

Topological Defects

18.354 L24

Order Parameters, Broken Symmetry, and Topology

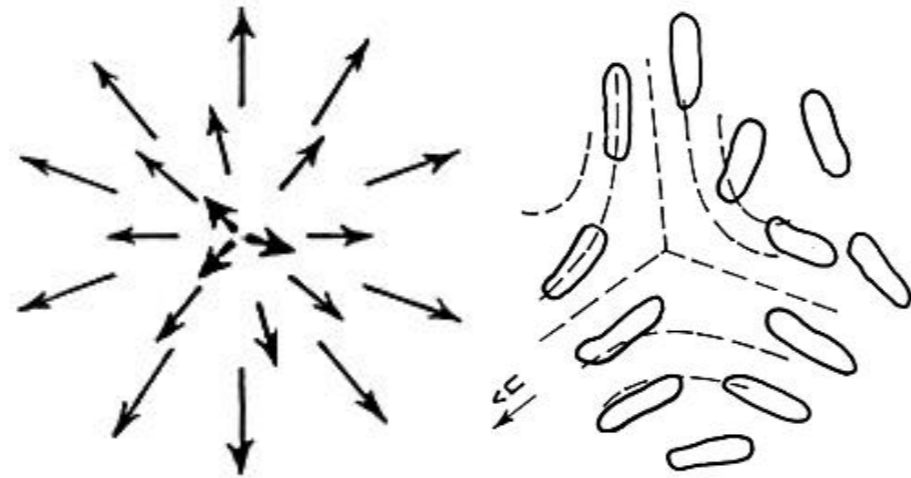
James P. Sethna

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DK-2800 Lyngby, DENMARK, and NORDITA, DK-2100 Copenhagen Ø,
DENMARK and Laboratory of Atomic and Solid State Physics (LASSP),
Clark Hall, Cornell University, Ithaca, NY 14853-2501, USA*

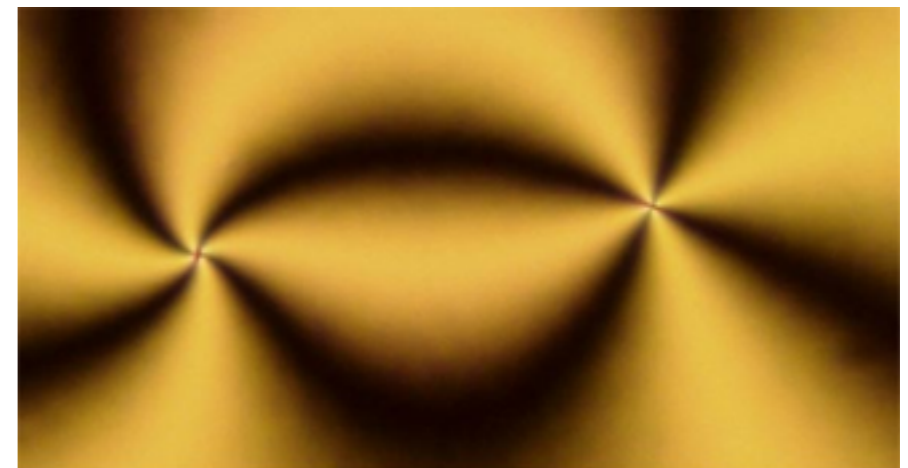
(Dated: May 27, 2003, 10:27 pm)

dunkel@mit.edu

Topological defects are discontinuities in order-parameter fields



- optical effects
- work hardening, etc



"umbilic defects" in a nematic liquid crystal

order = symmetry = invariance

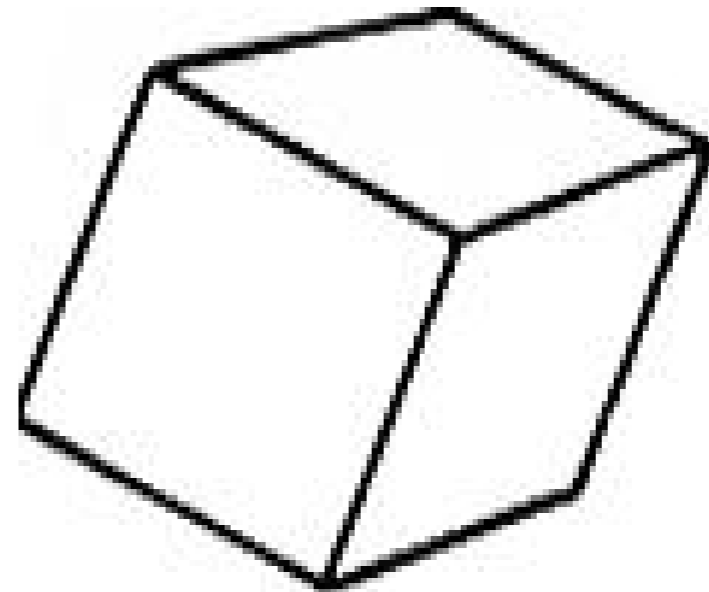
(under certain group actions)

symmetry groups can be discrete,
continuous, Lie-groups,

More or less symmetric ?



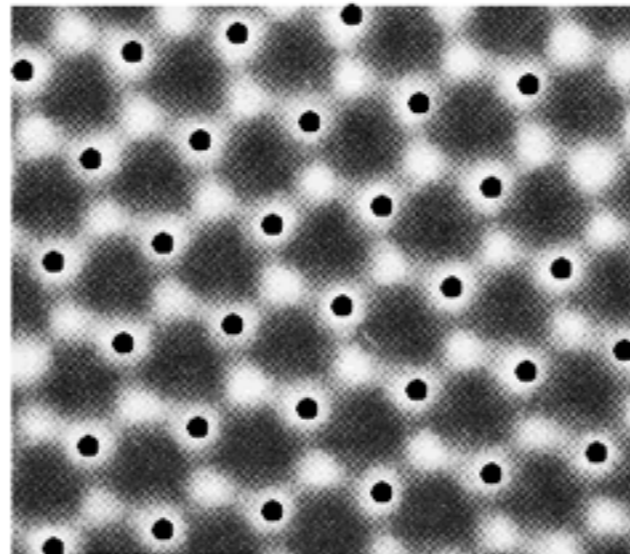
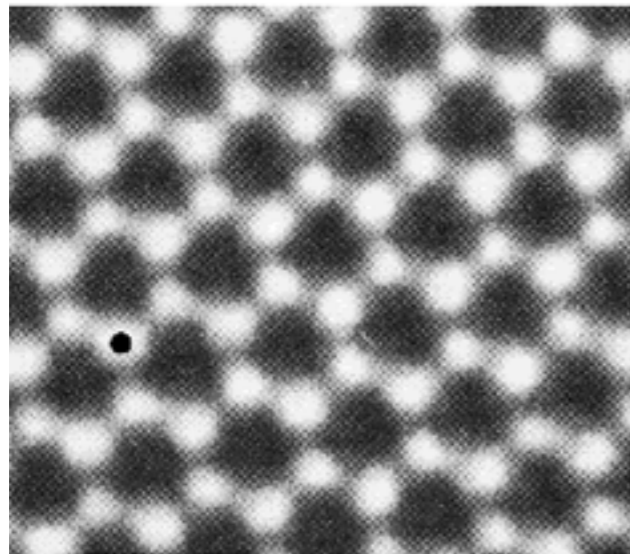
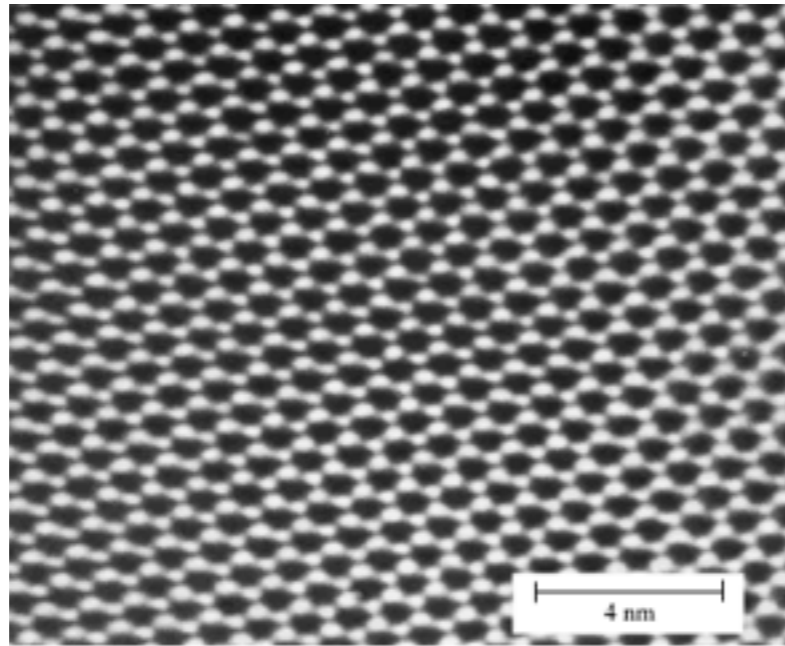
A



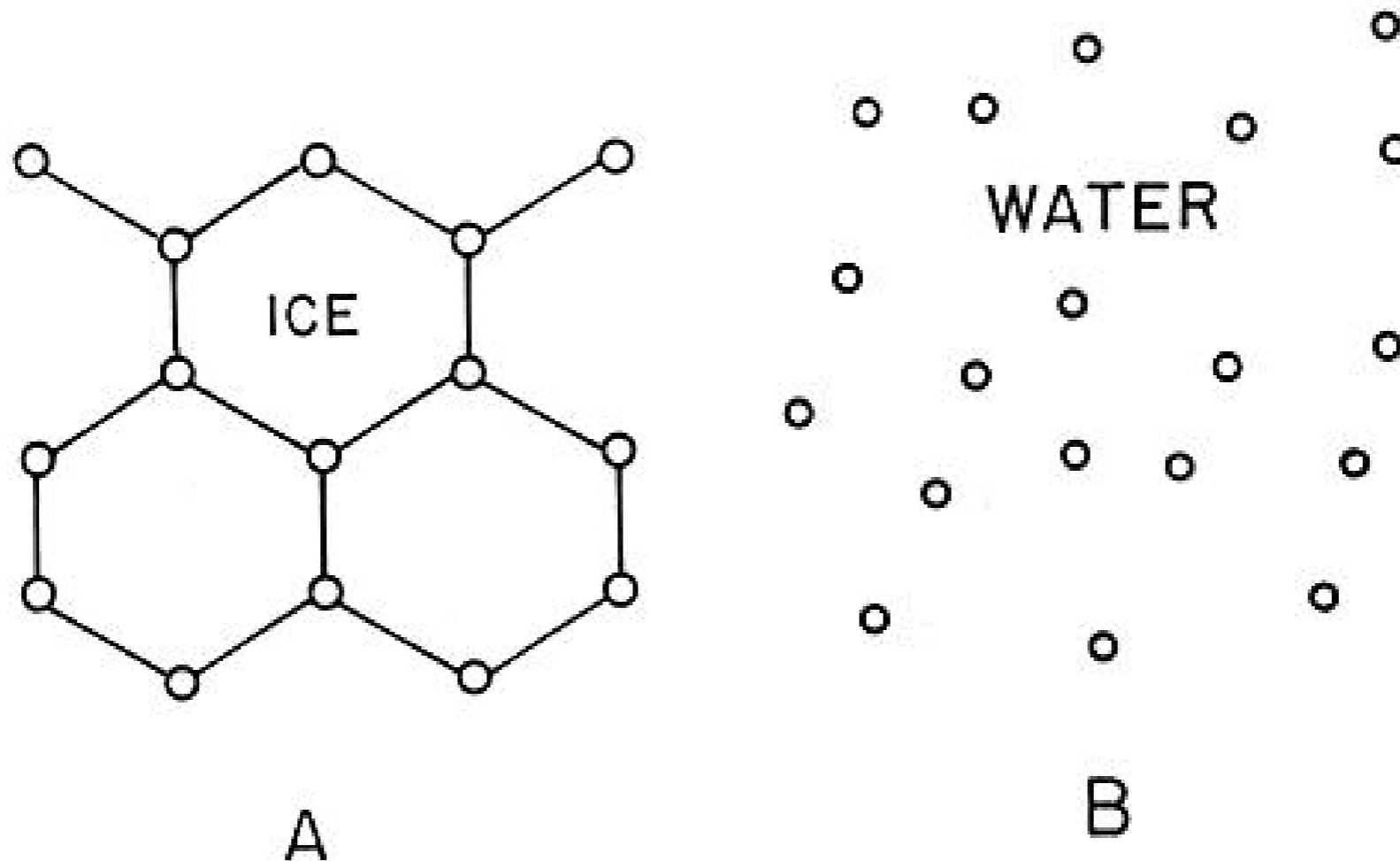
B

More or less symmetric ?

$Mg_2Al_4Si_5O_{18}$

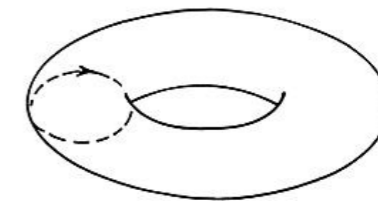
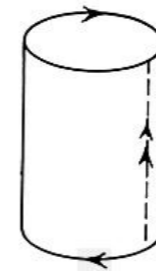
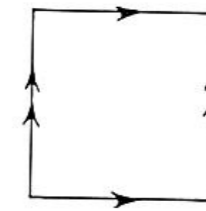
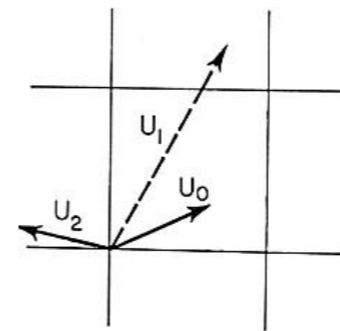
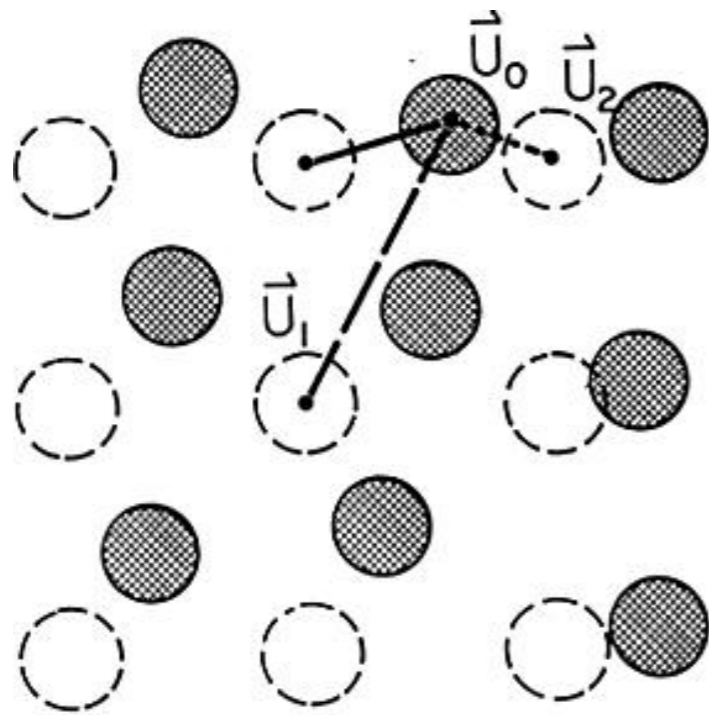


More or less symmetric ?



broken continuous
translation/rotation
symmetry (invariance)

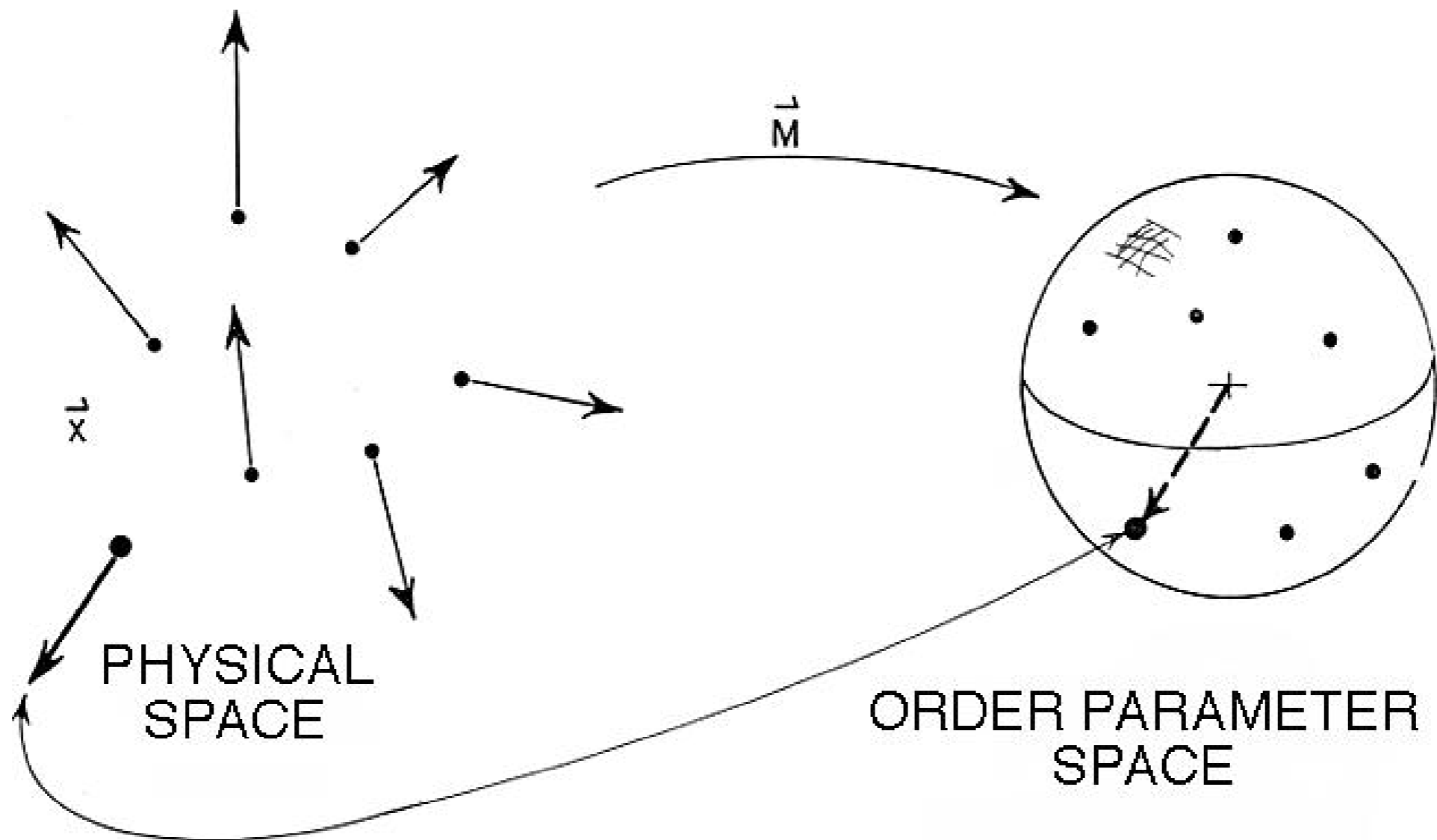
Order parameters: 2D crystal



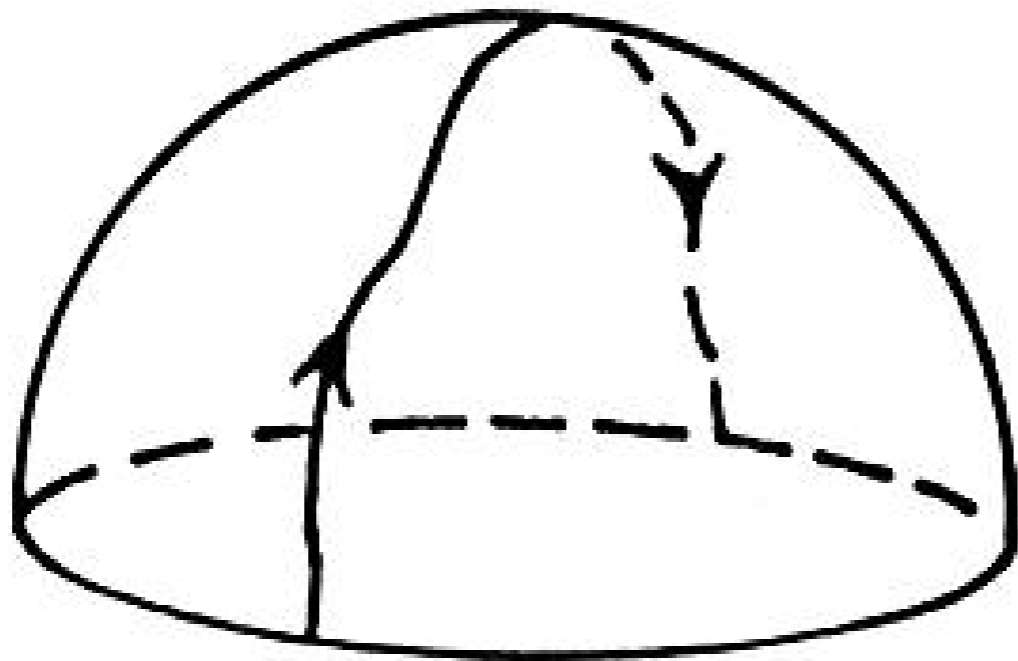
$$\vec{u} \equiv \vec{u} + a\hat{x} = \vec{u} + ma\hat{x} + na\hat{y}.$$

$$\mathcal{E} = \int dx (\kappa/2)(du/dx)^2.$$

Order parameters: magnets

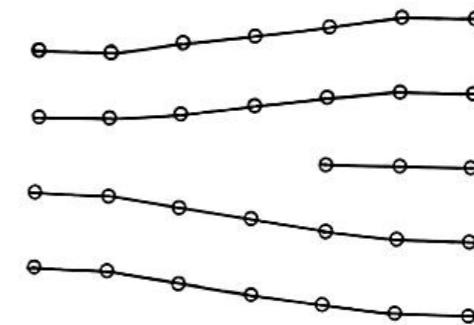
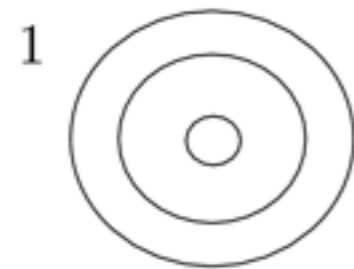
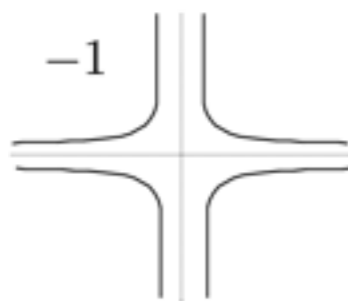
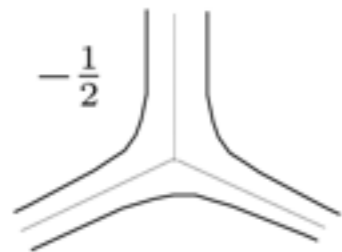
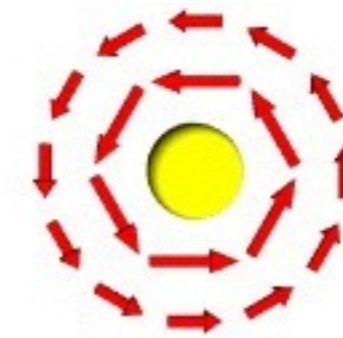
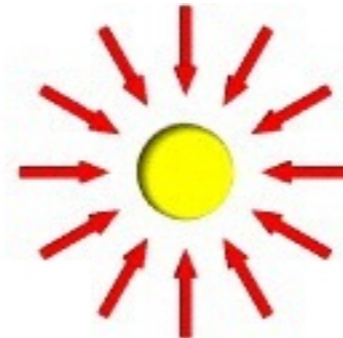
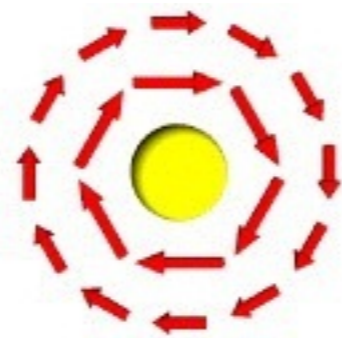
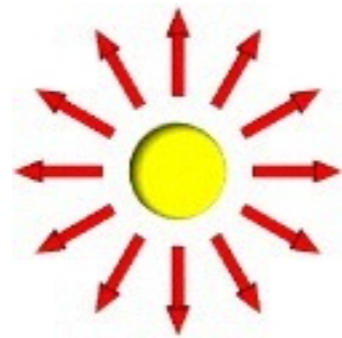


Order parameters: nematic liquid crystals

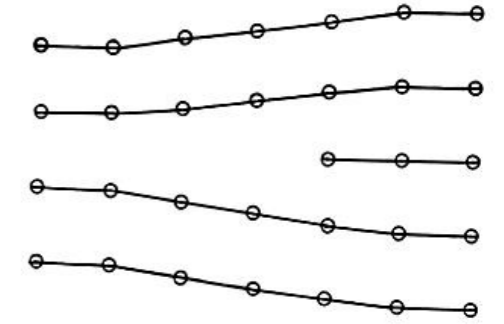


“projective plane” =
half-sphere
with opposite points on
equator identified

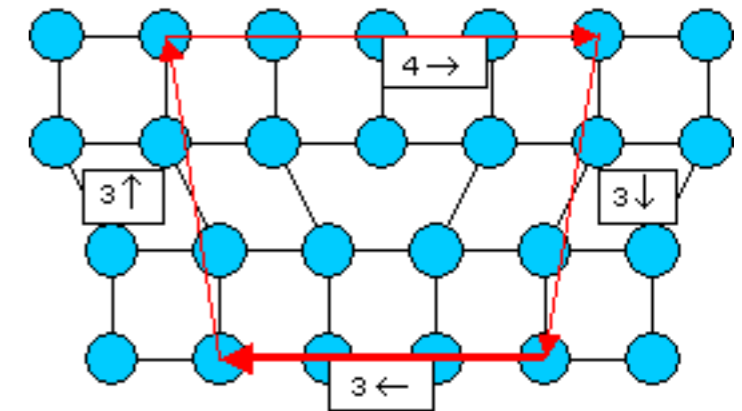
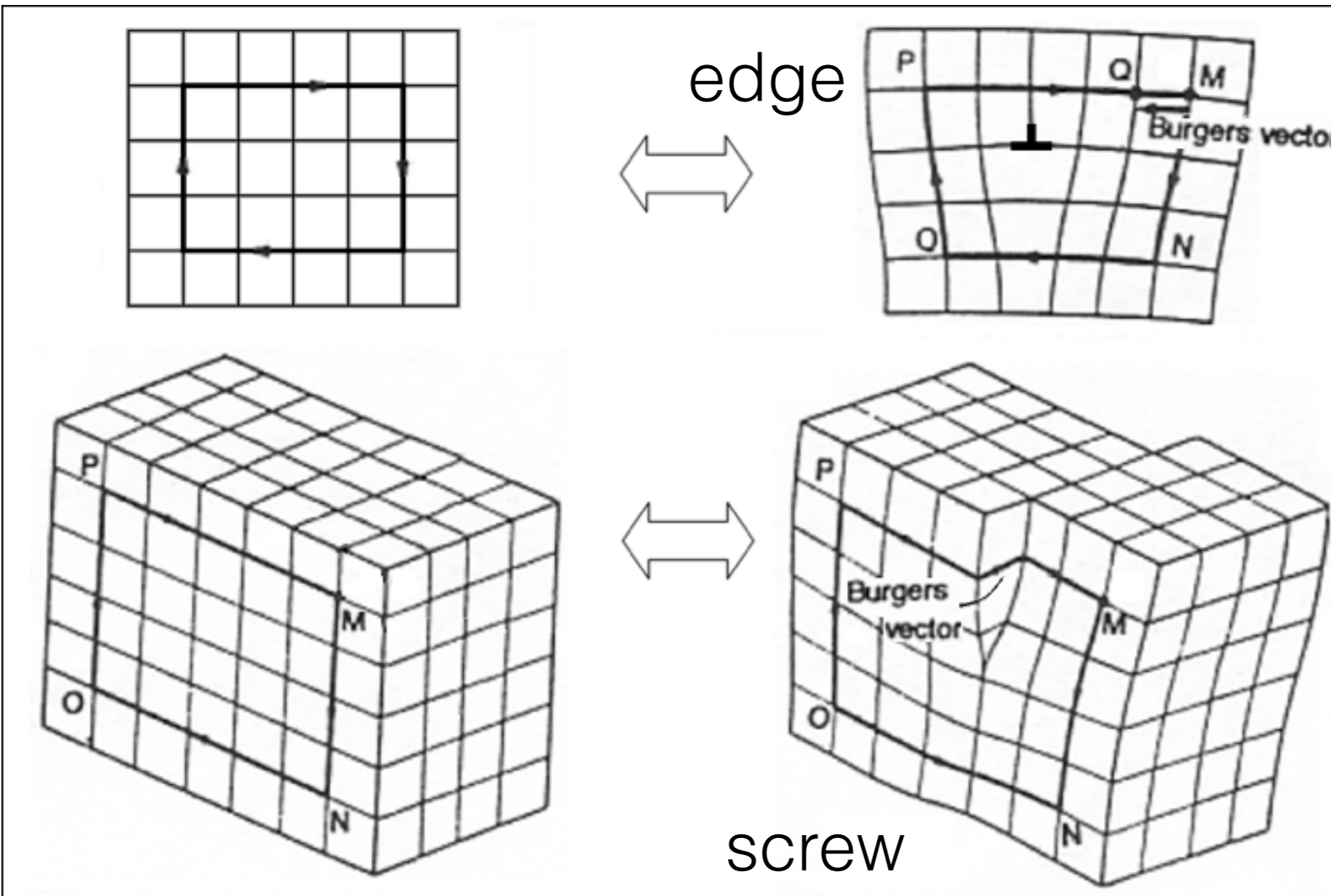
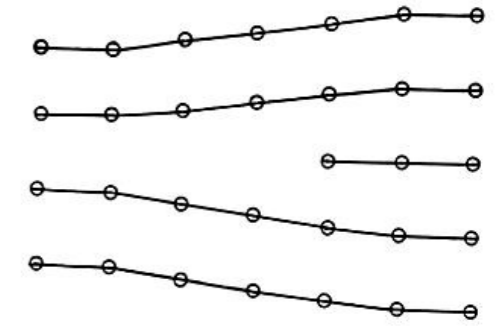
Topological defects



Work hardening

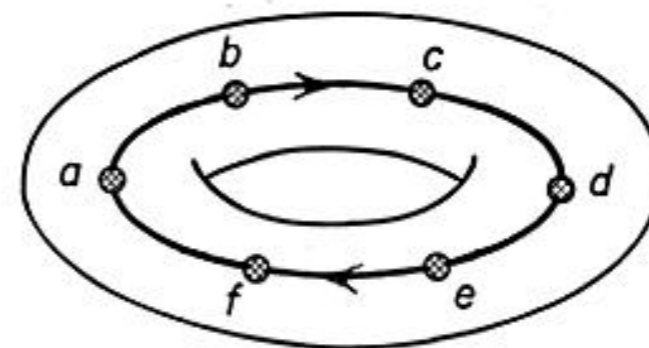
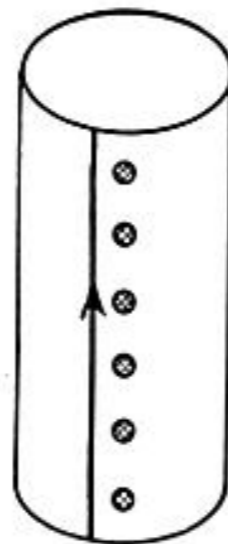
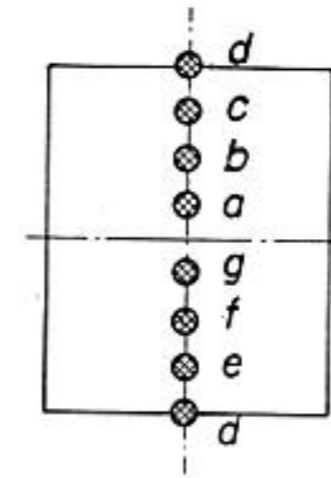
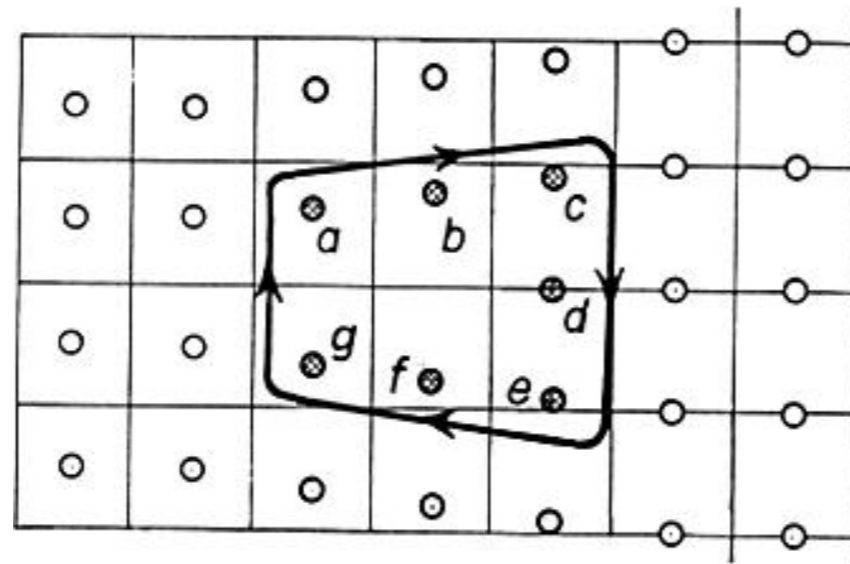
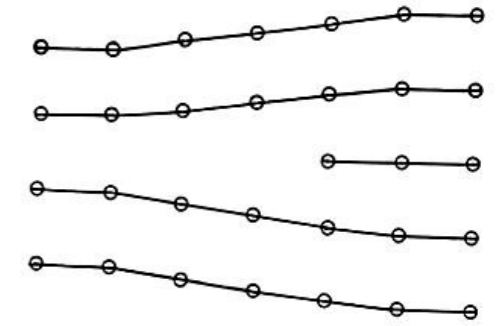


Disclinations



$$\|\mathbf{b}\| = (a/2)\sqrt{h^2 + k^2 + l^2}$$

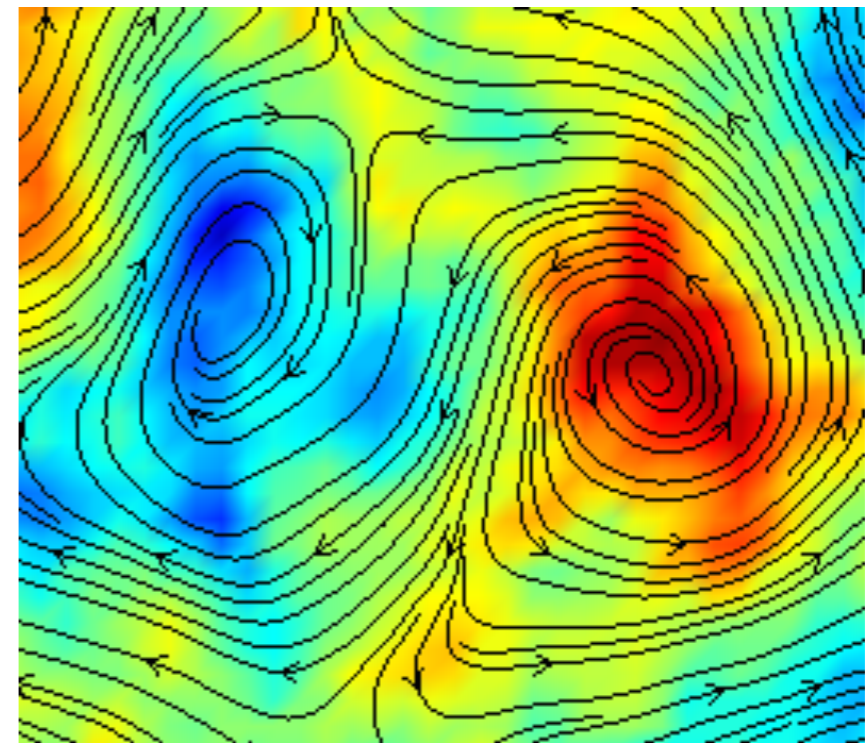
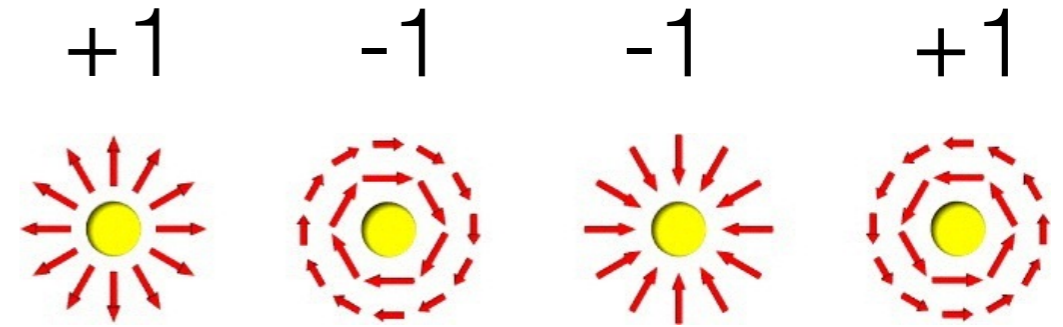
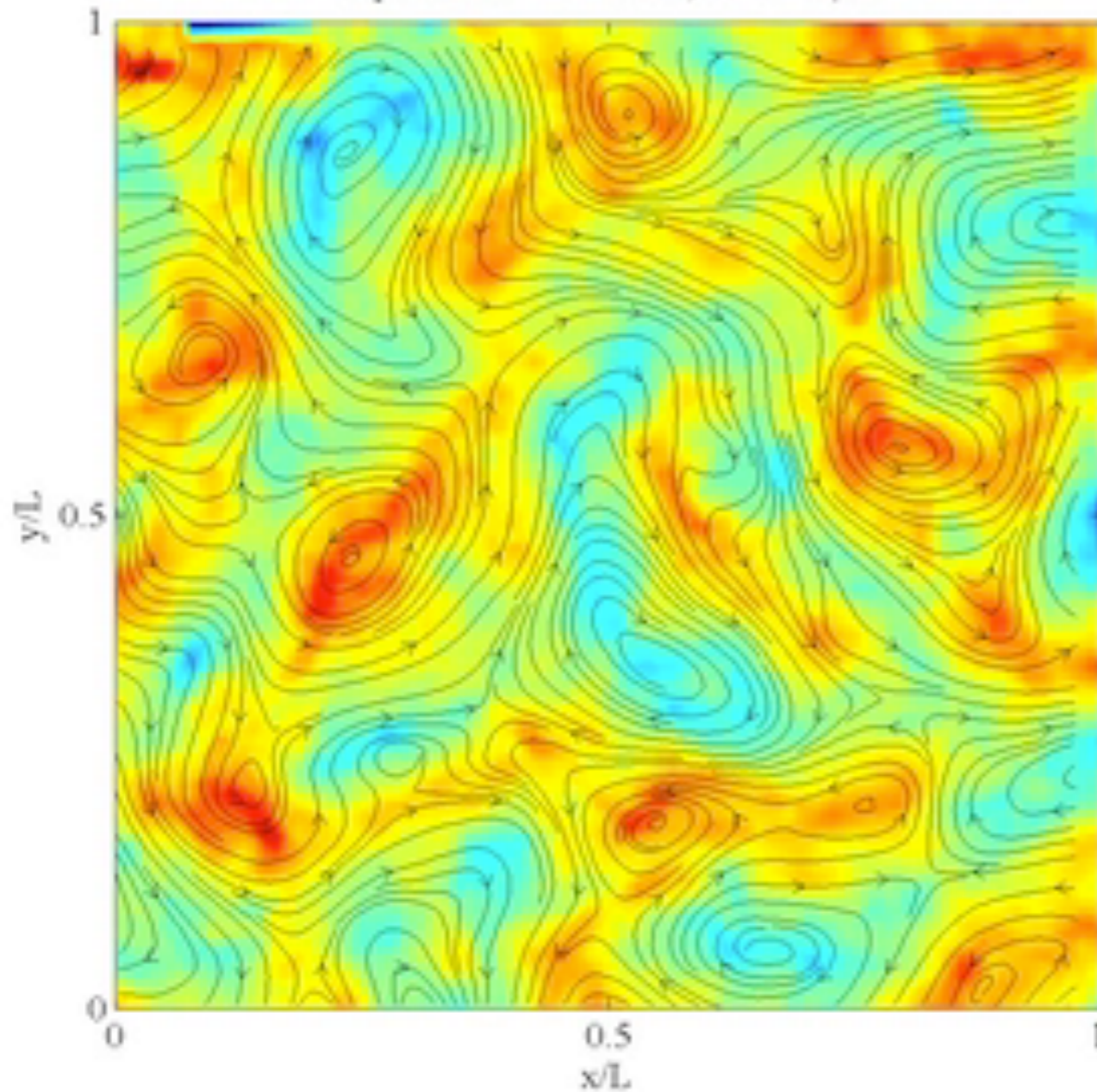
Disclinations



Bacterial vortices

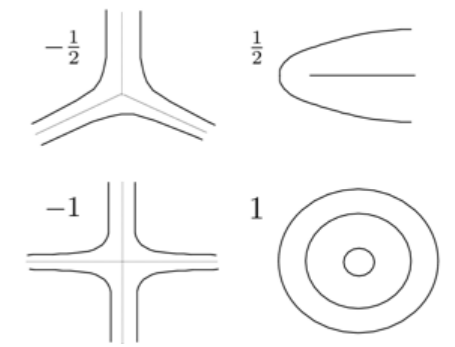
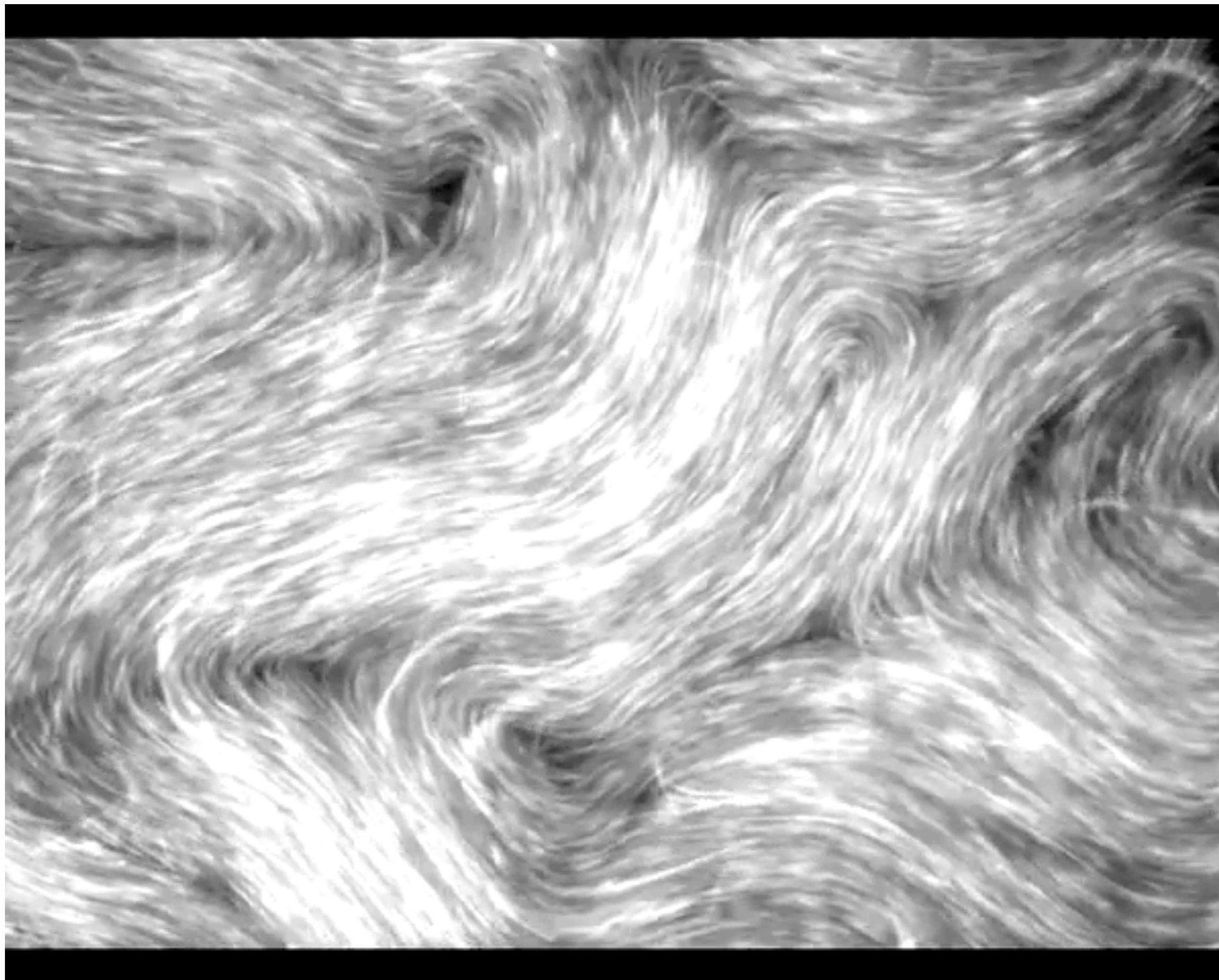
PIV

Experiment: $t = 0.1 \text{ s}$, $L = 276 \mu\text{m}$



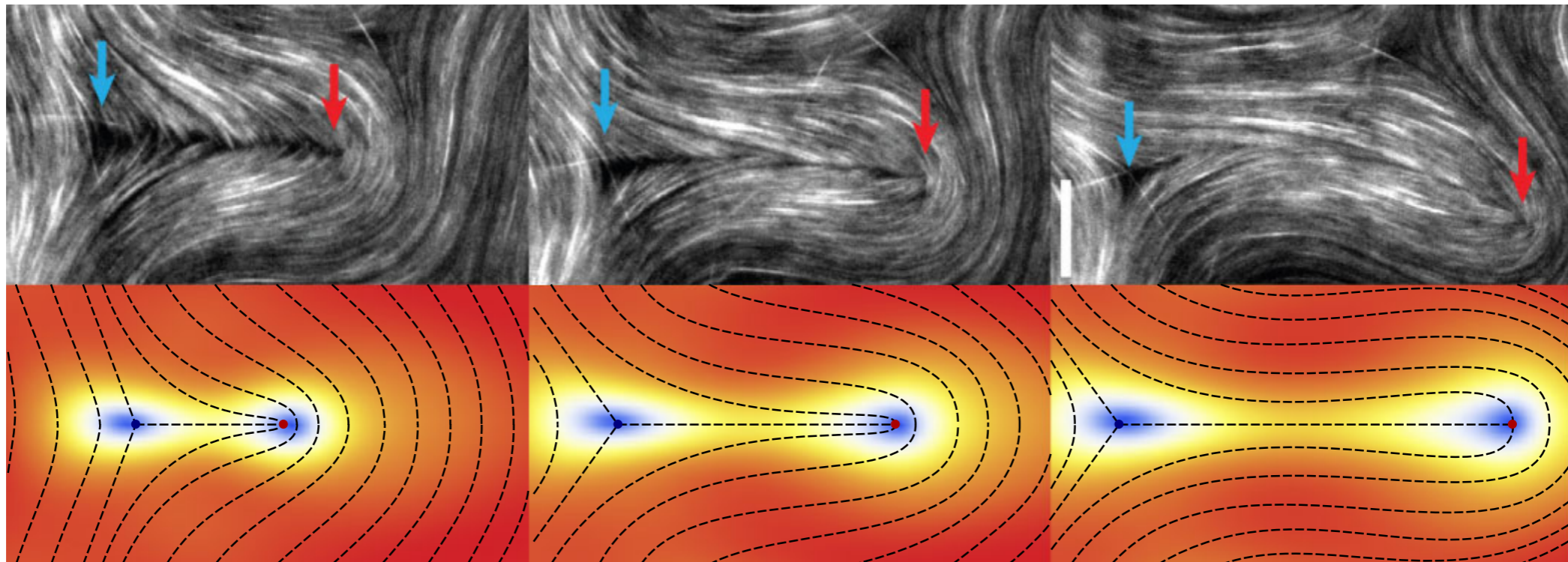
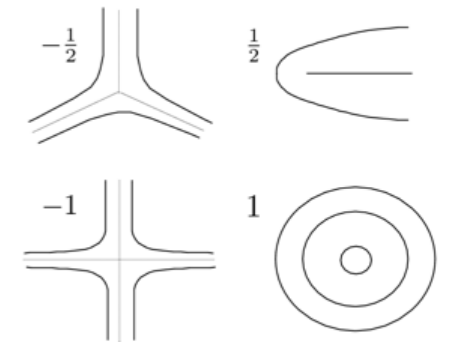
Dunkel et al PRL 2013

Active nematics



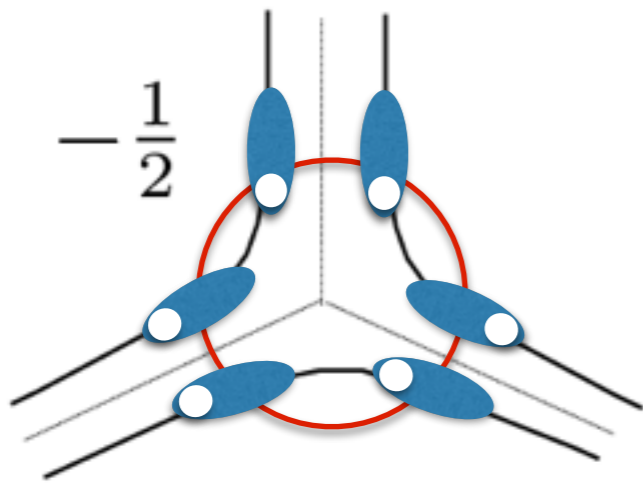
Dogic lab (Brandeis) Nature 2012

Active nematics

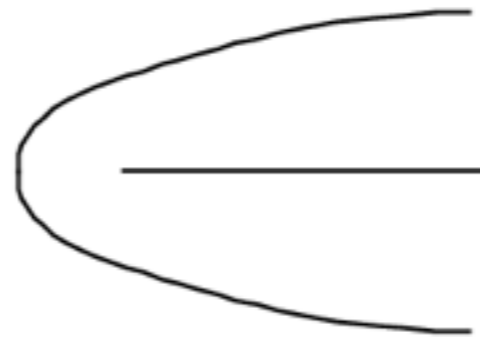


Giomi et al PRL 2012

Defects in nematics

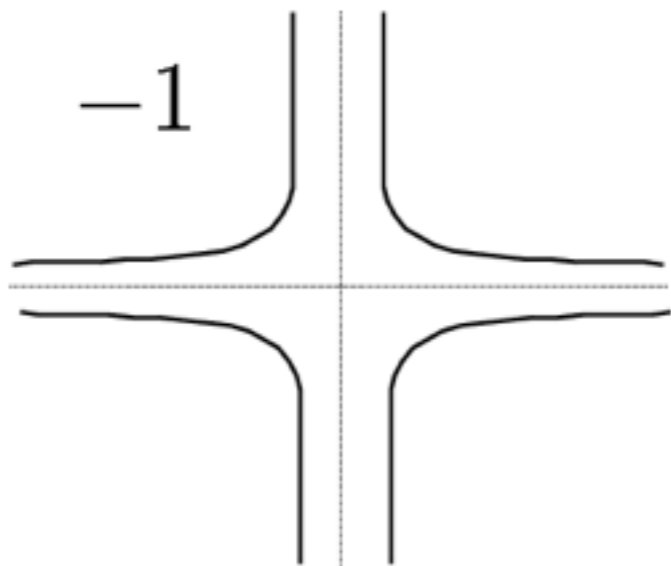


$\frac{1}{2}$

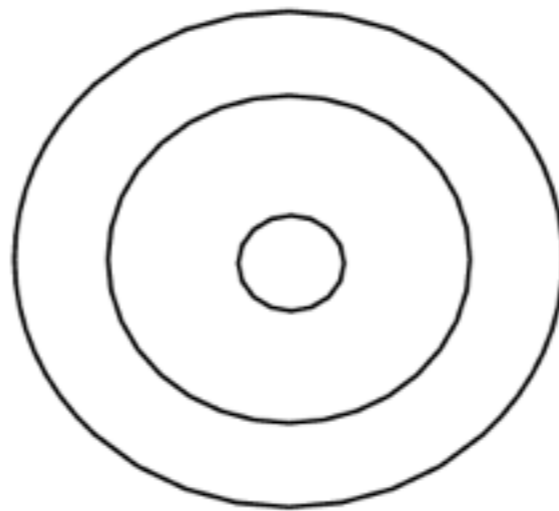


winding
number

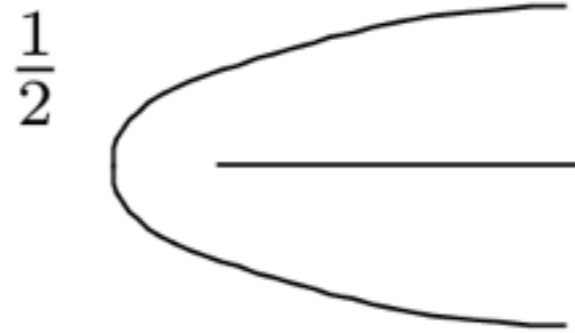
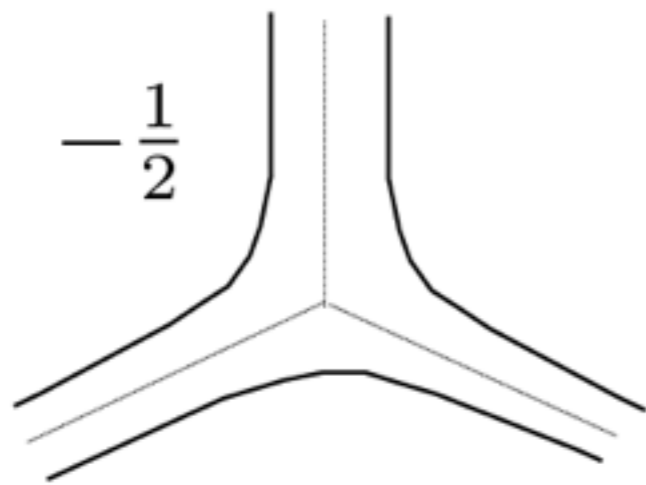
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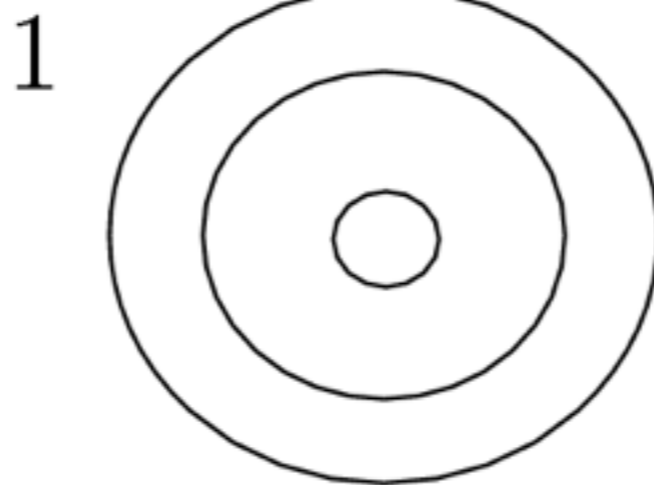
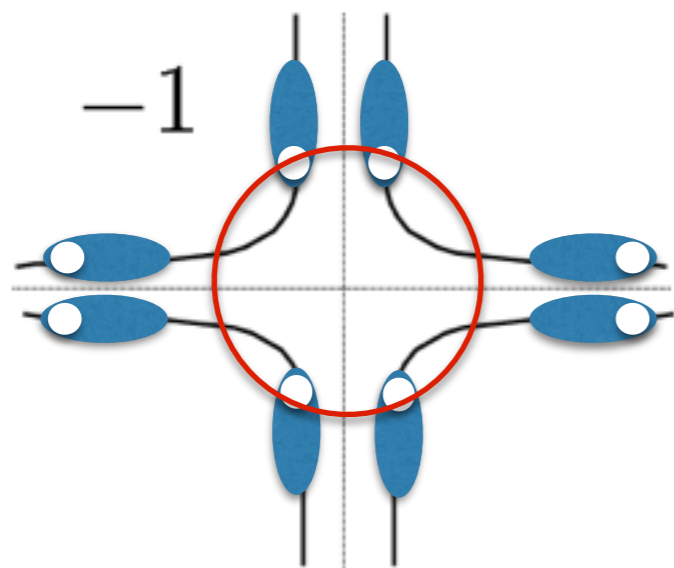
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Defects in nematics



winding
number

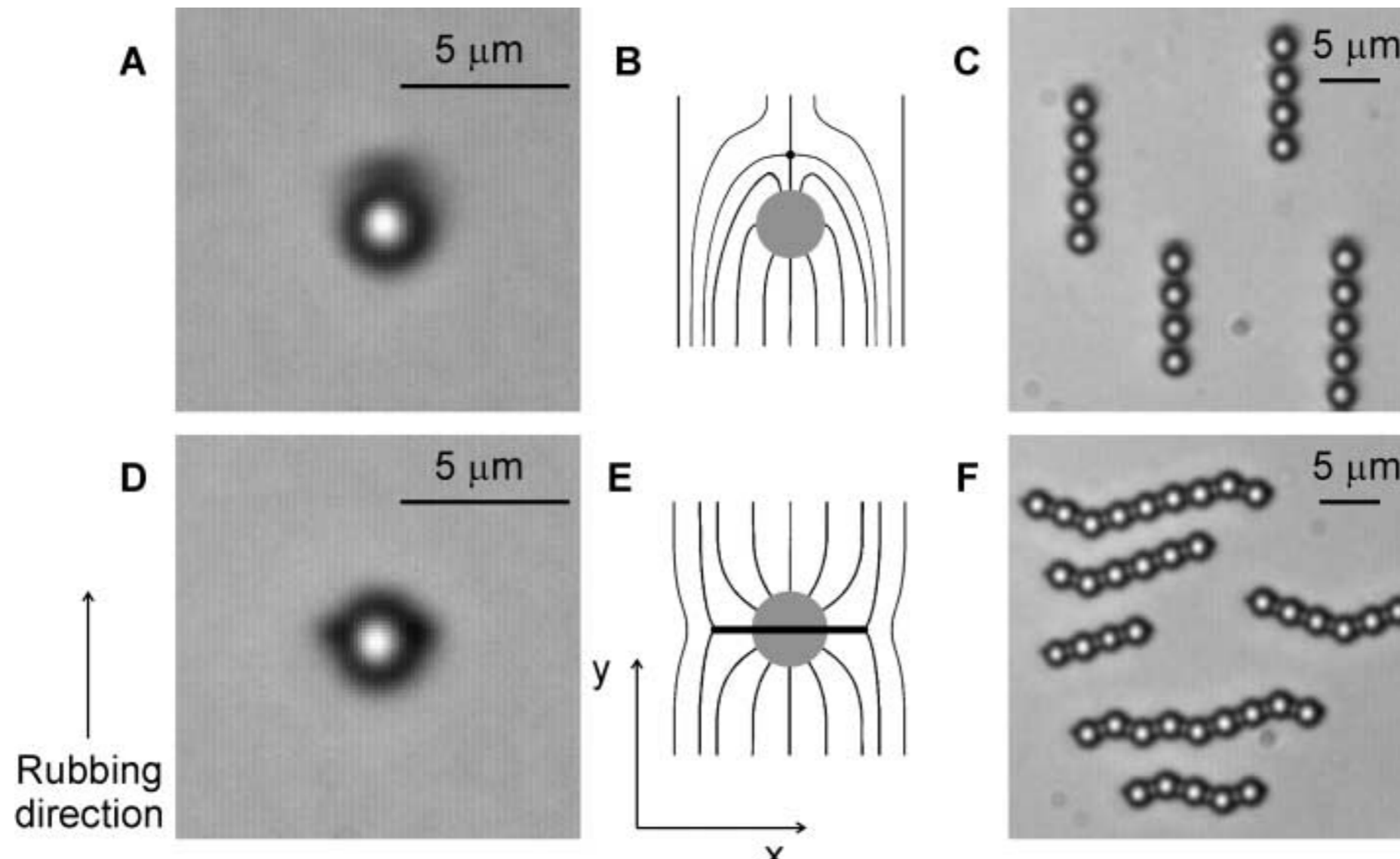


Two-Dimensional Nematic Colloidal Crystals Self-Assembled by Topological Defects

Igor Musevic *et al.*

Science **313**, 954 (2006);

DOI: 10.1126/science.1129660

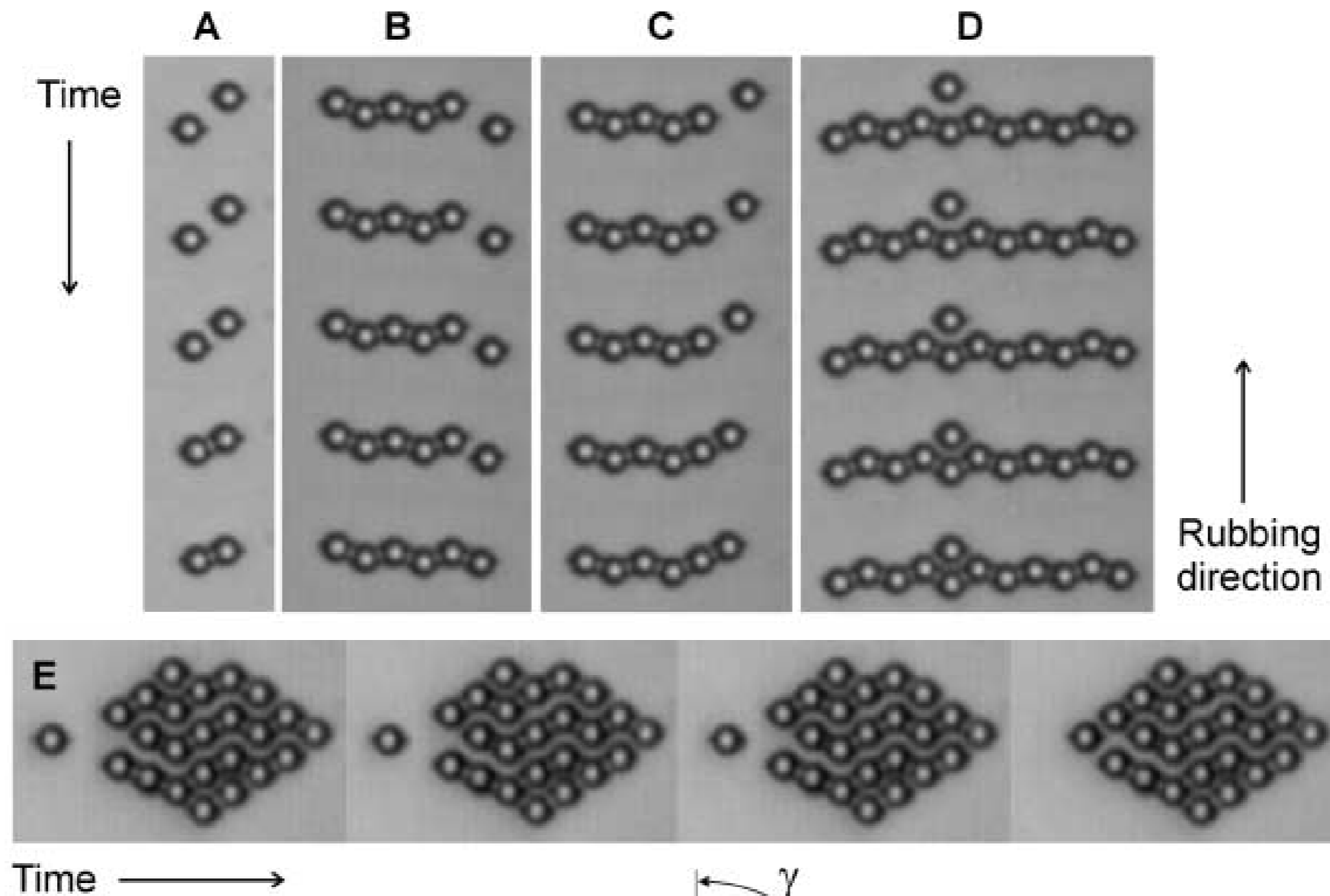


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Reconfigurable Knots and Links in Chiral Nematic Colloids

Uros Tkalec *et al.*

Science **333**, 62 (2011);

DOI: 10.1126/science.1205705

