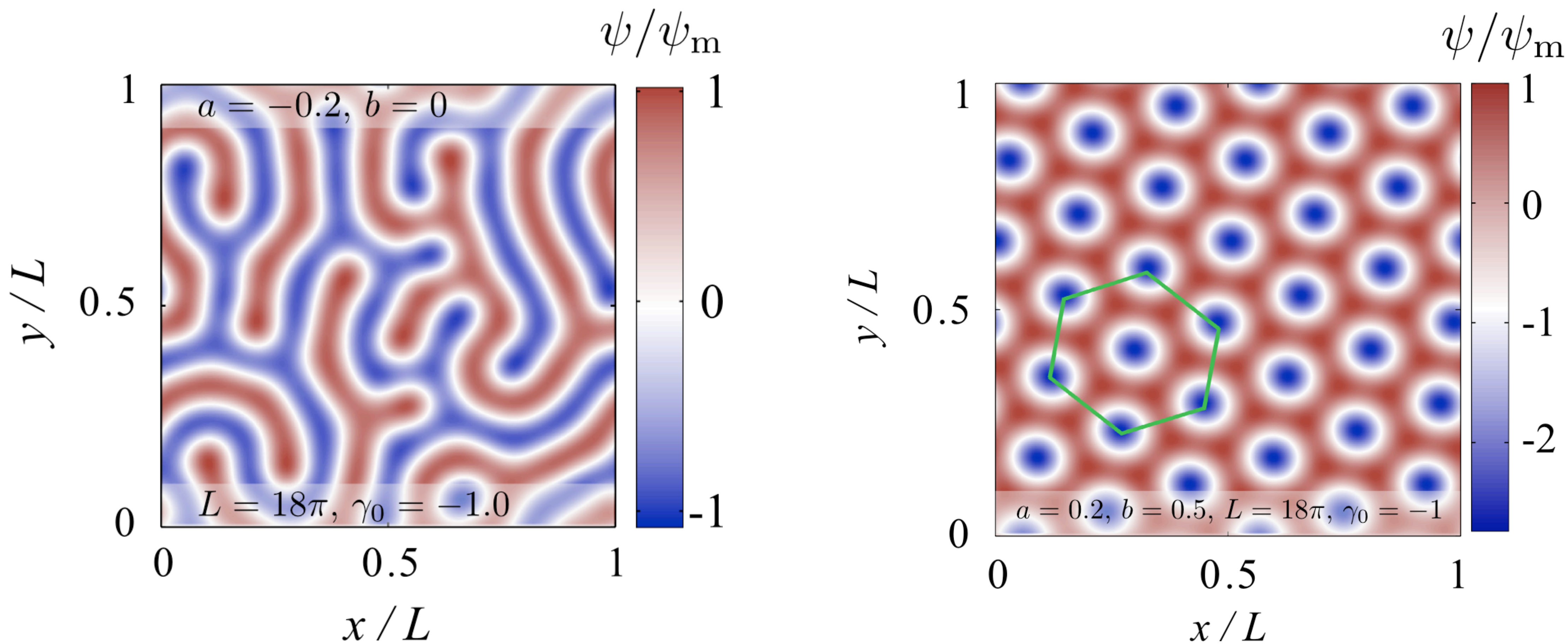
Scalar field theory

2d Swift-Hohenberg model

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

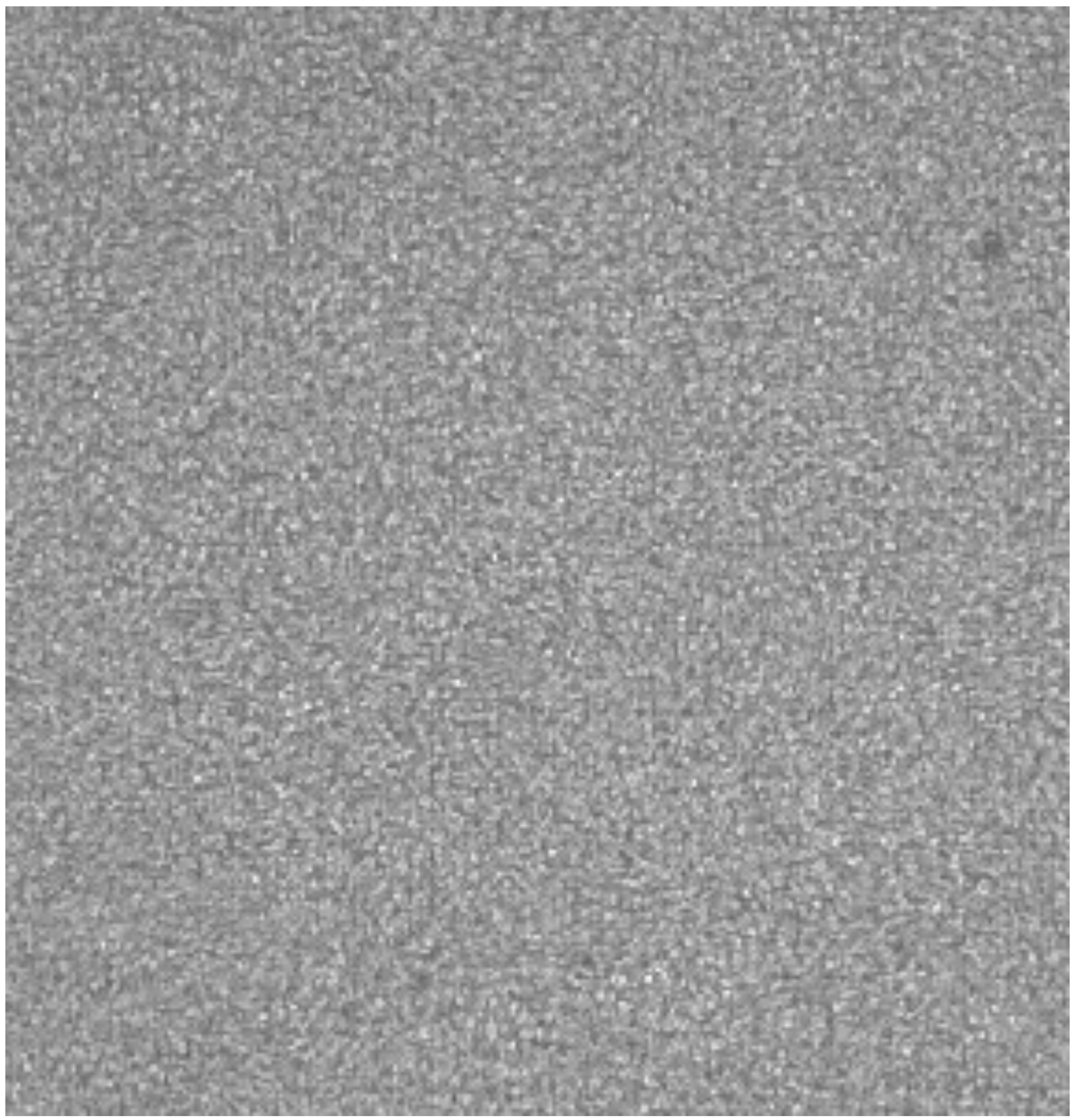
$$U(\psi) = \frac{a}{2}\psi^2 + \frac{b}{3}\psi^3 + \frac{c}{4}\psi^4$$



Active patterns

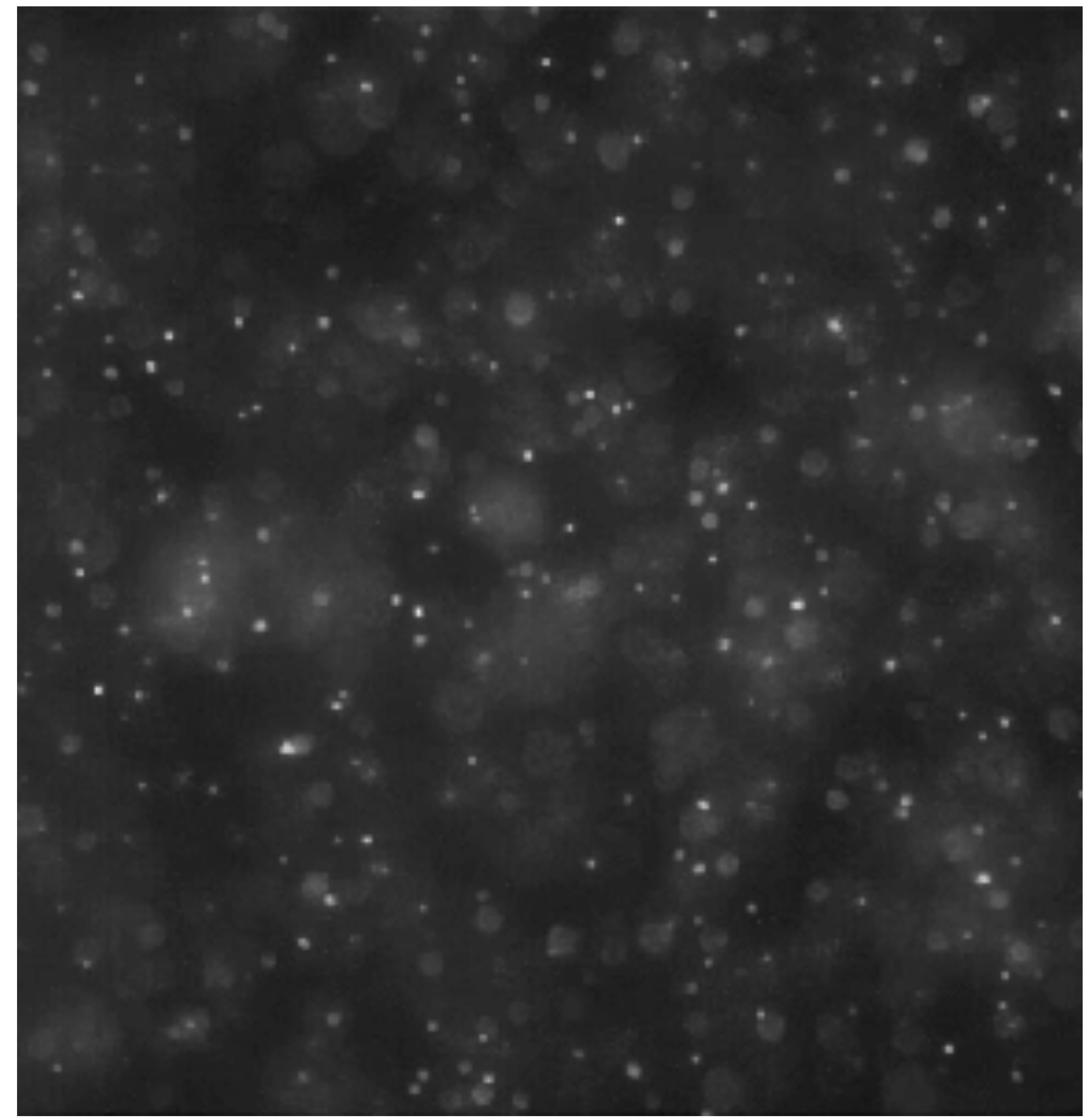
PRL (2013)

B. subtilis



bright field

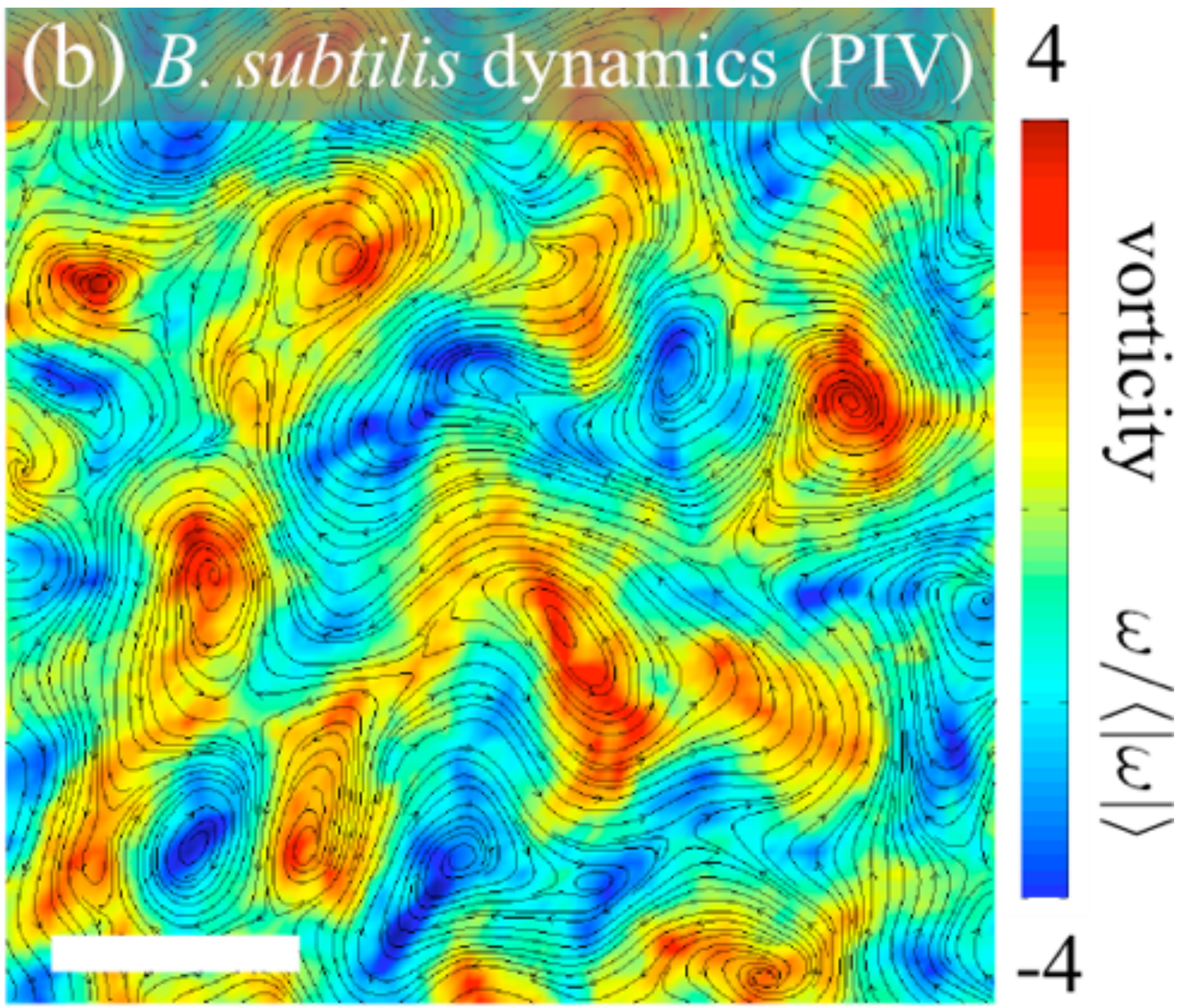
tracer



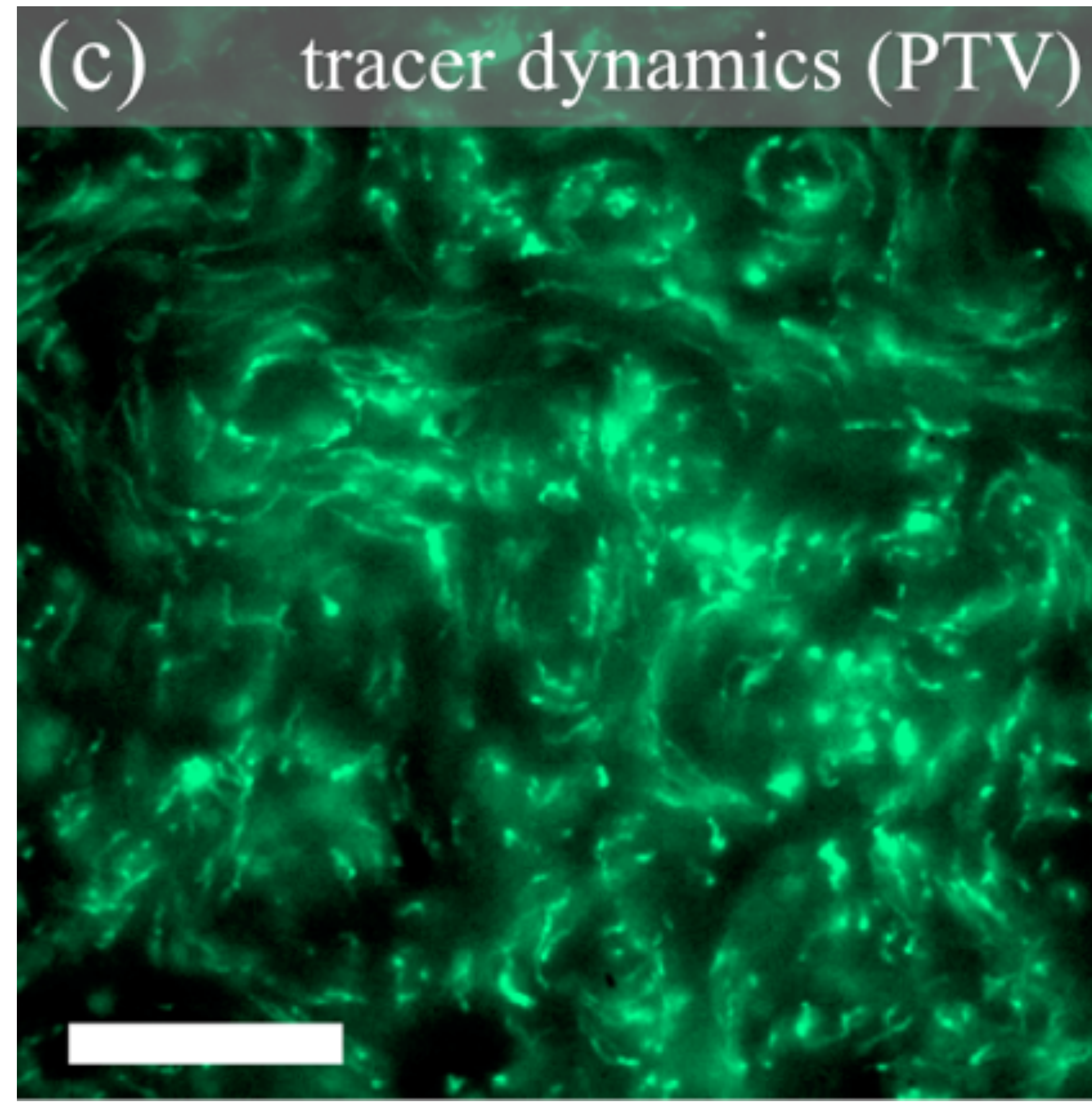
fluorescence

3D bacterial suspension

PRL (2013)



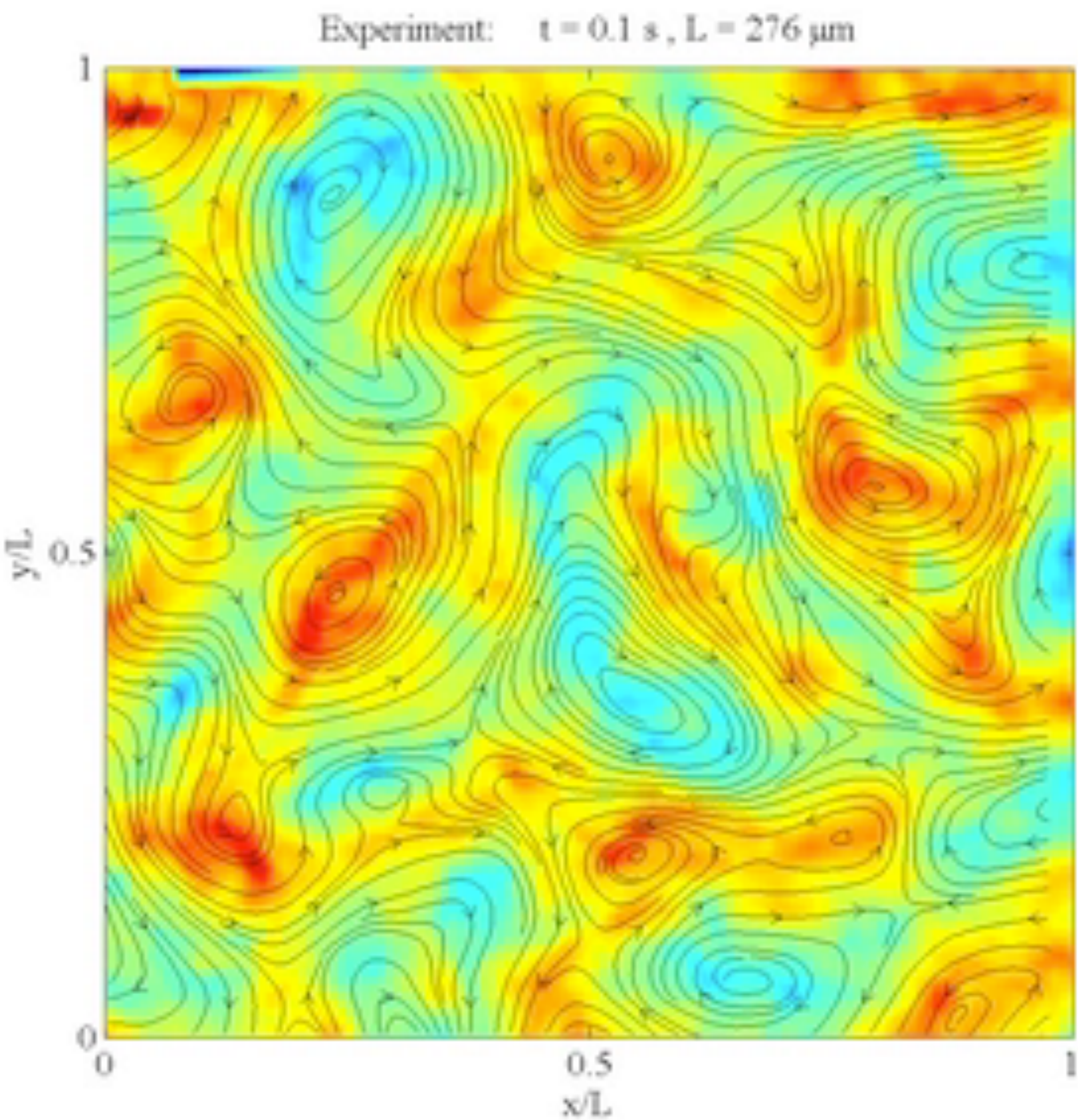
bright field



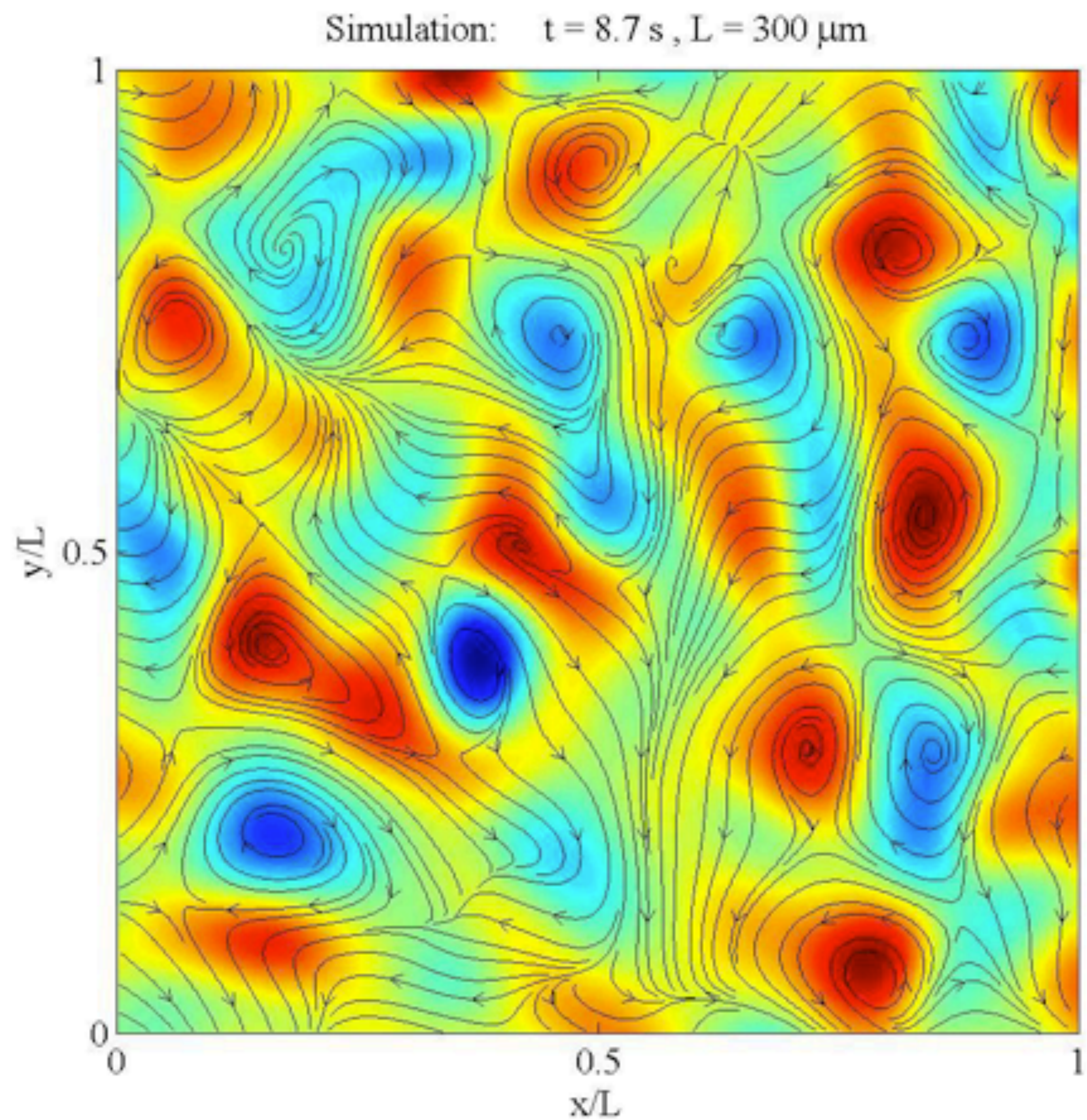
fluorescence

3D suspension

PRL (2013)



Experiment:
quasi-2D slice



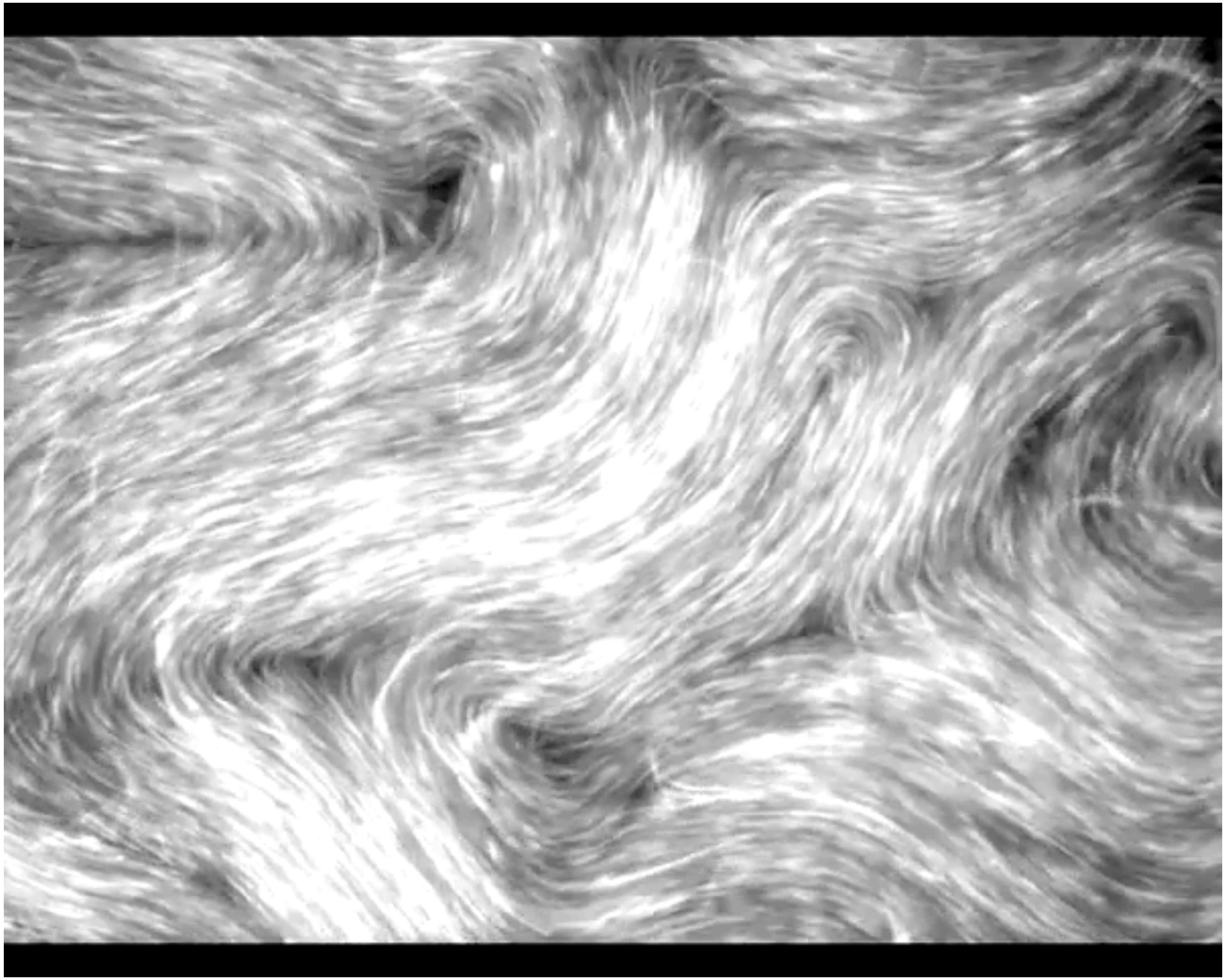
Theory:
2D slice

Vector field theory

incompressibility $\nabla \cdot \boldsymbol{v} = 0$

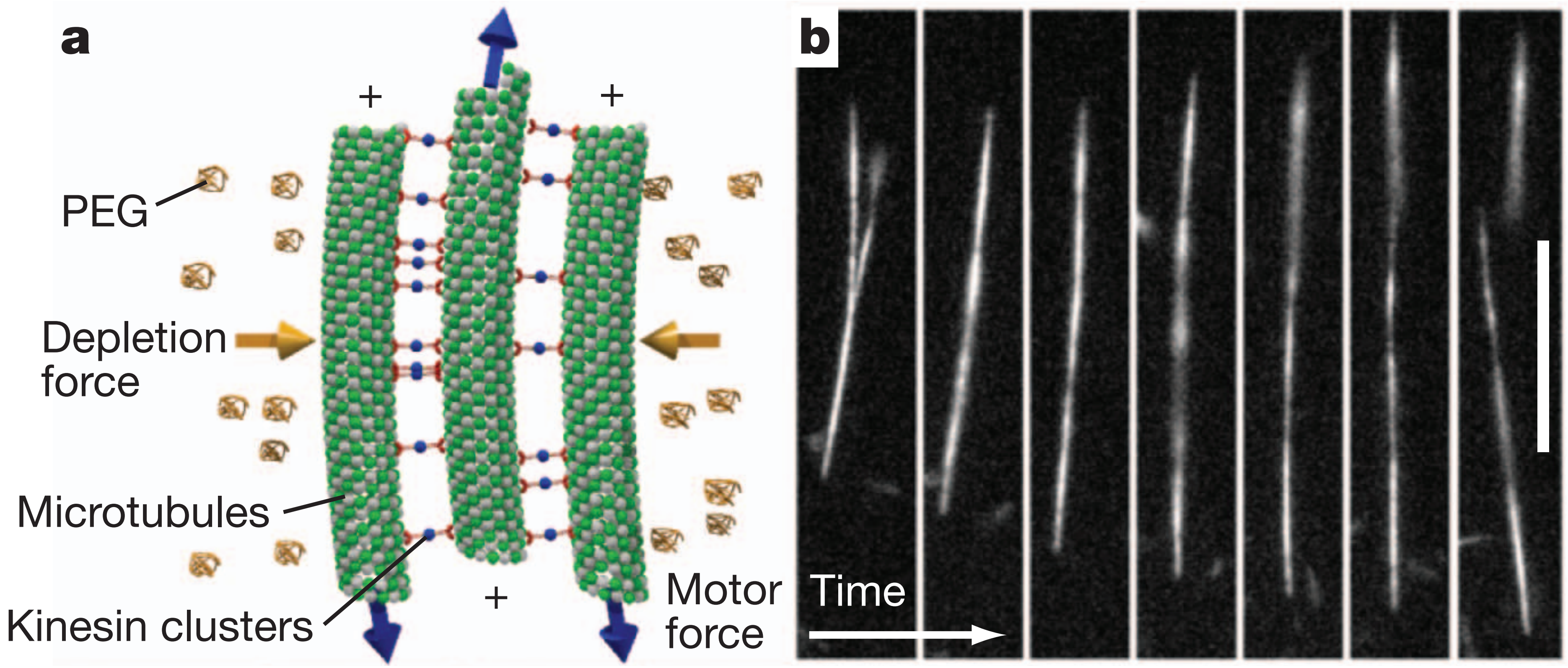
$$(\partial_t + \lambda_0 \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla(p + \lambda_1 \boldsymbol{v}^2) - (\beta \boldsymbol{v}^2 + \alpha)\boldsymbol{v} + \\ + \Gamma_0 \nabla^2 \boldsymbol{v} - \Gamma_2 (\nabla^2)^2 \boldsymbol{v}$$

Active nematics



Dogic lab (Brandeis) Nature 2012

Active nematics



Dogic lab (Brandeis) Nature 2012

no head or tail ⇒ Q-tensor order-parameter

$$Q_{ij} = Q_{ji}\ , \qquad \mathrm{Tr}\, Q = 0 \qquad Q = \begin{pmatrix} \lambda & \mu \\ \mu & -\lambda \end{pmatrix} .$$

$$\Delta = \sqrt{\lambda^2 + \mu^2}, \qquad \Lambda^{\pm} = \pm\Delta$$

Matrix field theory

$$\partial_t Q_{ij} + v_k \partial_k Q_{ij} = -\frac{\delta \mathcal{F}}{\delta Q_{ij}}$$

$$v_k \;=\; D\,\partial_n Q_{nk}$$

biological networks

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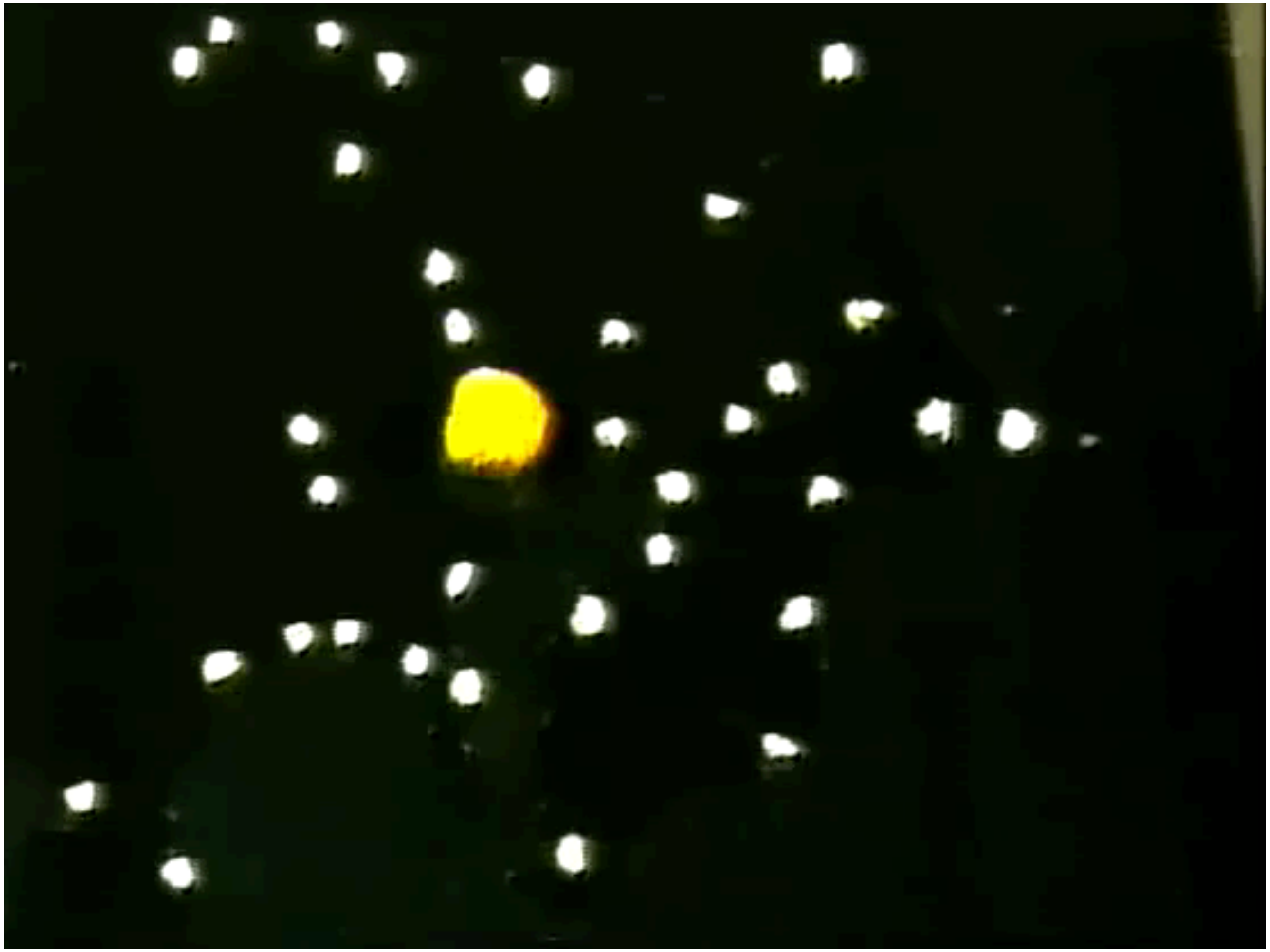
Tokyo rail network by *Physarum plasmodium*



Tero et al (2010) Science

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