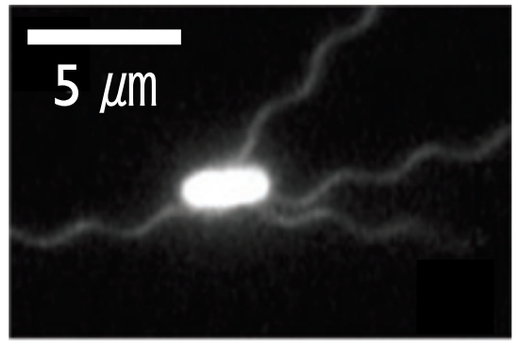
LowRe hydrodynamics
&
microbial locomotion

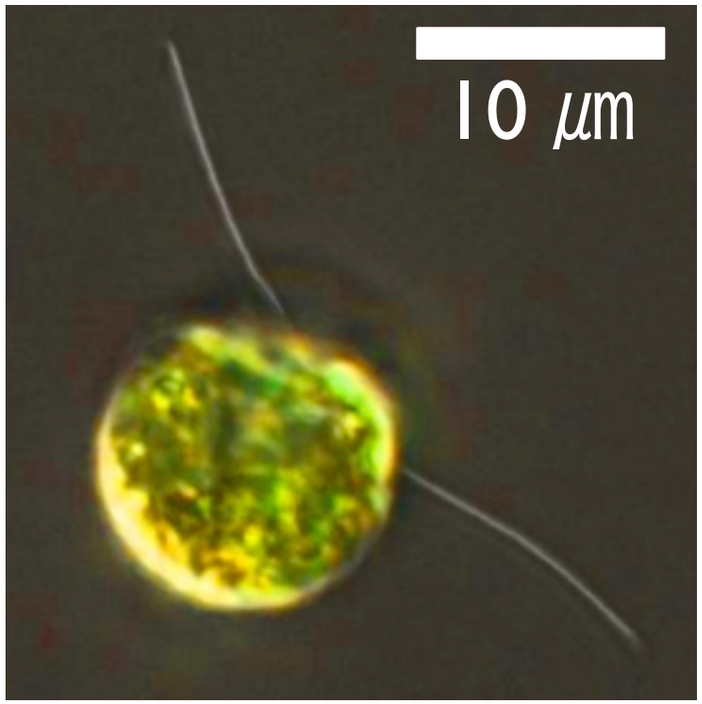
18.354 - L15

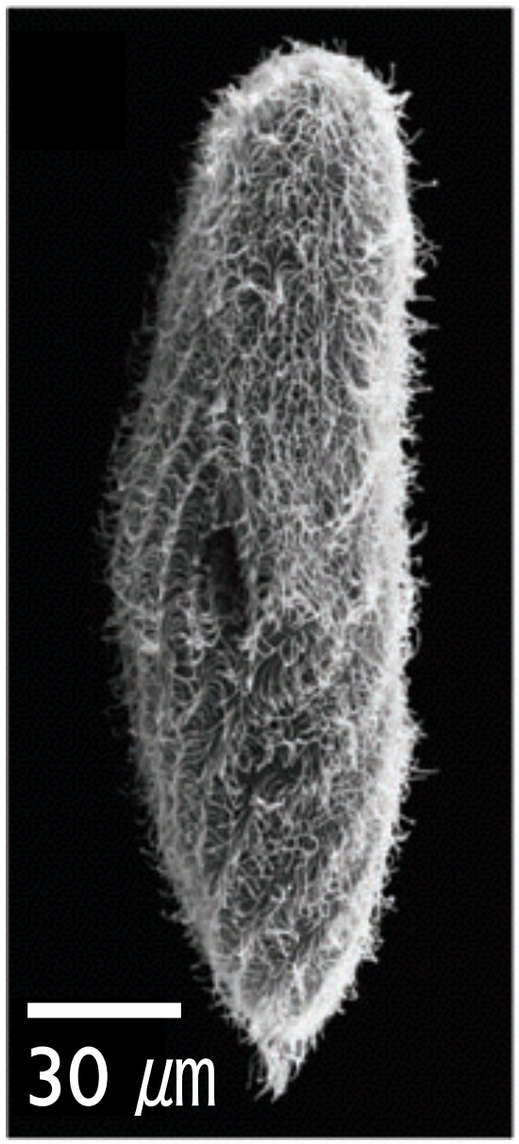
dunkel@mit.edu

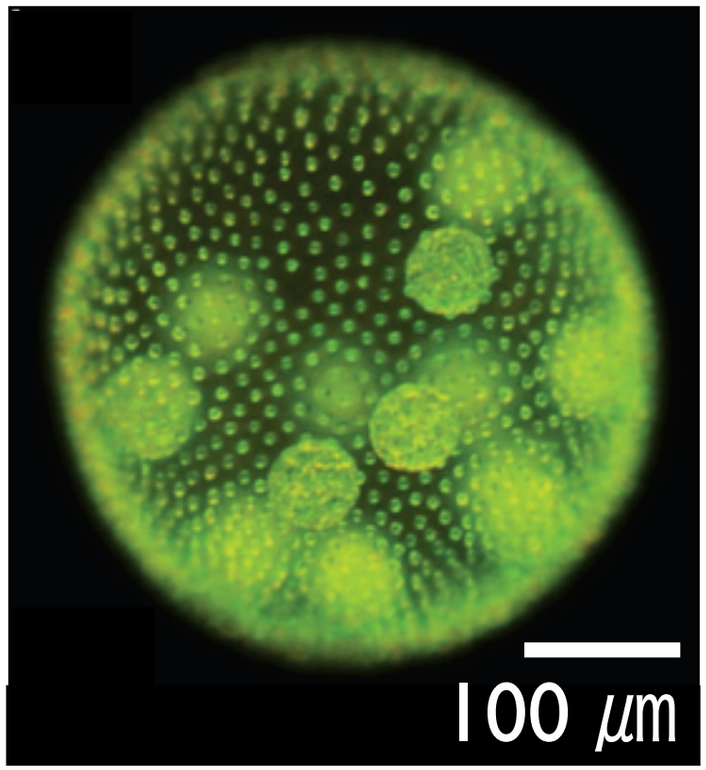
Why microbial hydrodynamics ?

- micro-machines
- hydrodynamic propulsion
- > 50% global biomass
- gut flora, biofilms, ...
- global food web
- > 50% global carbon fixation







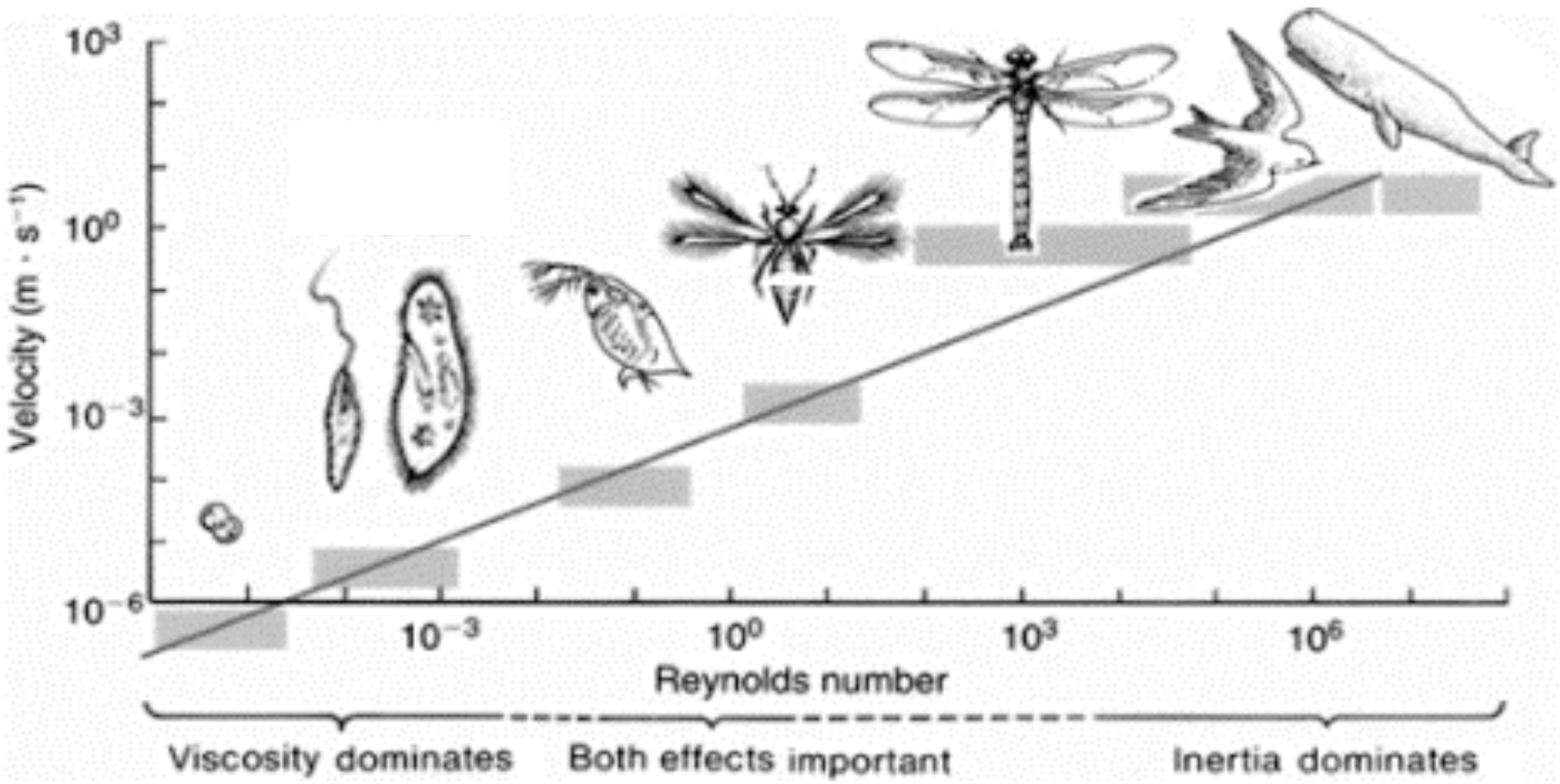


Whitman et al (1998) PNAS

Guasto et al (2012) Annu Rev Fluid Mech

Reynolds numbers

$$Re = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$



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Swimming at low Reynolds number

Navier - Stokes:

$$-\nabla p + \eta \nabla^2 \vec{v} = \rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v}$$

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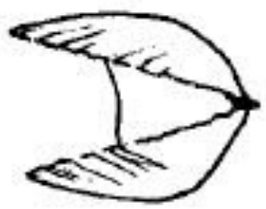
The Scallop Theorem



American Journal of Physics, Vol. 45, No. 1, January 1977

Geoffrey Ingram Taylor

James Lighthill

$$0 = \mu \nabla^2 \boldsymbol{u} - \nabla p + \boldsymbol{f},$$
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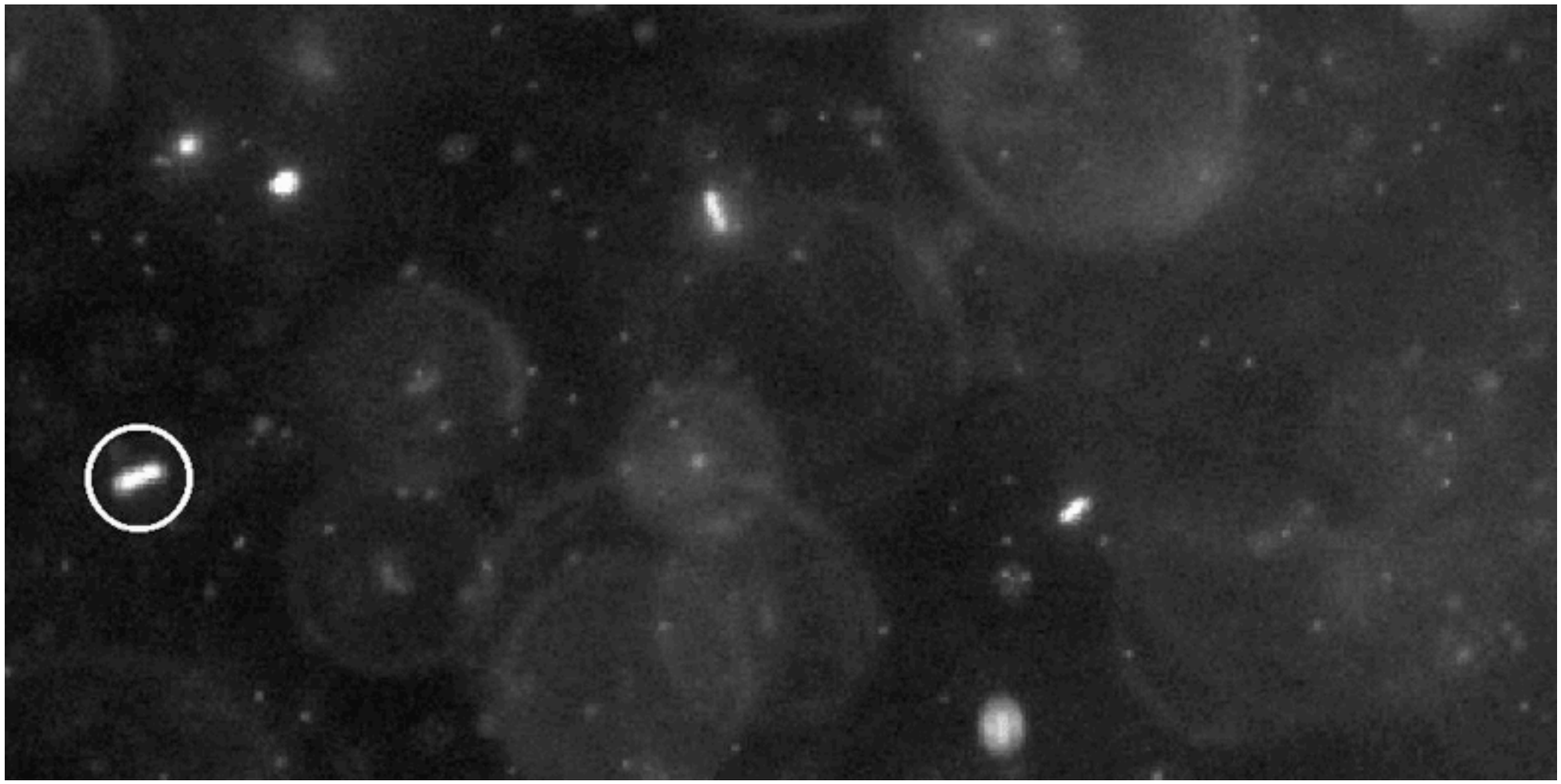
\+ time-dependent BCs

Edward Purcell

Shapere & Wilczek (1987) PRL

Zero-Re flow

E.coli (non-tumbling HCB 437)



Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

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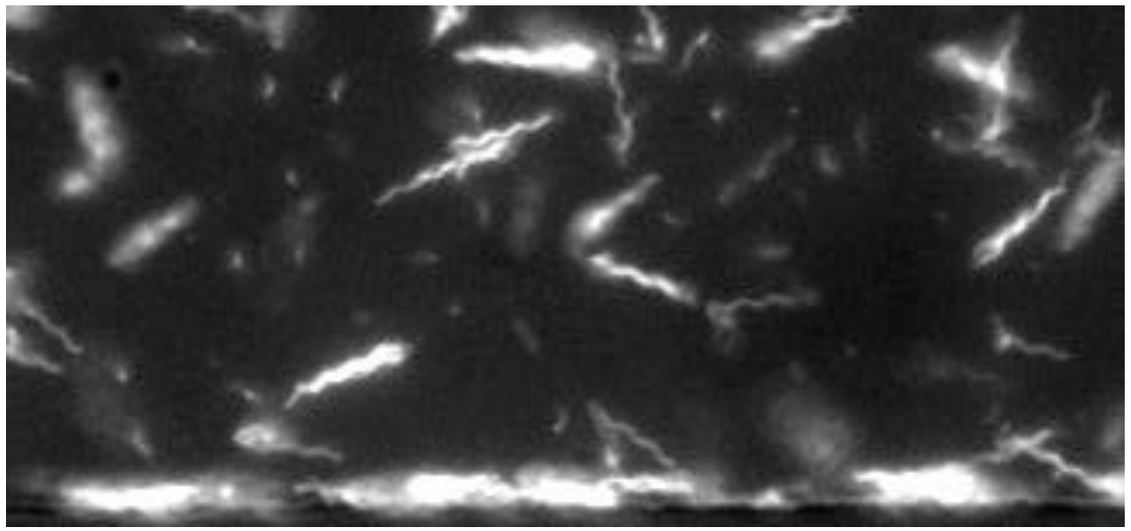
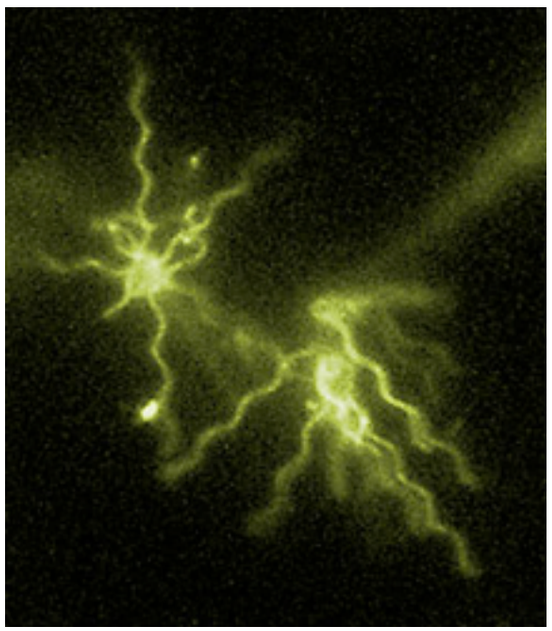
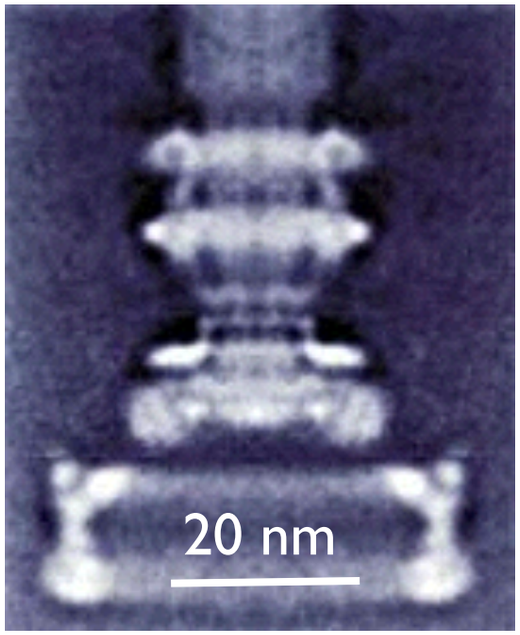
Bacterial motors

movie: V. Kantsler



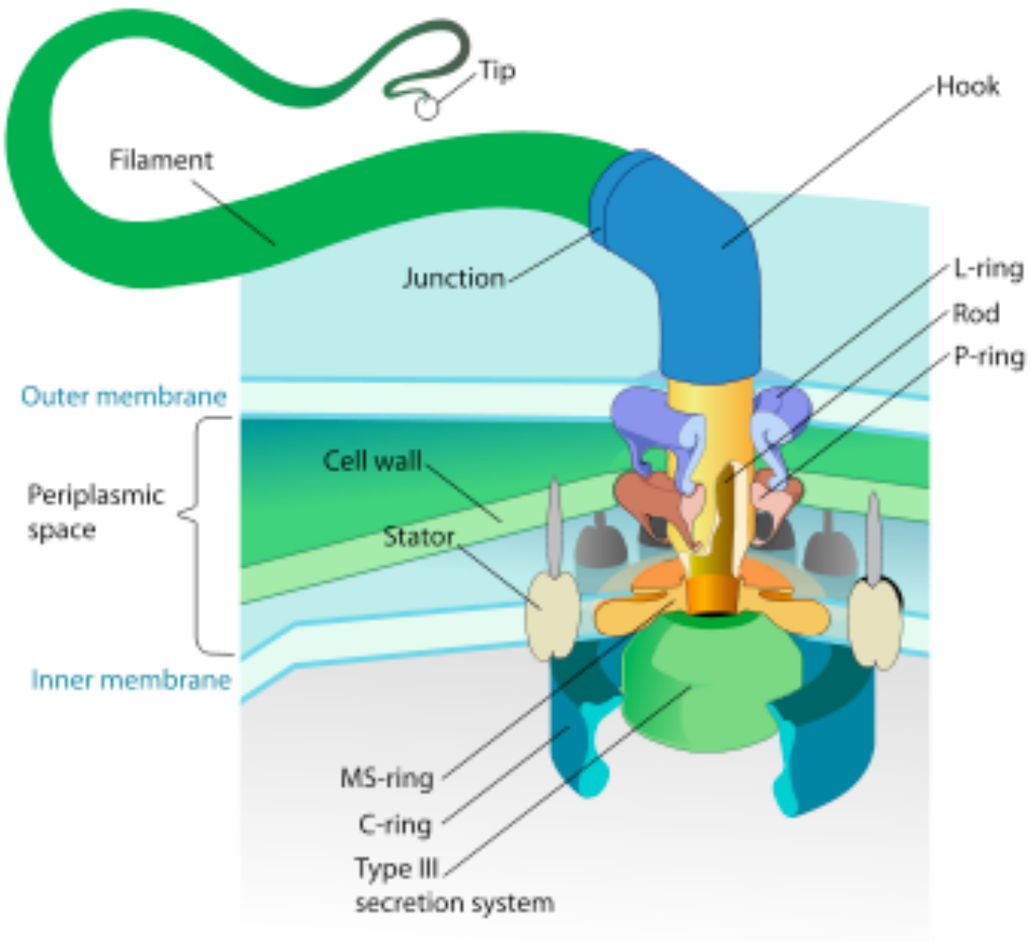


Berg (1999) Physics Today



Chen et al (2011) EMBO Journal

~20 parts



source: wiki

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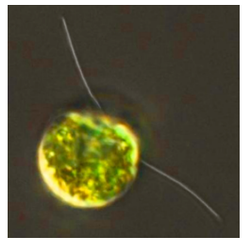
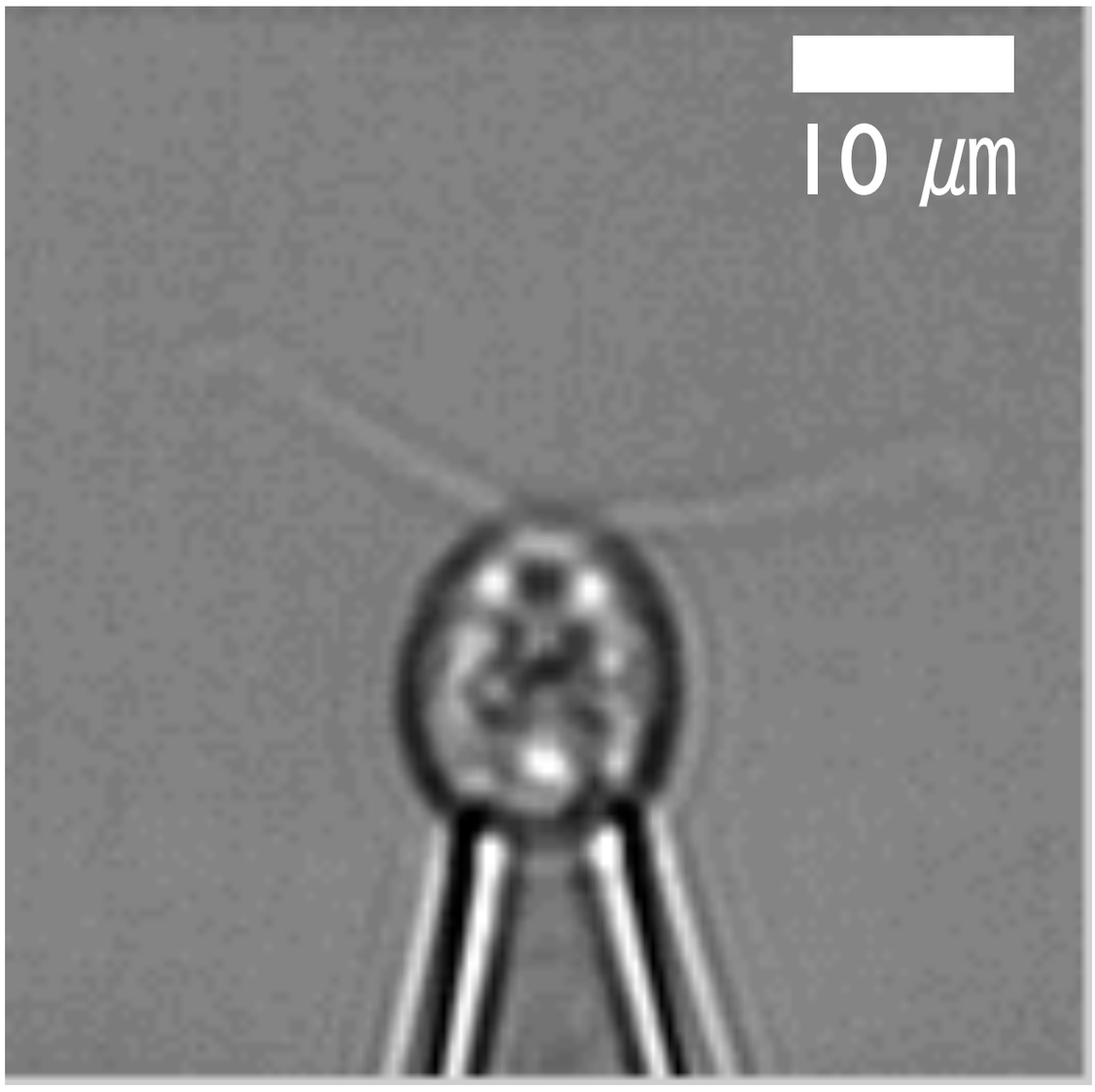
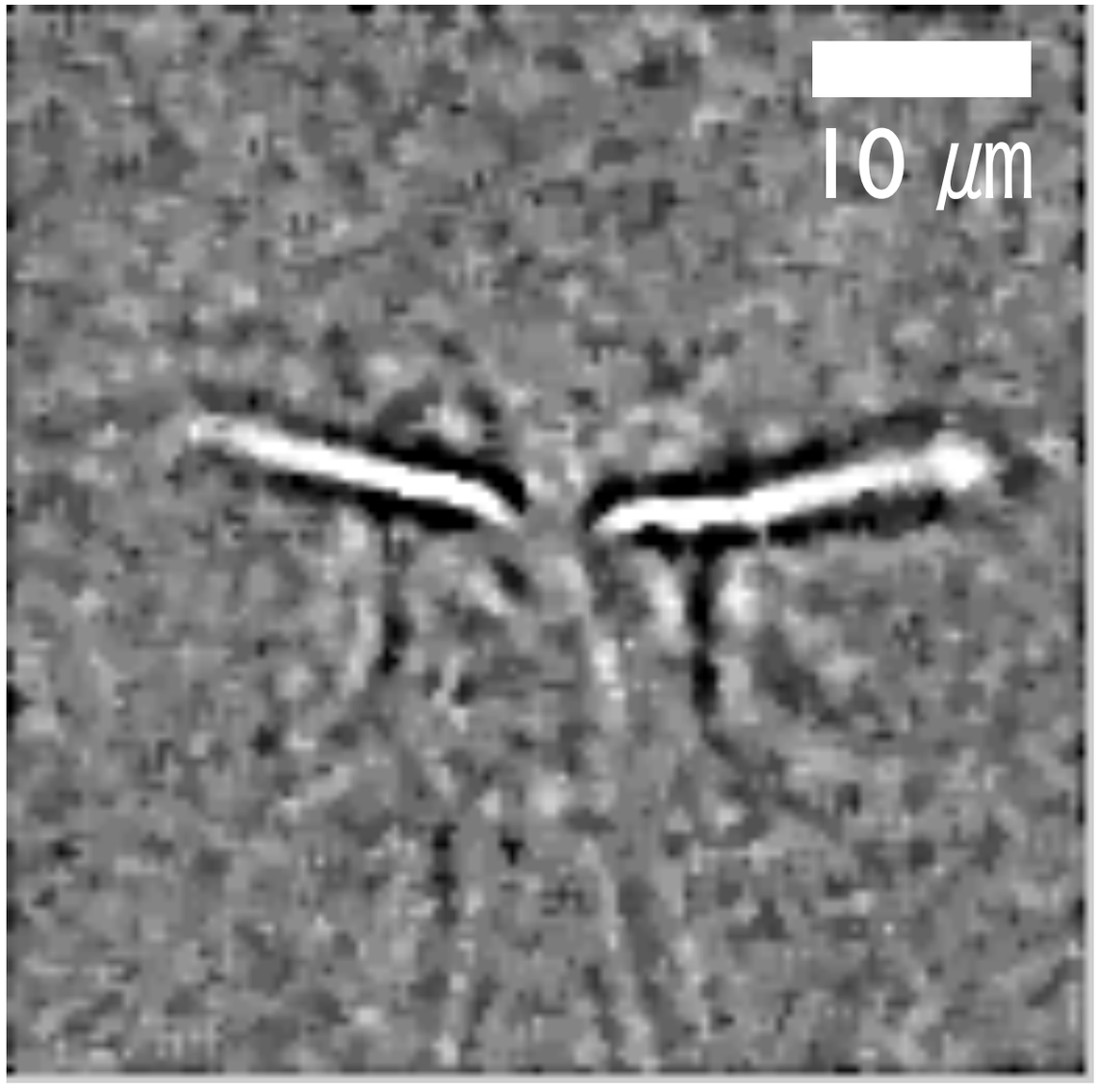
Chlamydomonas alga







~ 50 beats / sec

speed ~100 μm/s

Goldstein et al (2011) PRL

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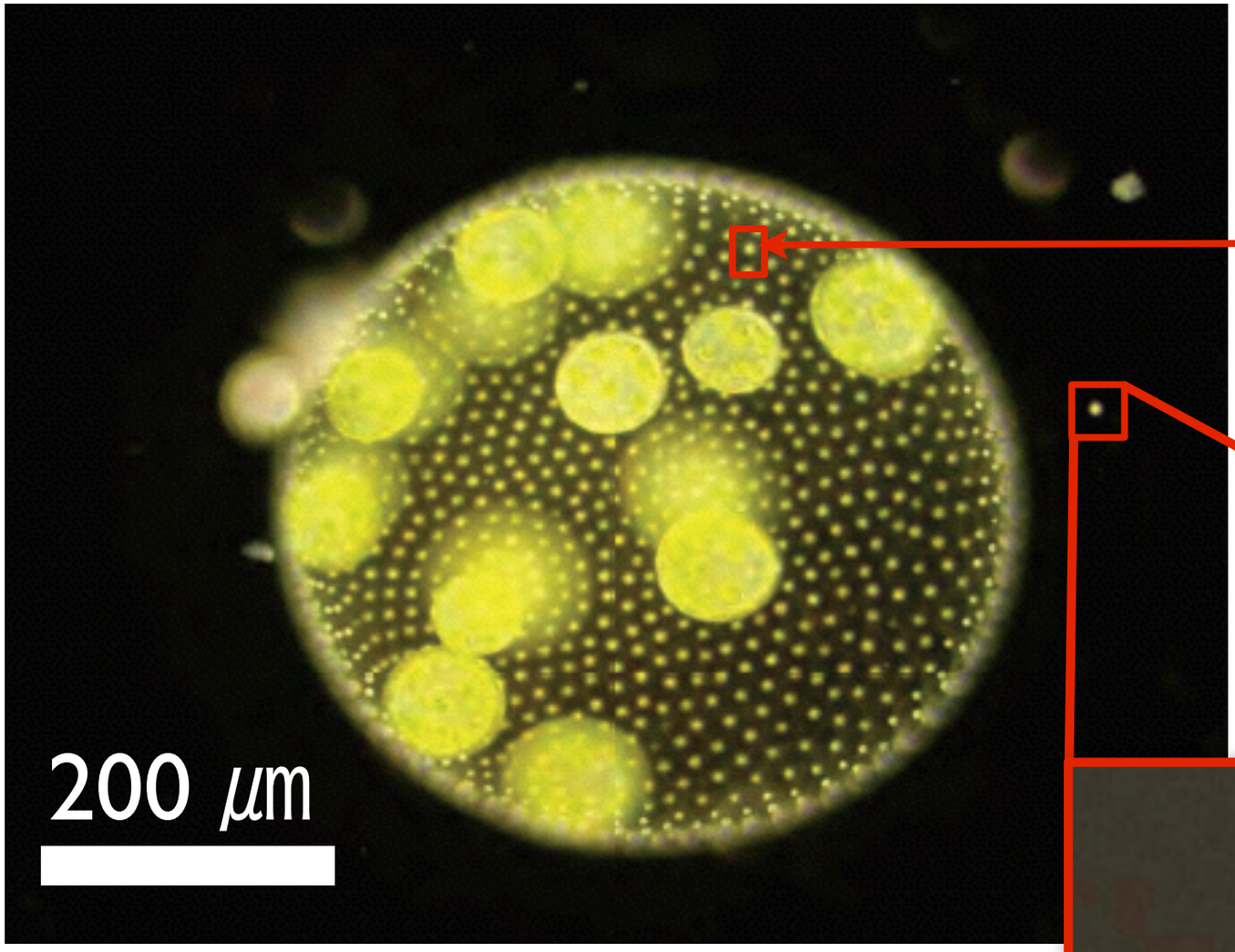
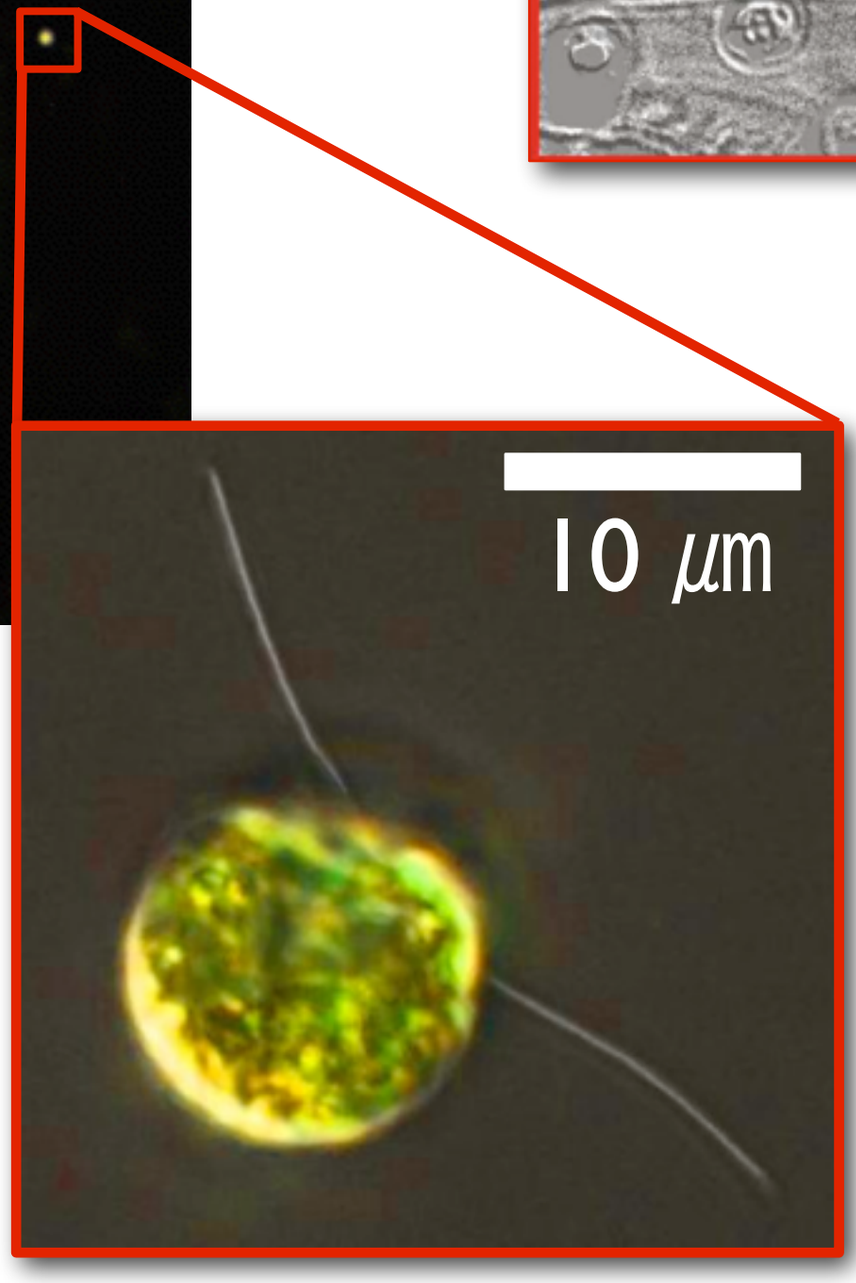
Volvox carteri





Chlamydomonas reinhardtii

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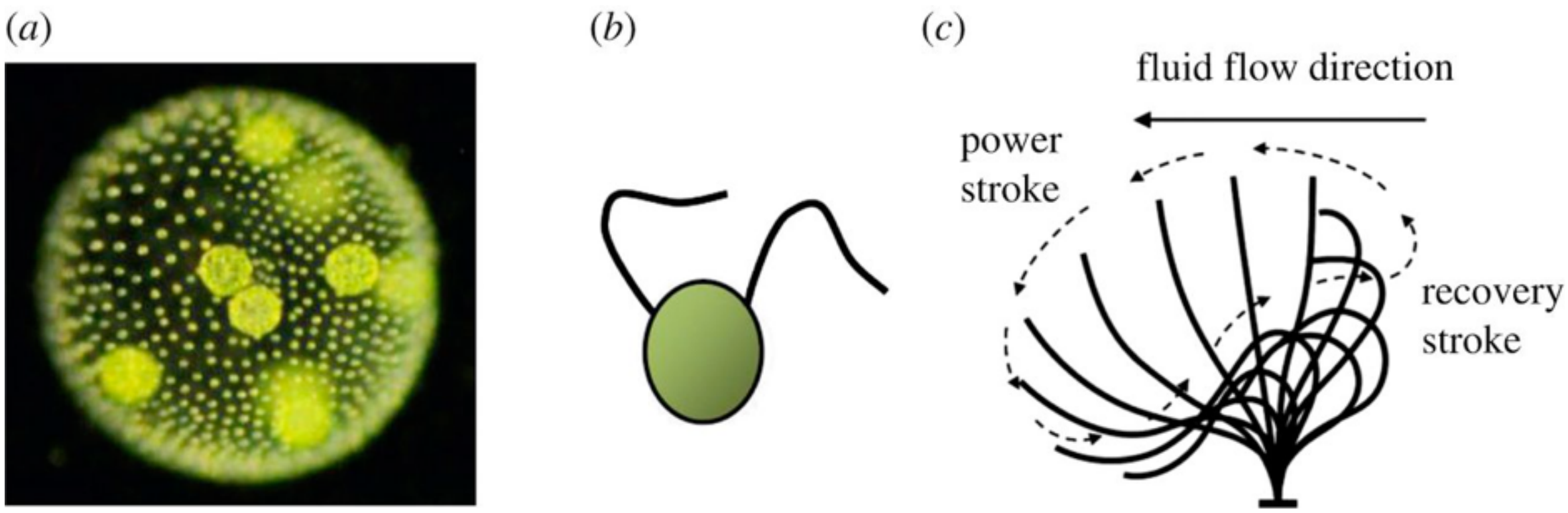
Stroke



Sareh et al (2013) J Roy Soc Interface

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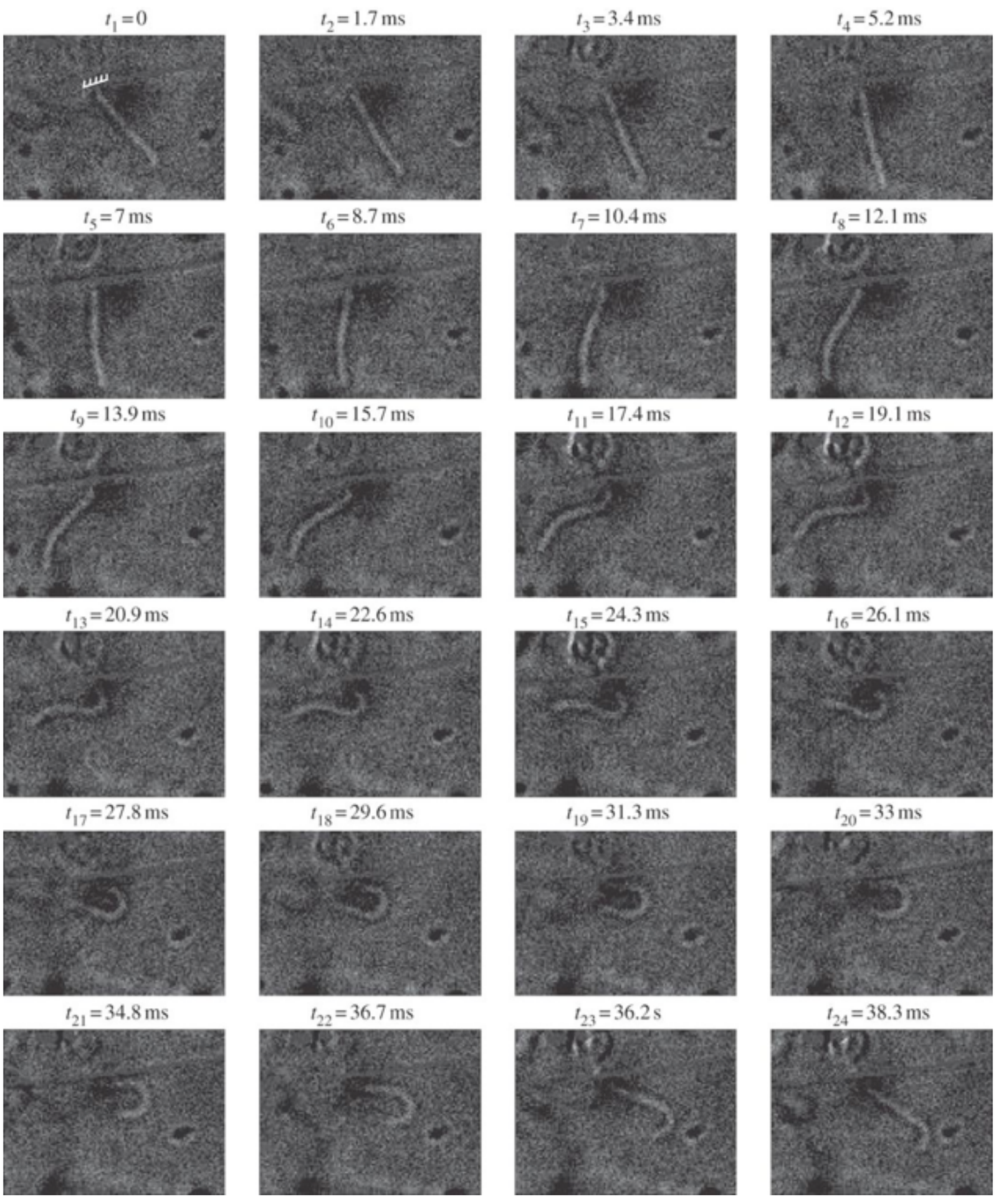

Volvox carteri

beating frequency

25Hz

Sareh et al (2013)
J Roy Soc Interface

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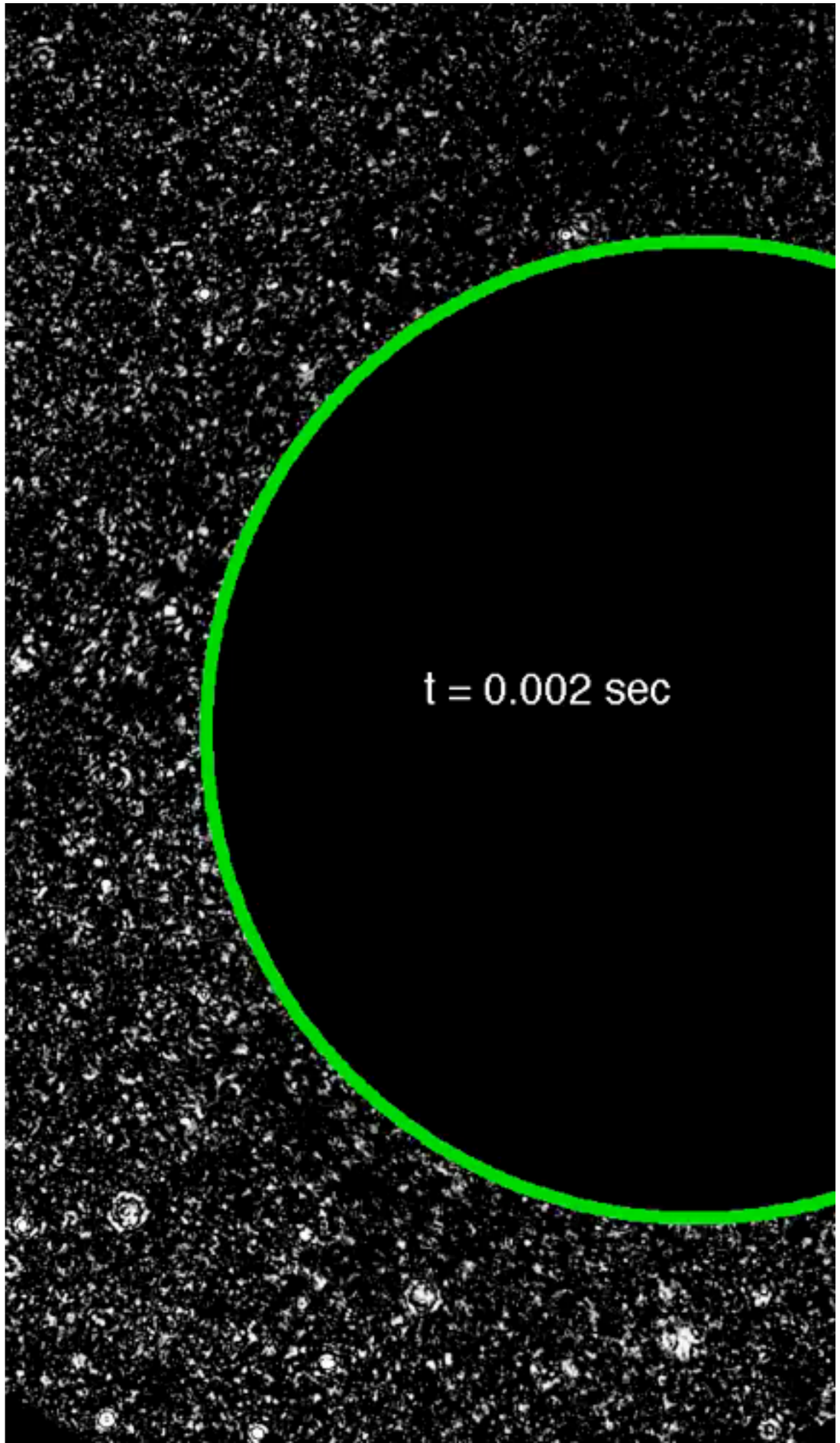

Meta-chronal waves

Brumley et al (2012) PRL

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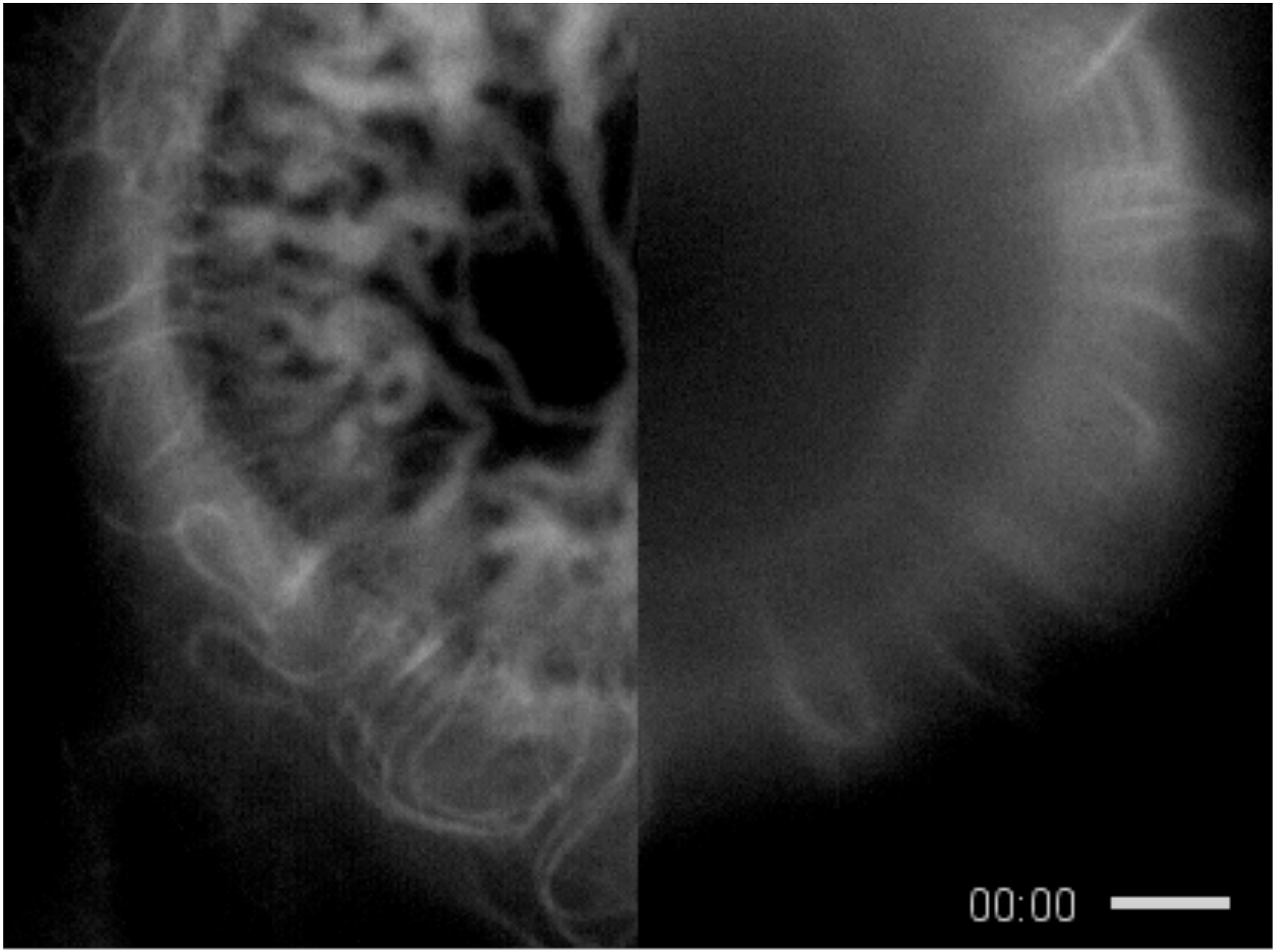
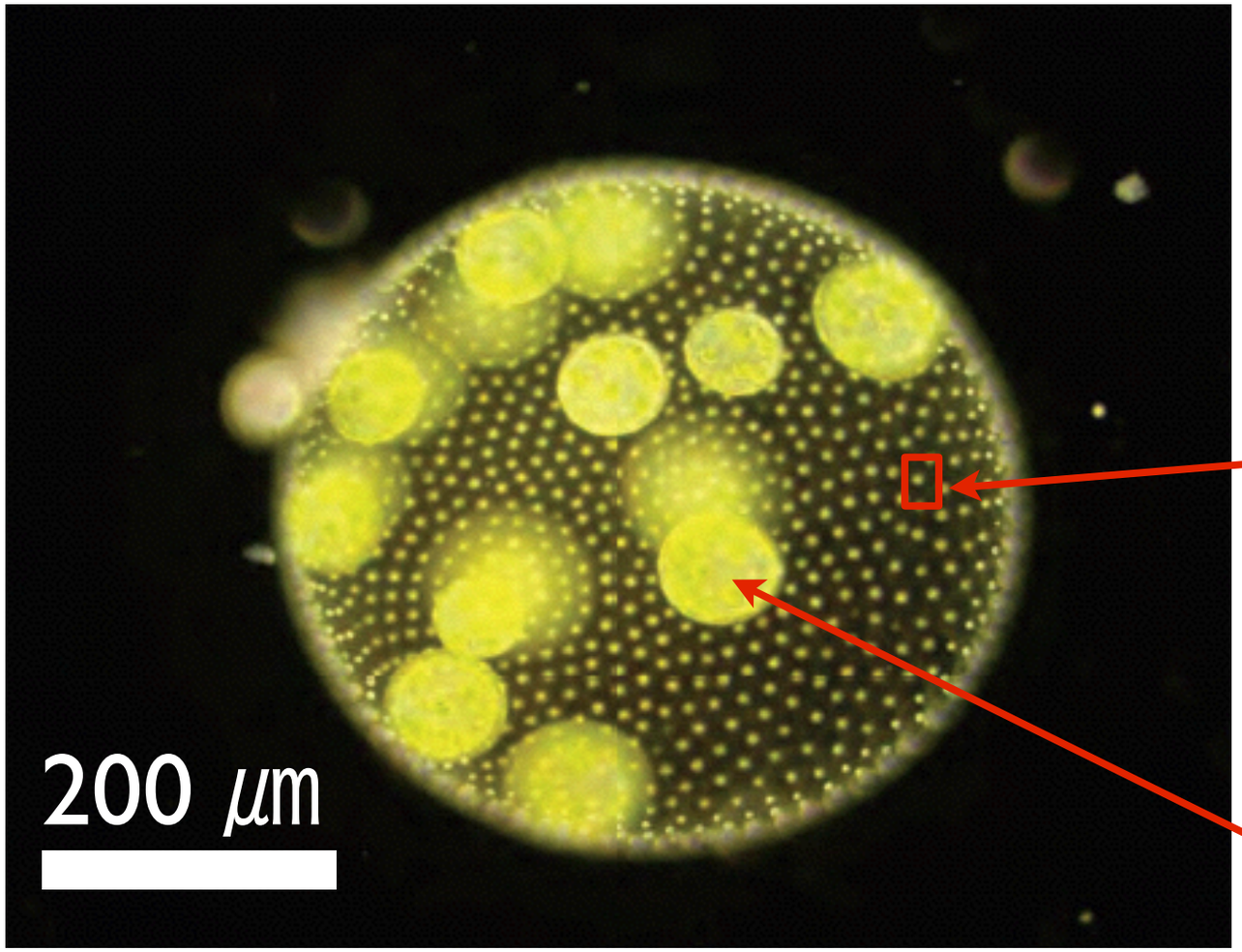
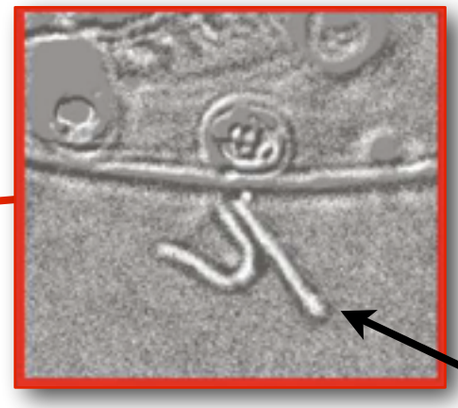


Dogic lab (Brandeis)

dunkel@math.mit.edu

Volvox carteri

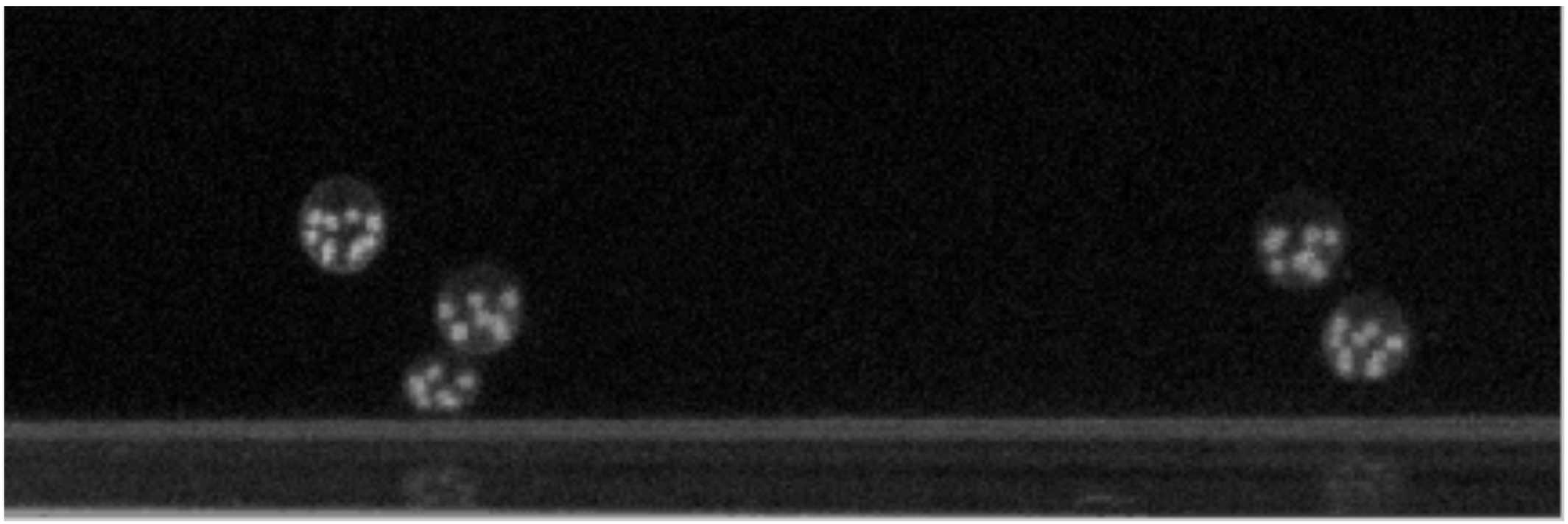




somatic cell

cilia

daughter colony



Drescher et al (2010) PRL

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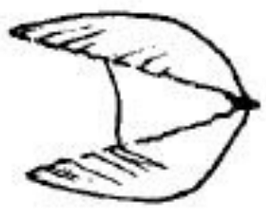

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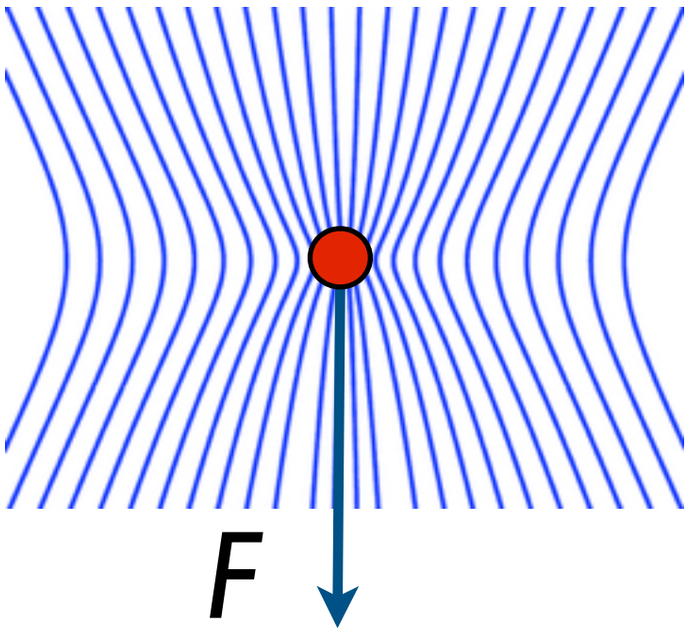
Edward Purcell

Shapere & Wilczek (1987) PRL

Superposition of singularities

stokeslet

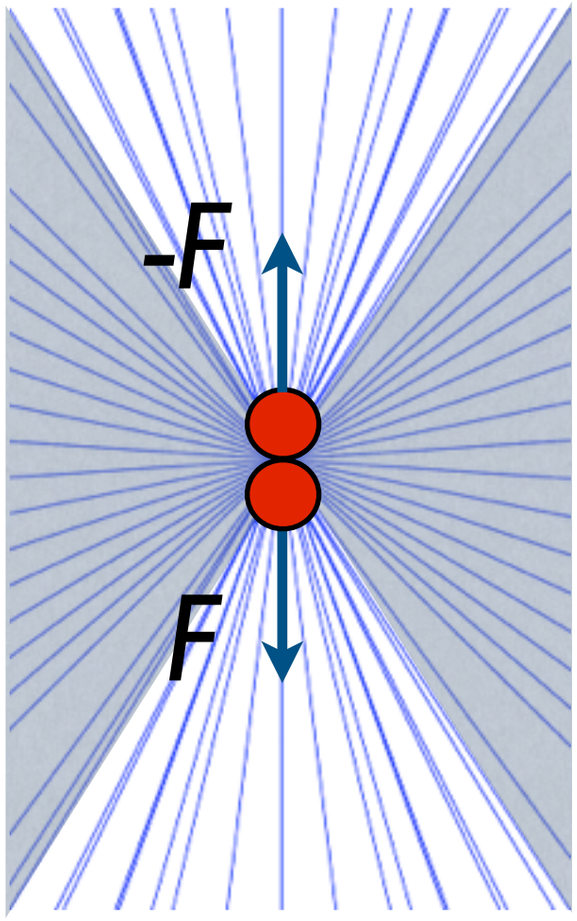


$$p(\boldsymbol{r}) = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{F}}{4\pi r^2} + p_0$$

$$v_i(\boldsymbol{r}) = \frac{(8\pi\mu)^{-1}}{r}[\delta_{ij} + \hat{r}_i\hat{r}_j]F_j$$

flow ~ r^{-1}

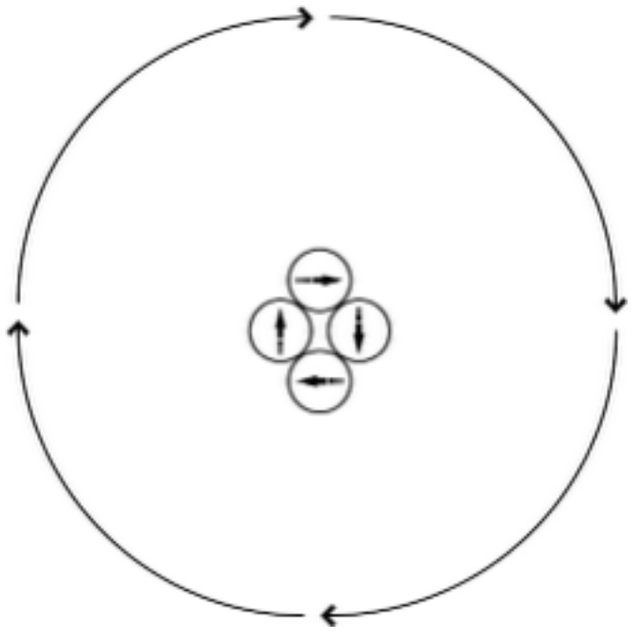
2x stokeslet =
symmetric dipole



r^{-2}

‘pusher’

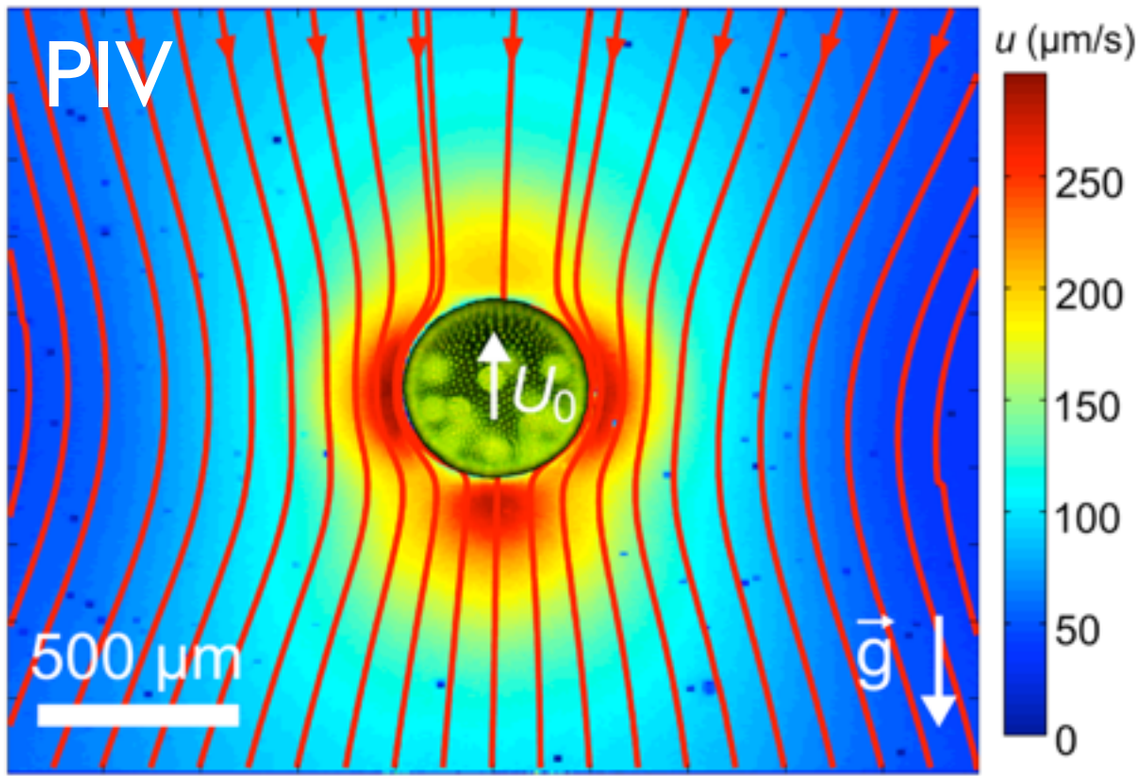
rotlet



r^{-2}

Volvox

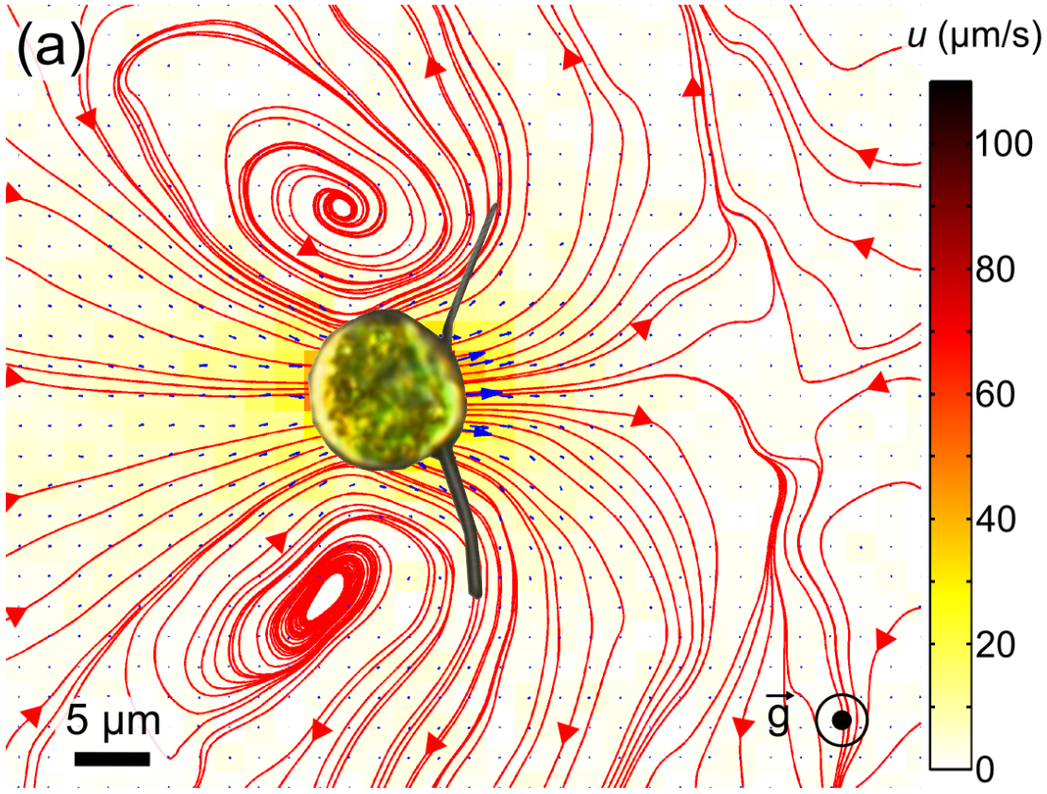
swimming speed ~ 100 μm/sec



$$\mathbf{v}_{fit}(\mathbf{r}) = -\ U_0\,\hat{\mathbf{y}} - \frac{A_{St}}{r}\,(\mathbf{I}+\hat{\mathbf{r}}\hat{\mathbf{r}})\cdot\hat{\mathbf{y}} \qquad (1)$$
$$+\ \frac{A_{str}}{r^2}\,(1-3(y/r)^2)\,\hat{\mathbf{r}} - \frac{A_{sd}}{r^3}\left(\frac{\mathbf{I}}{3}-\hat{\mathbf{r}}\hat{\mathbf{r}}\right)\cdot\hat{\mathbf{y}}$$

Chlamy

swimming speed ~ 50 μm/sec



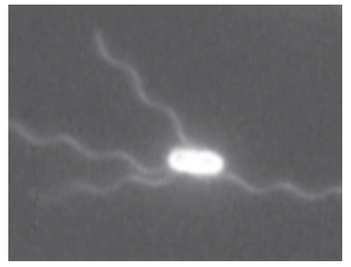
$$3\mathrm{D}: \quad \boldsymbol{v} \sim 1/r^2$$
$$2\mathrm{D}: \quad \boldsymbol{v} \sim 1/r$$

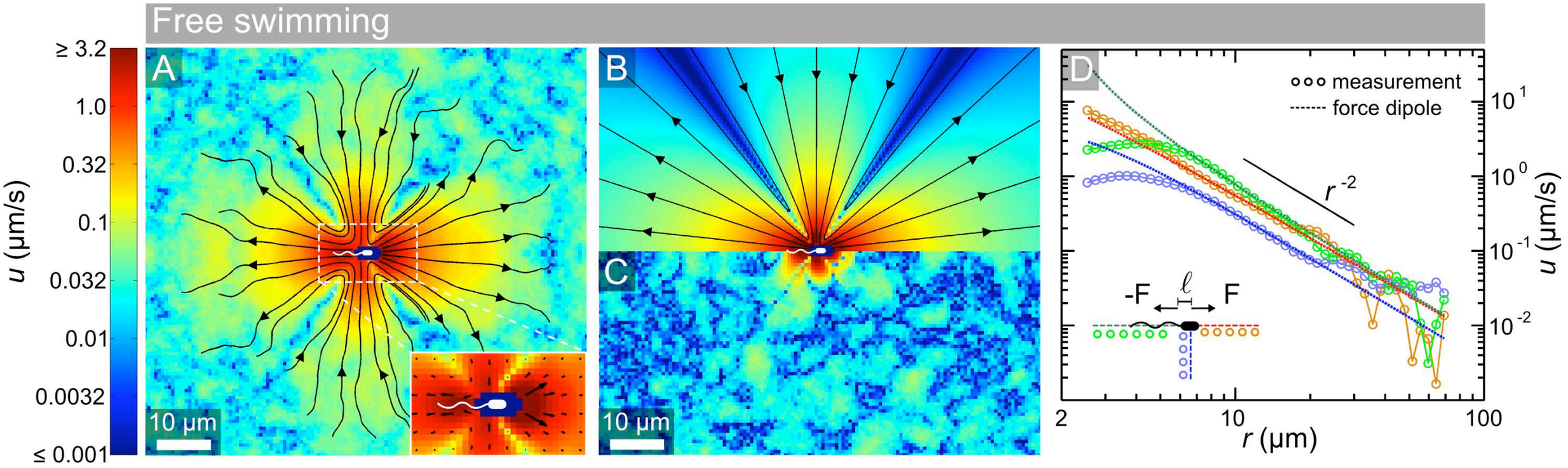
... no dipoles !

Drescher et al (2010) PRL

Guasto et al (2010) PRL

E.coli (non-tumbling HCB 437)





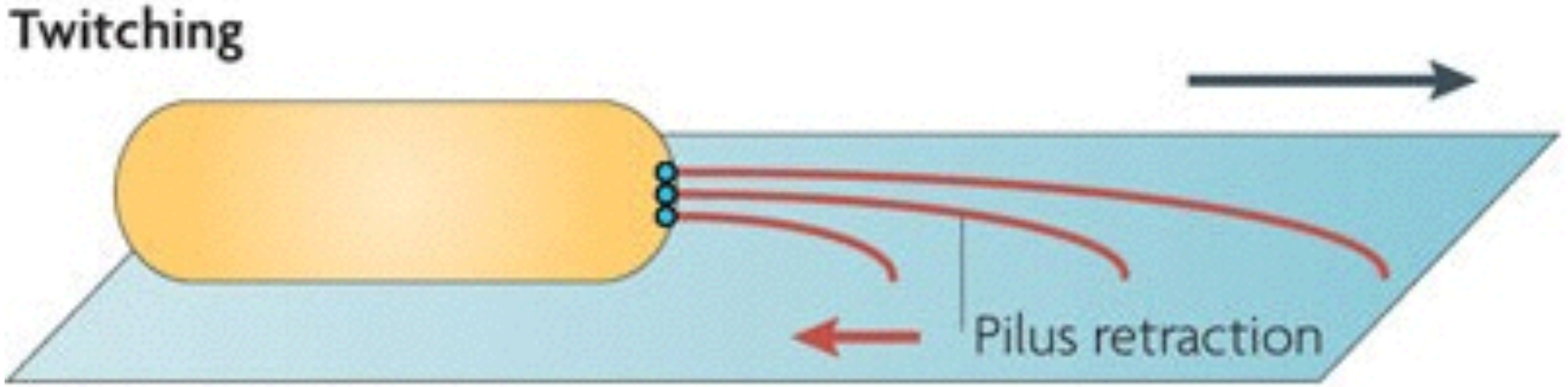
$$\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$V_0 = 22 \pm 5 \ \ \mu\mathrm{m/s}$$
$$\ell = 1.9 \ \ \mu\mathrm{m}$$
$$F = 0.42 \ \ \mathrm{pN}$$

weak ‘pusher’ dipole

Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

Twitching motility

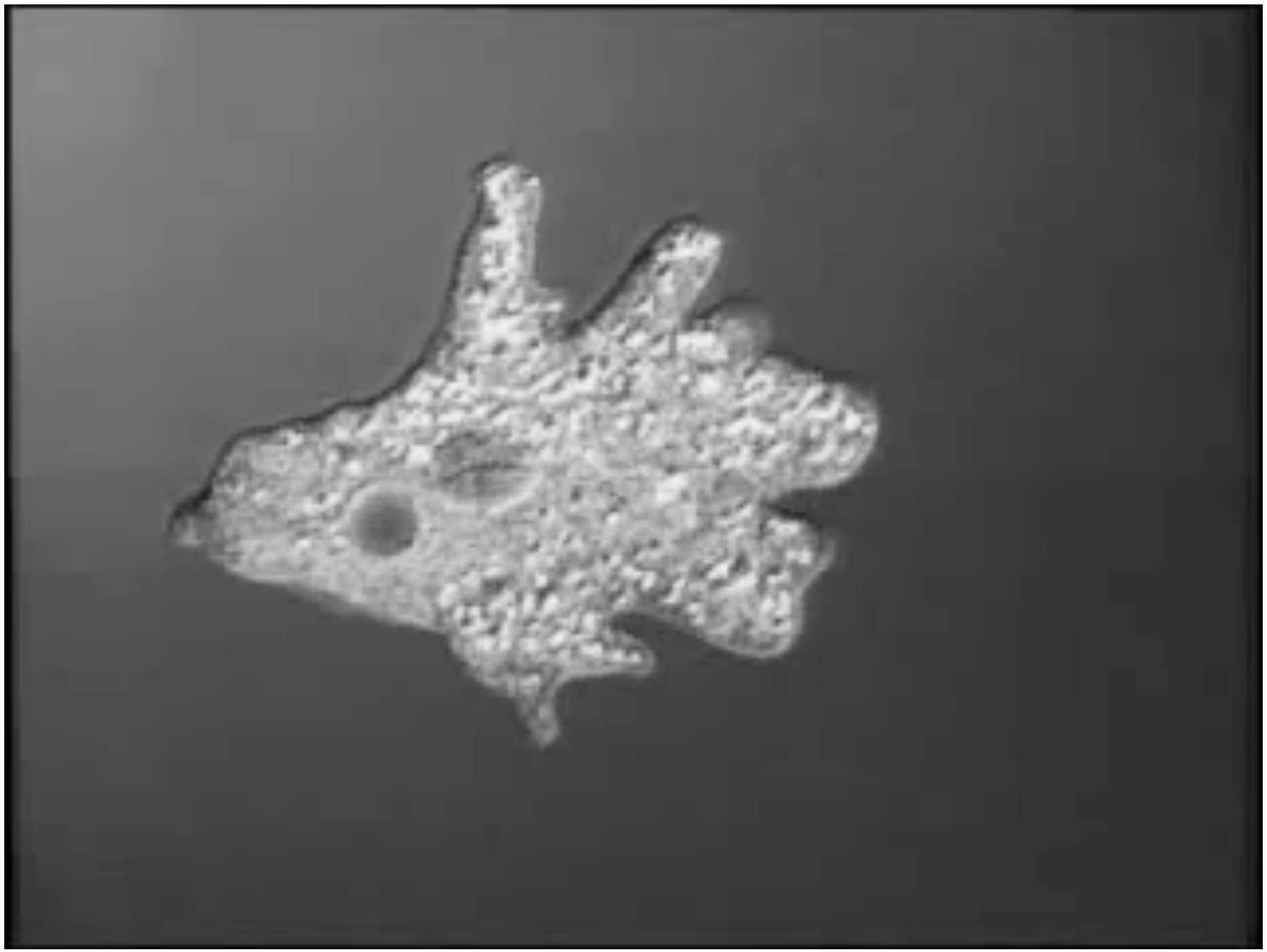


Type-IV Pili

Twitching motility

Pseudomonas

Amoeboid locomotion



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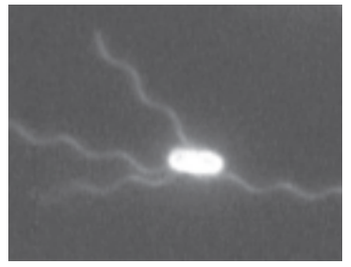

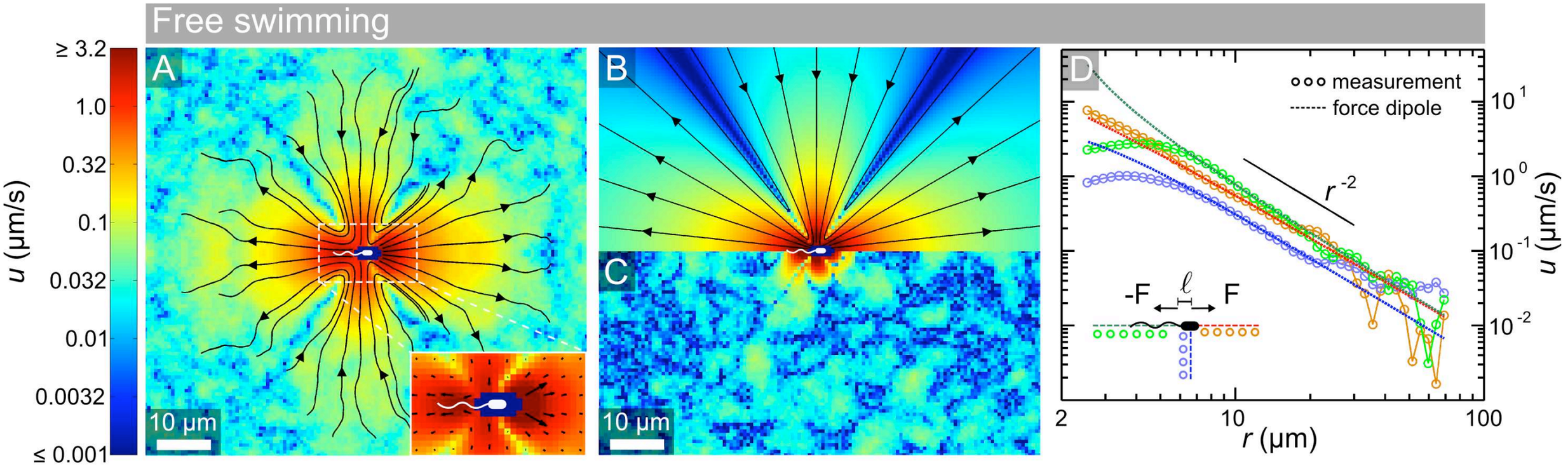

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