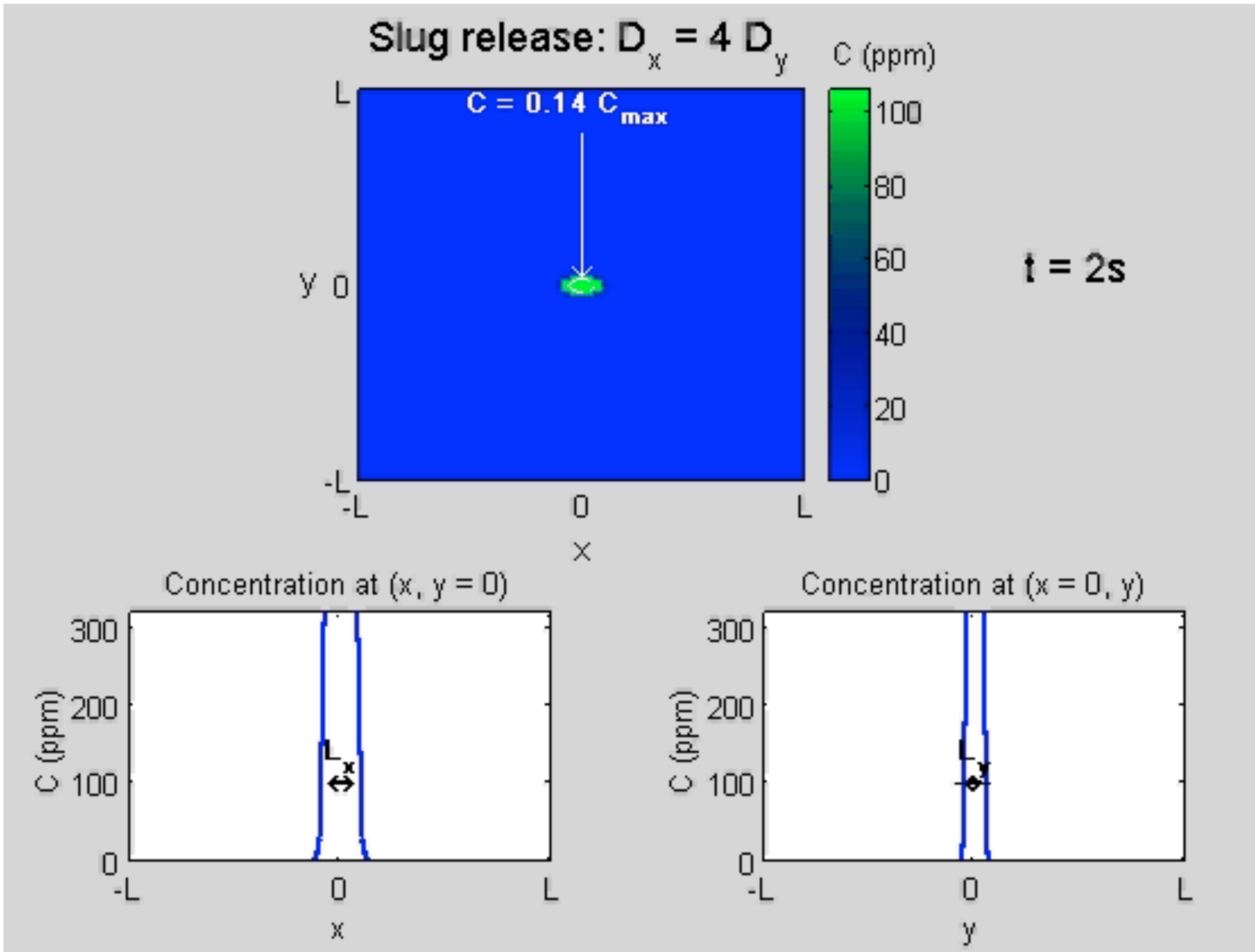
Beyond diffusion: Pattern formation

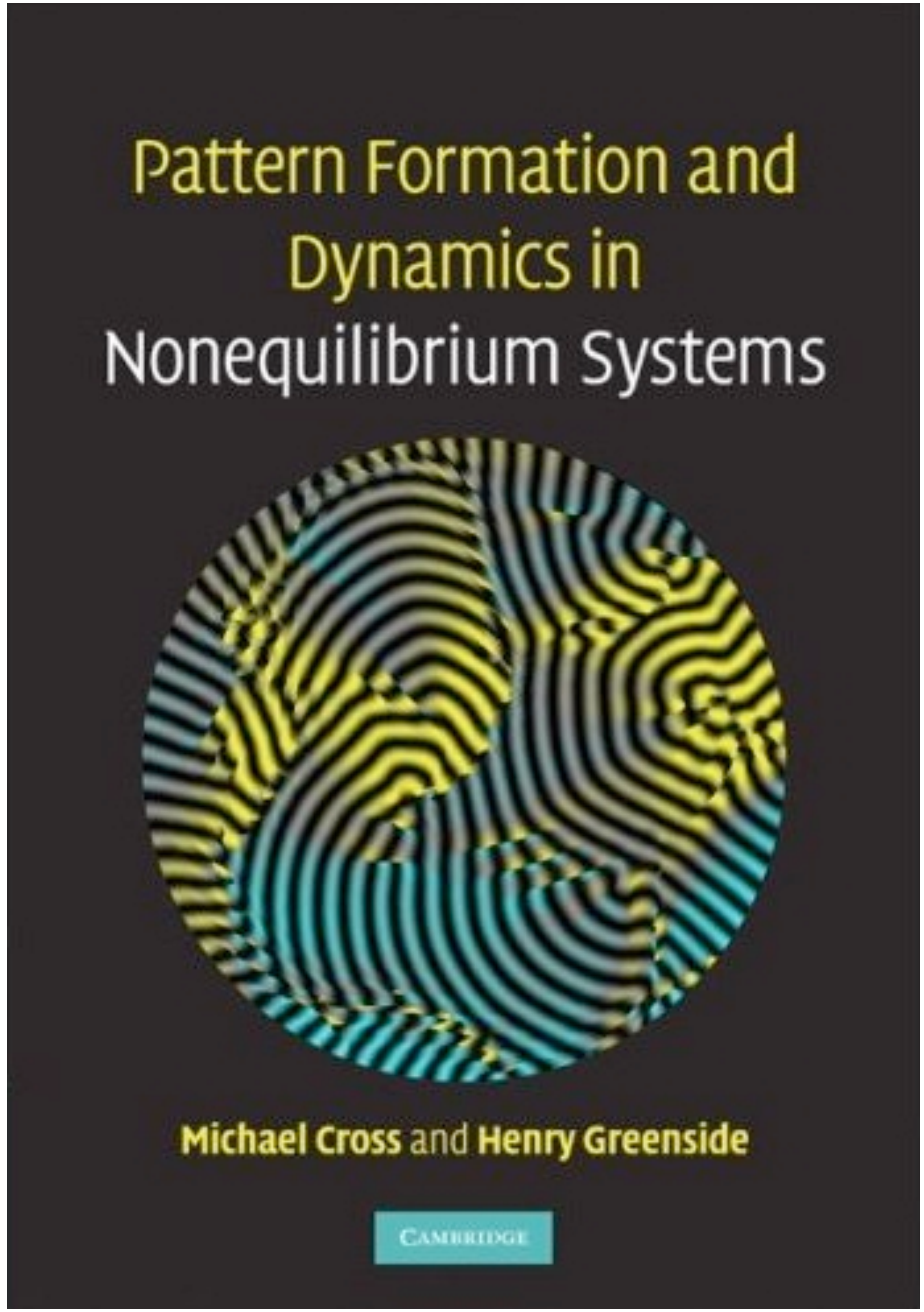
18.354 - L07

dunkel@mit.edu

2D diffusion equation



dunkel@math.mit.edu



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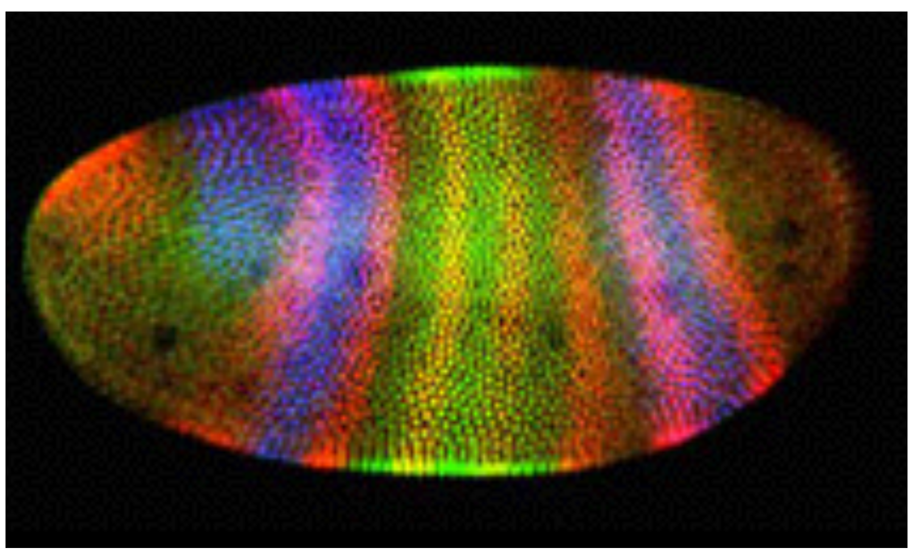
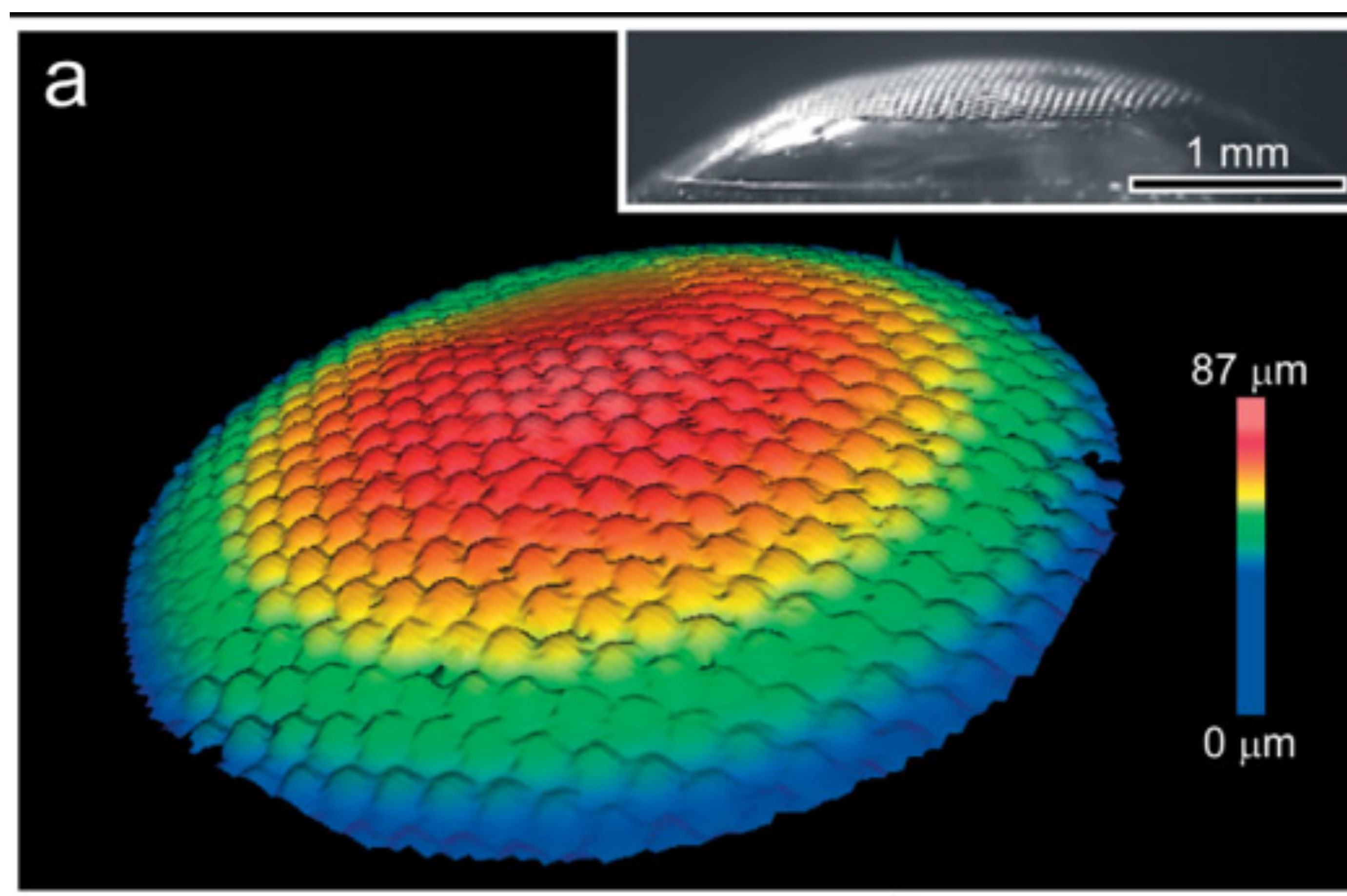
Natural & artificial patterns



Paddock (2001) Biotechniques



Chan & Crosby (2006) Advanced Materials

Plankton

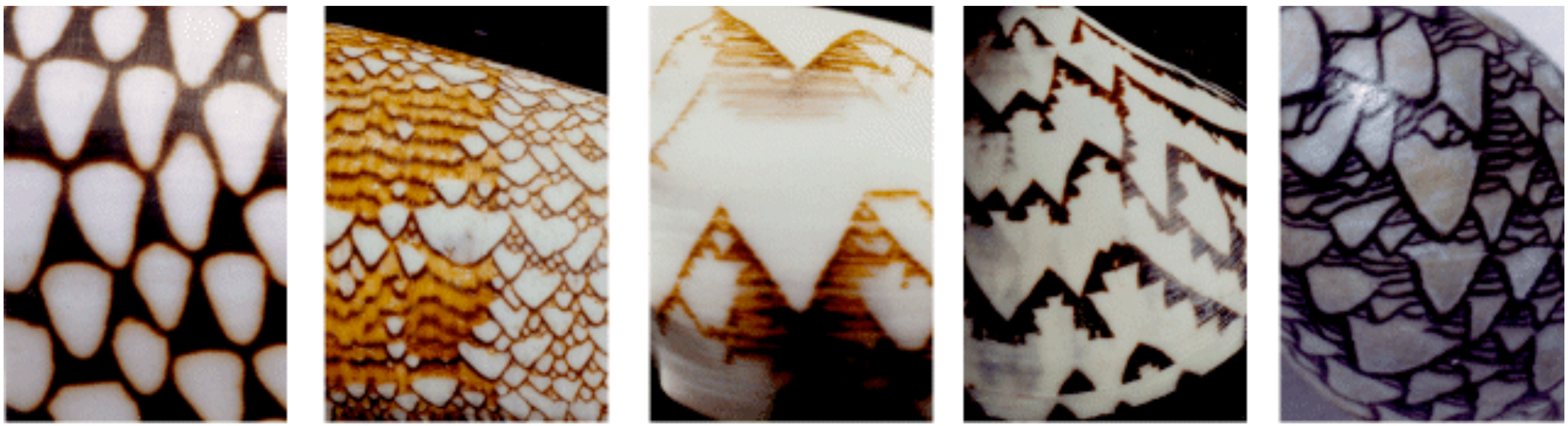
ESA cost of Ireland

dunkel@math.mit.edu

Animals

dunkel@math.mit.edu

Shells



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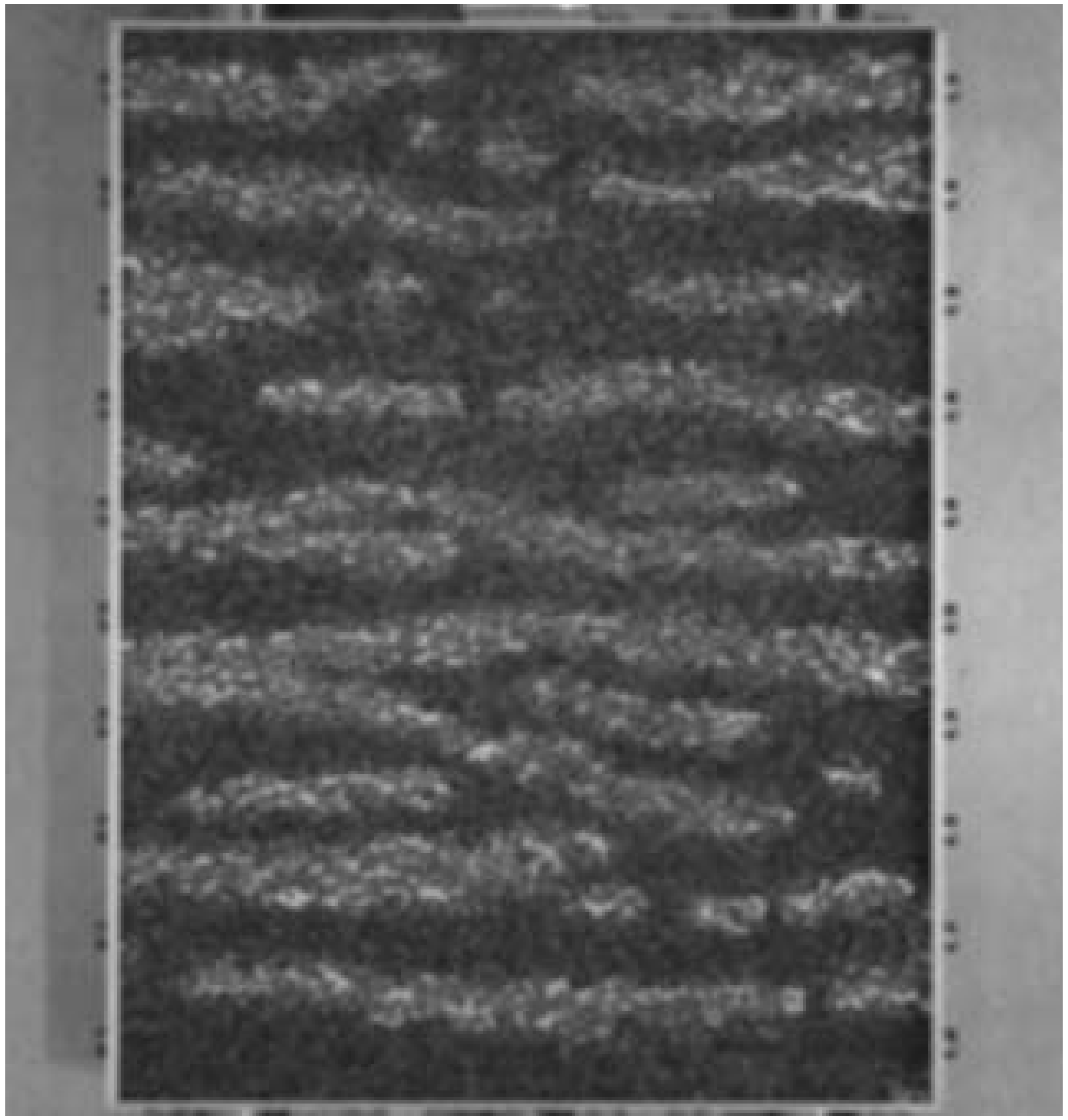
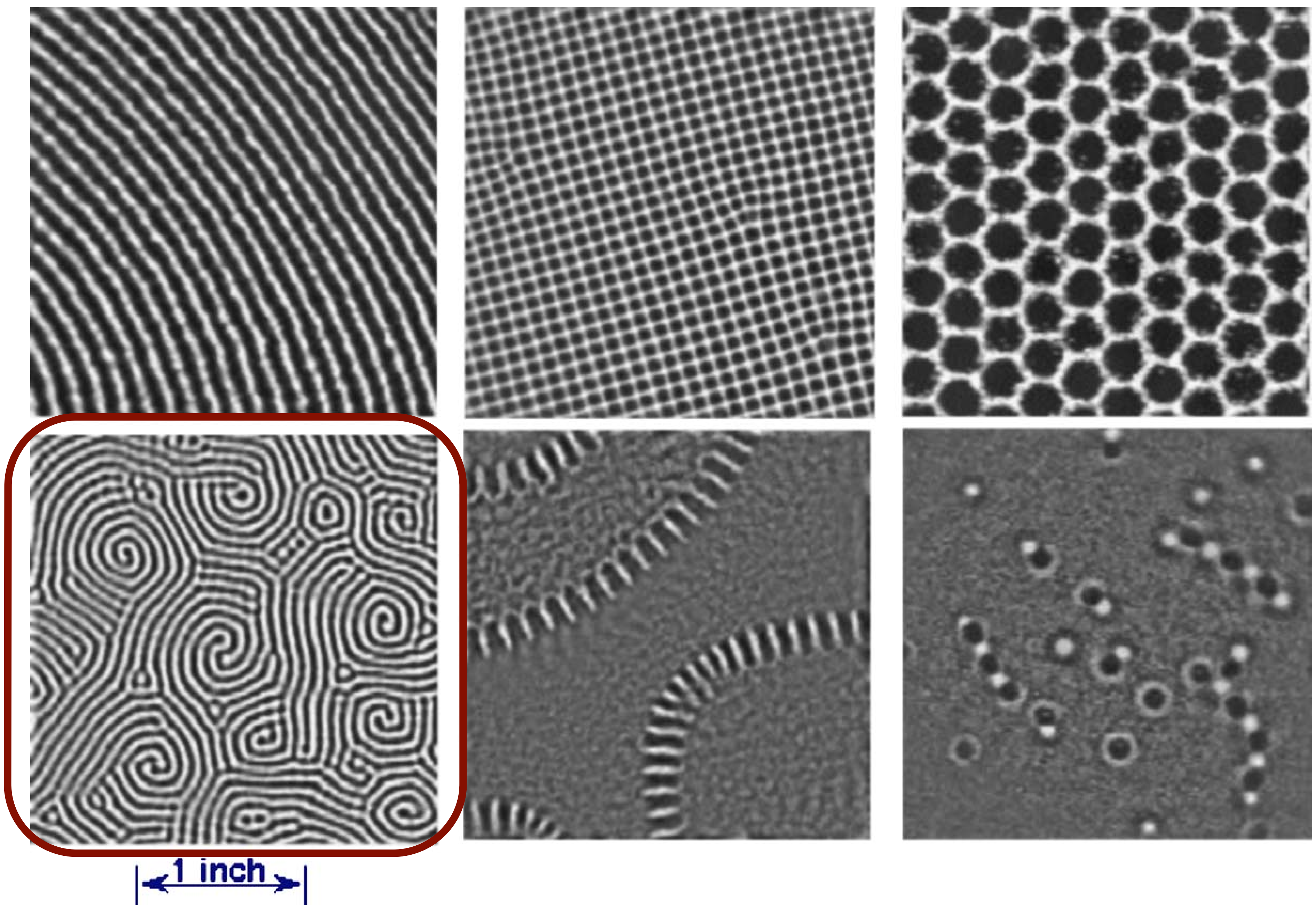
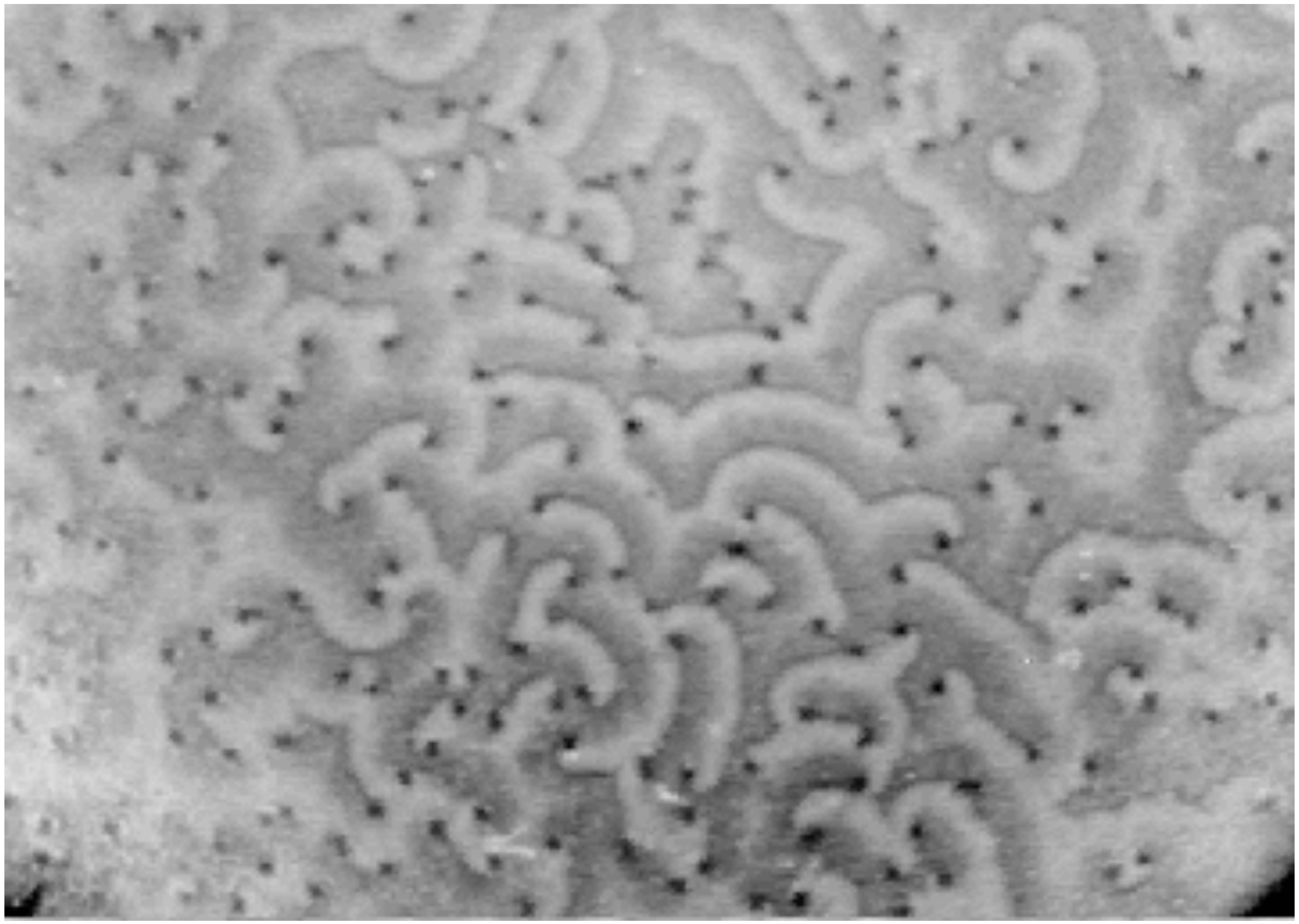
Compare: vibrated granular media



Vibrated granular media

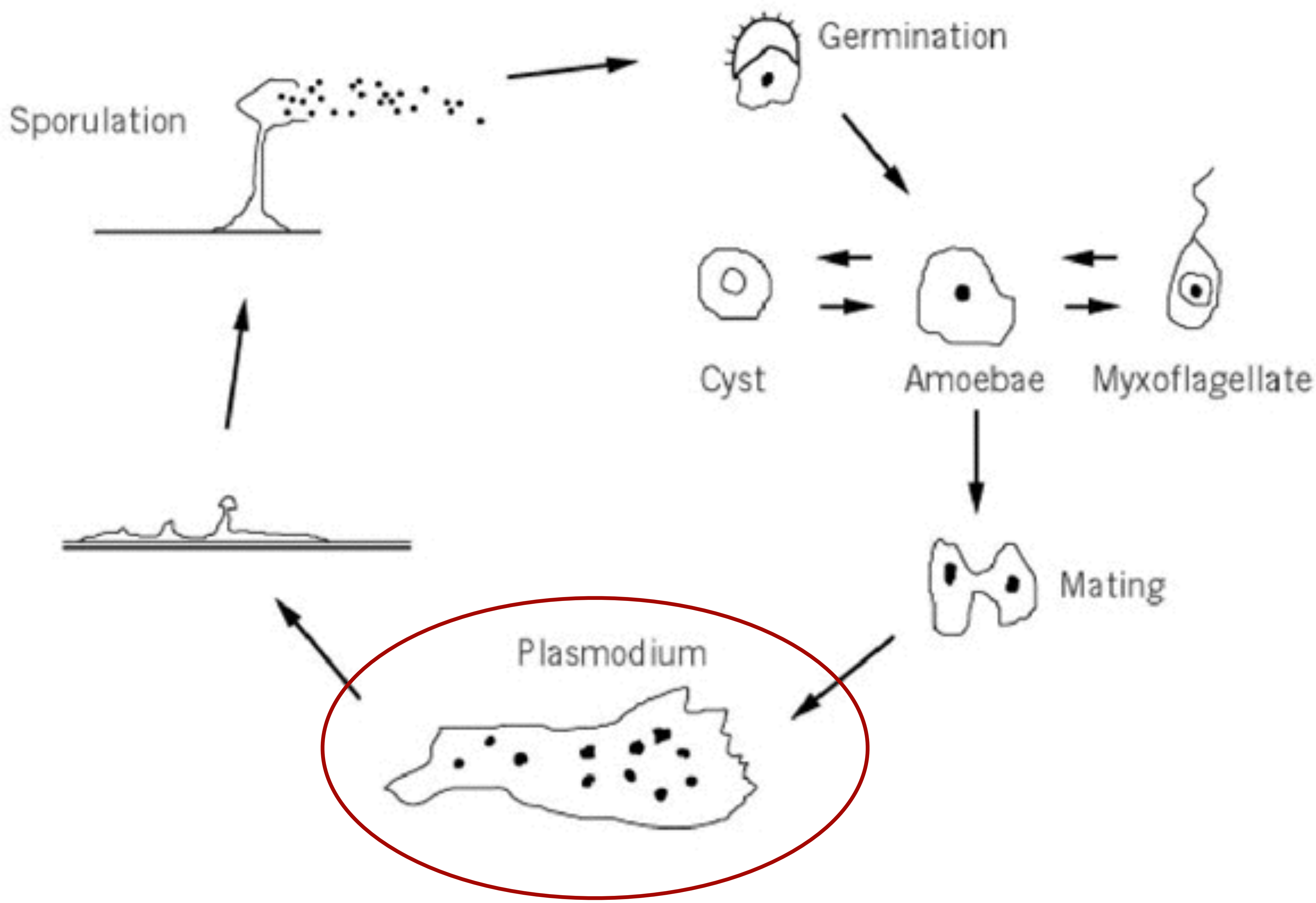


Slime mold



aggregation of a starving slime mold (credit: Florian Siegert)

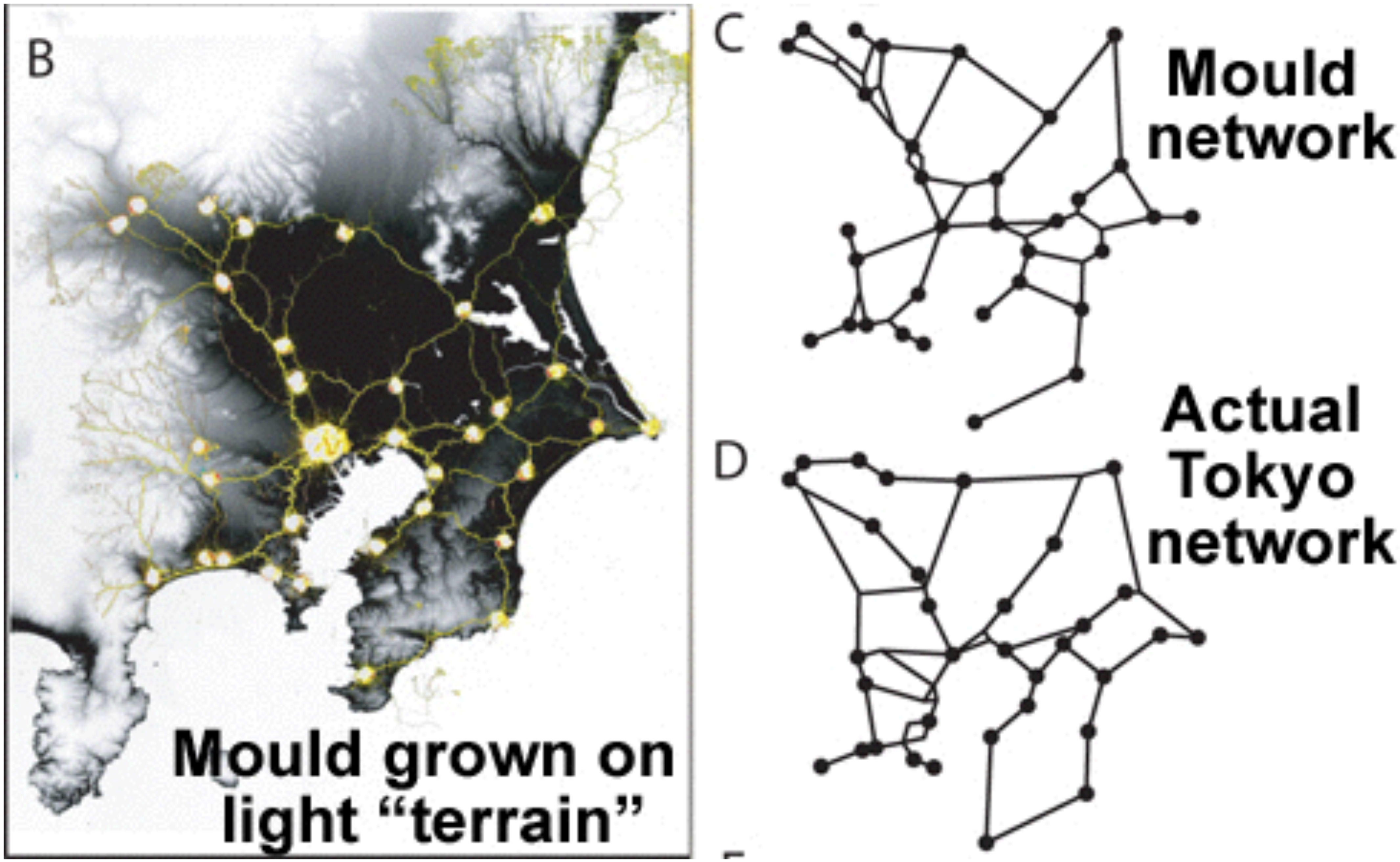
Physarum developmental cycle



UNIVERSITY OF CAMBRIDGE

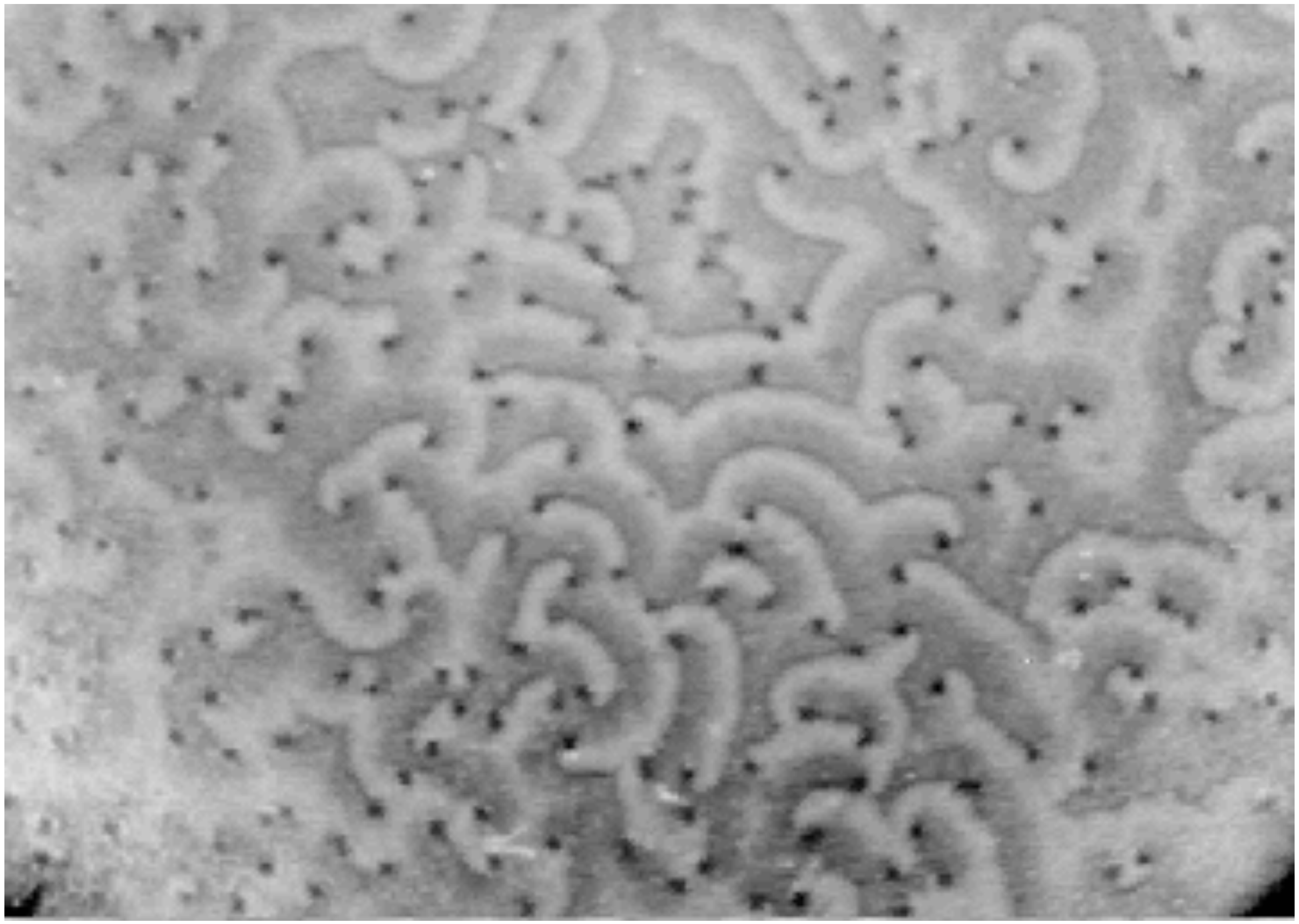
Slime mold (physarum plasmodium)

Tero *et al* (2010) Science



Tero *et al* (2010) Science

Slime mold



aggregation of a starving slime mold (credit: Florian Siegert)

Belousov-Zhabotinsky reaction

Movie credit: Stephen Morris

Mix of potassium bromate, cerium(IV) sulfate, malonic acid and citric acid in dilute sulfuric acid.

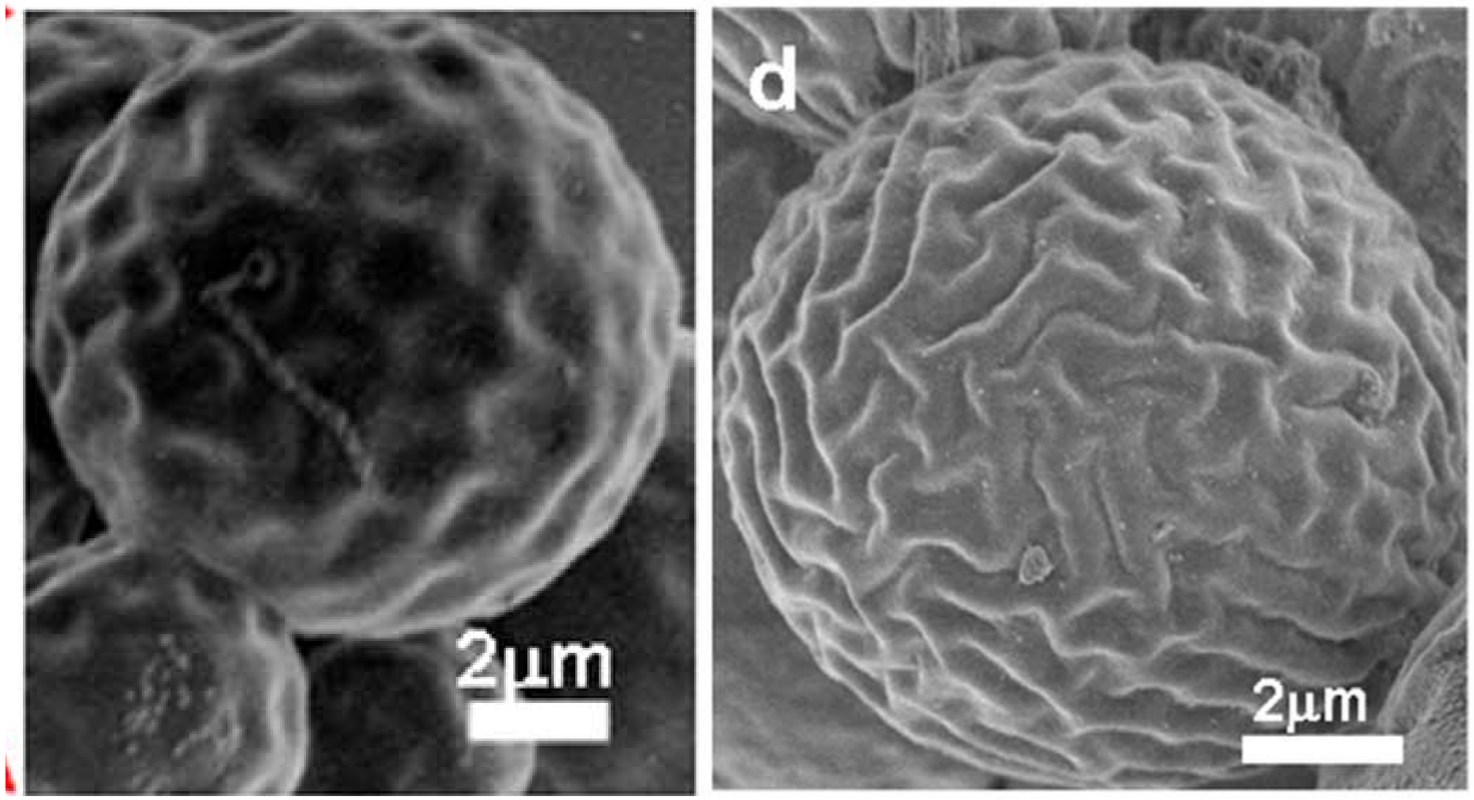
The ratio of concentration of the cerium(IV) and cerium(III) ions oscillates.

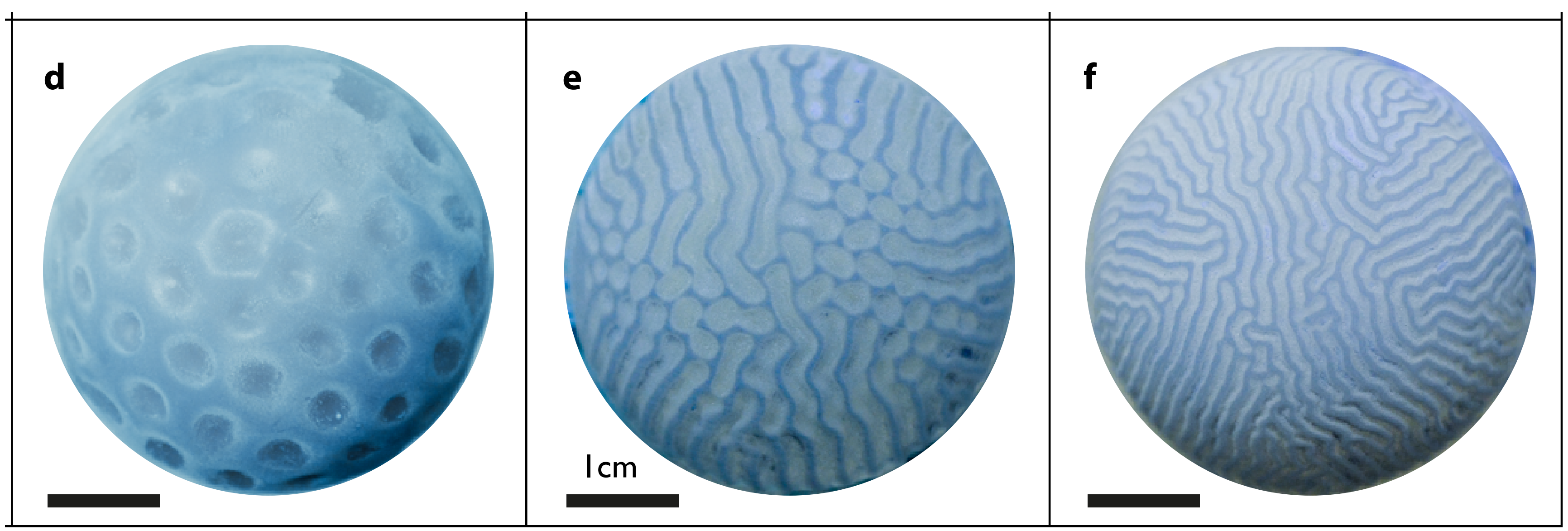
This is due to the cerium(IV) ions being reduced by malonic acid to cerium(III) ions, which are then oxidized back to cerium(IV) ions by bromate(V) ions.

dunkel@math.mit.edu

Surface buckling

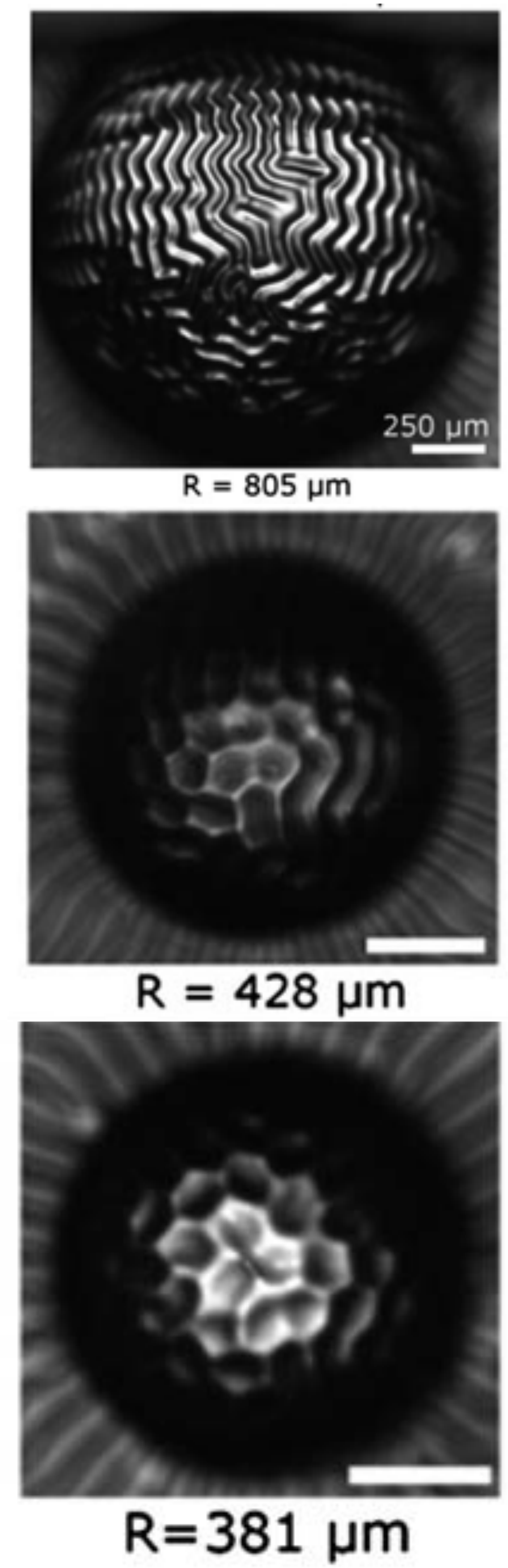
Yin et al (2014) Sci Rep

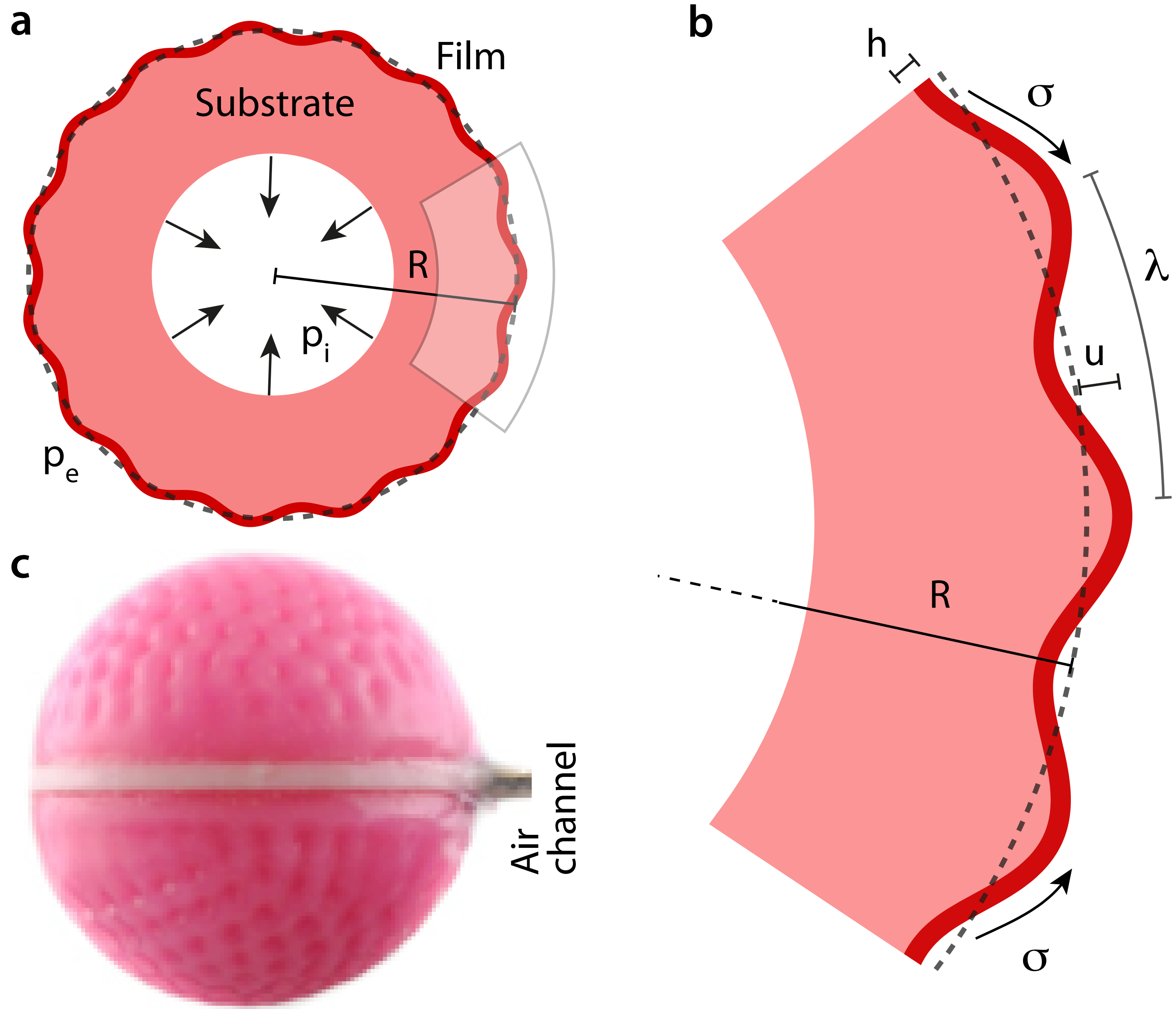




Stoop, Lagrange, Terwagne, Reis & Dunkel, 2014

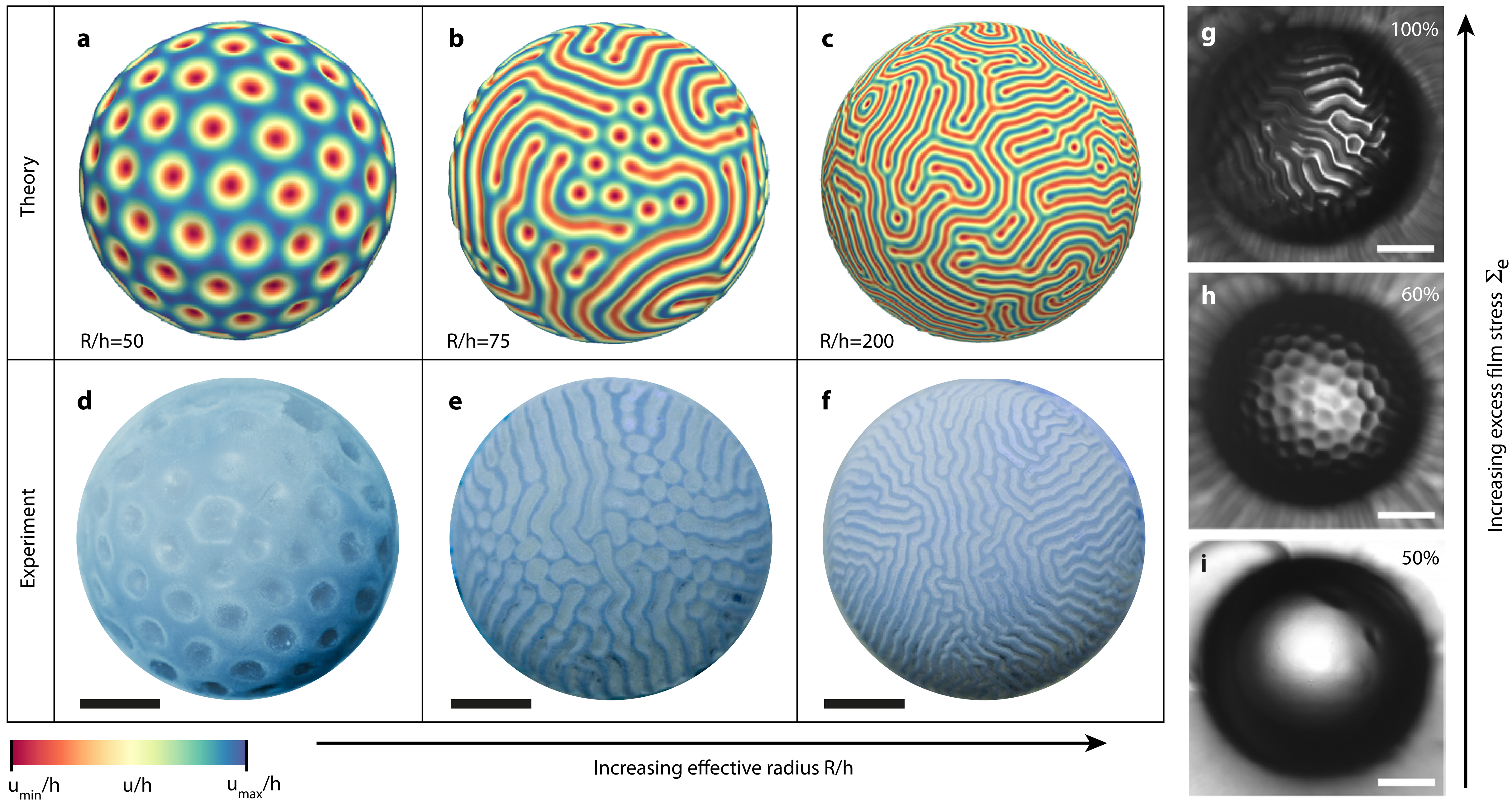
Breid & Crosby (2013) Soft Matter





Reis lab @ MIT

Theory *vs.* experiment



Stoop et al., Nature Materials, 2015

Higher-order PDE models
&
symmetry-breaking

Mathematical description

1. higher-than-second-order PDEs

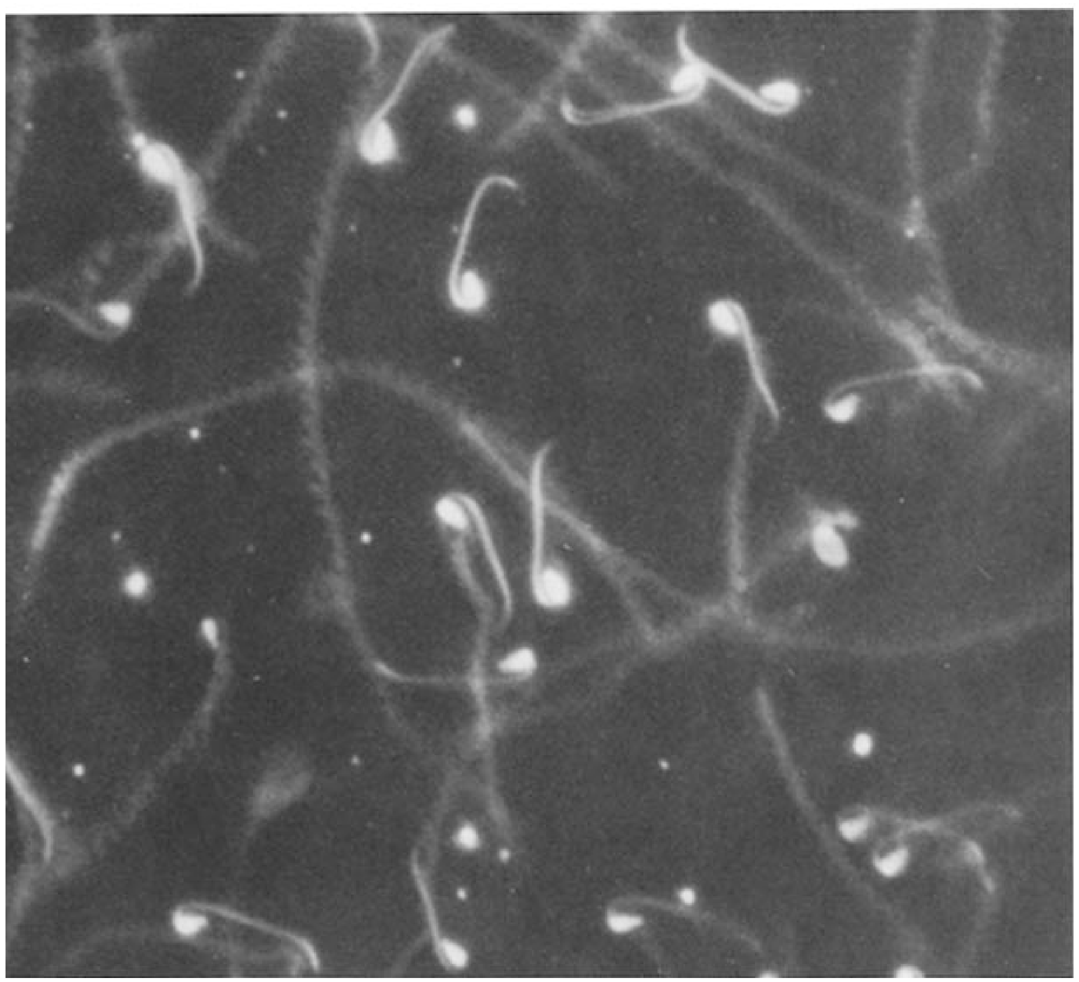
2. many coupled second-order PDEs

Broken reflection-symmetry at surfaces

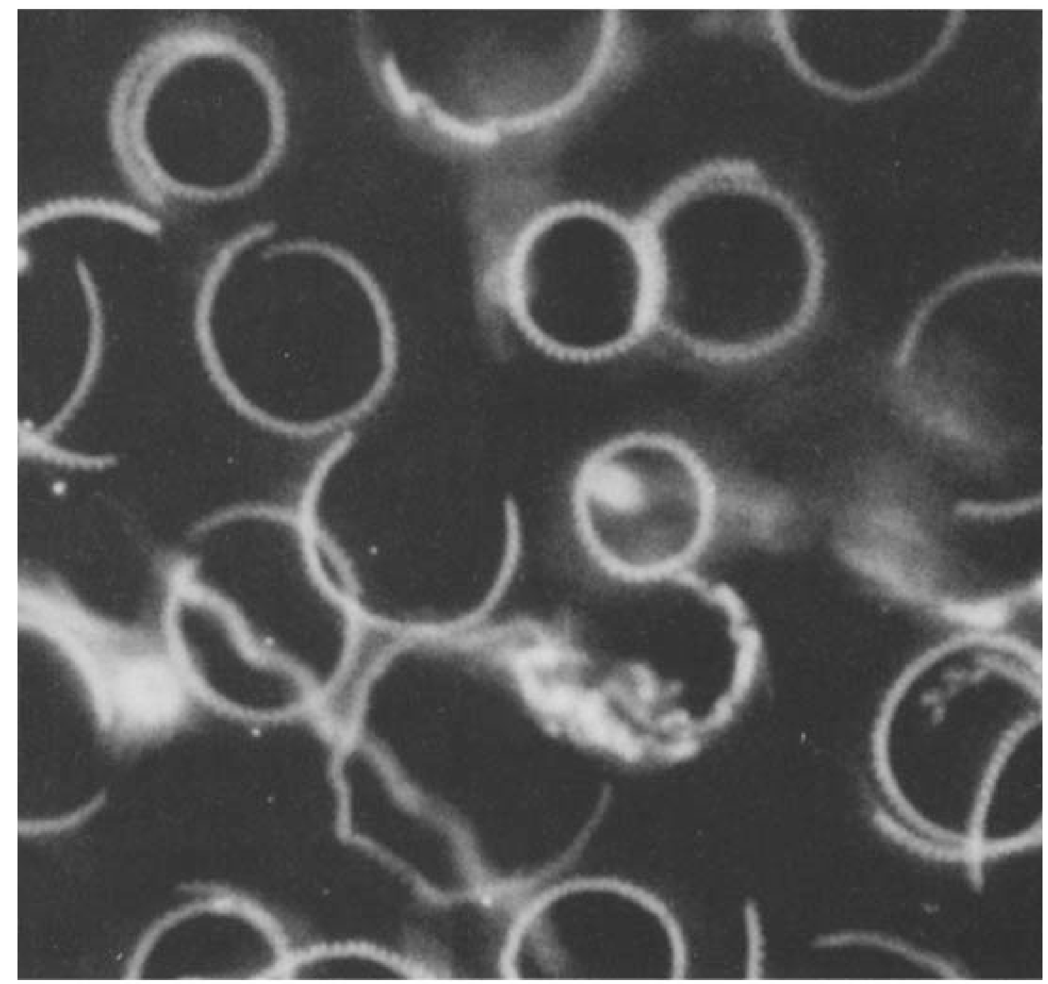
Gibbons (1980) JCB

Sea urchin sperm

in bulk (dilute)



near surface (dilute)



similar for bacteria (E. coli): Di Luzio et al (2005) Nature

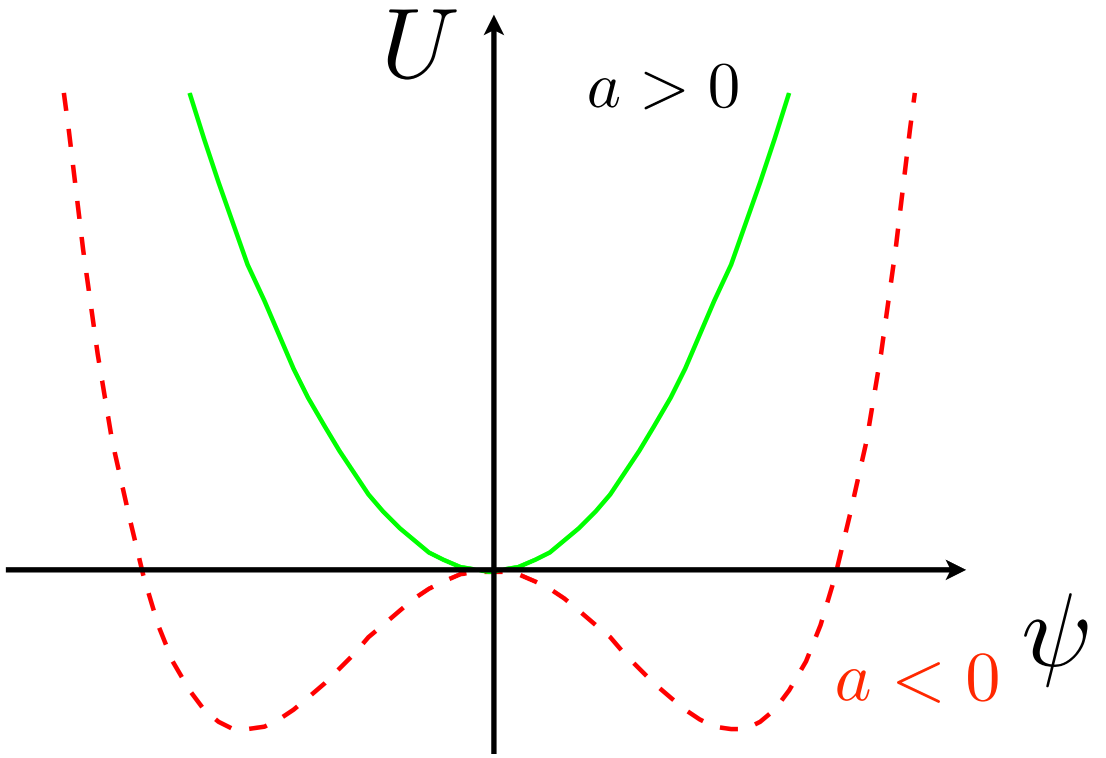
2d Swift-Hohenberg model

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

$$U(\psi) = \frac{a}{2}\psi^2 + \cancel{\frac{b}{3}\psi^3} + \frac{c}{4}\psi^4$$

reflection-symmetry

$$\psi \mapsto -\psi$$



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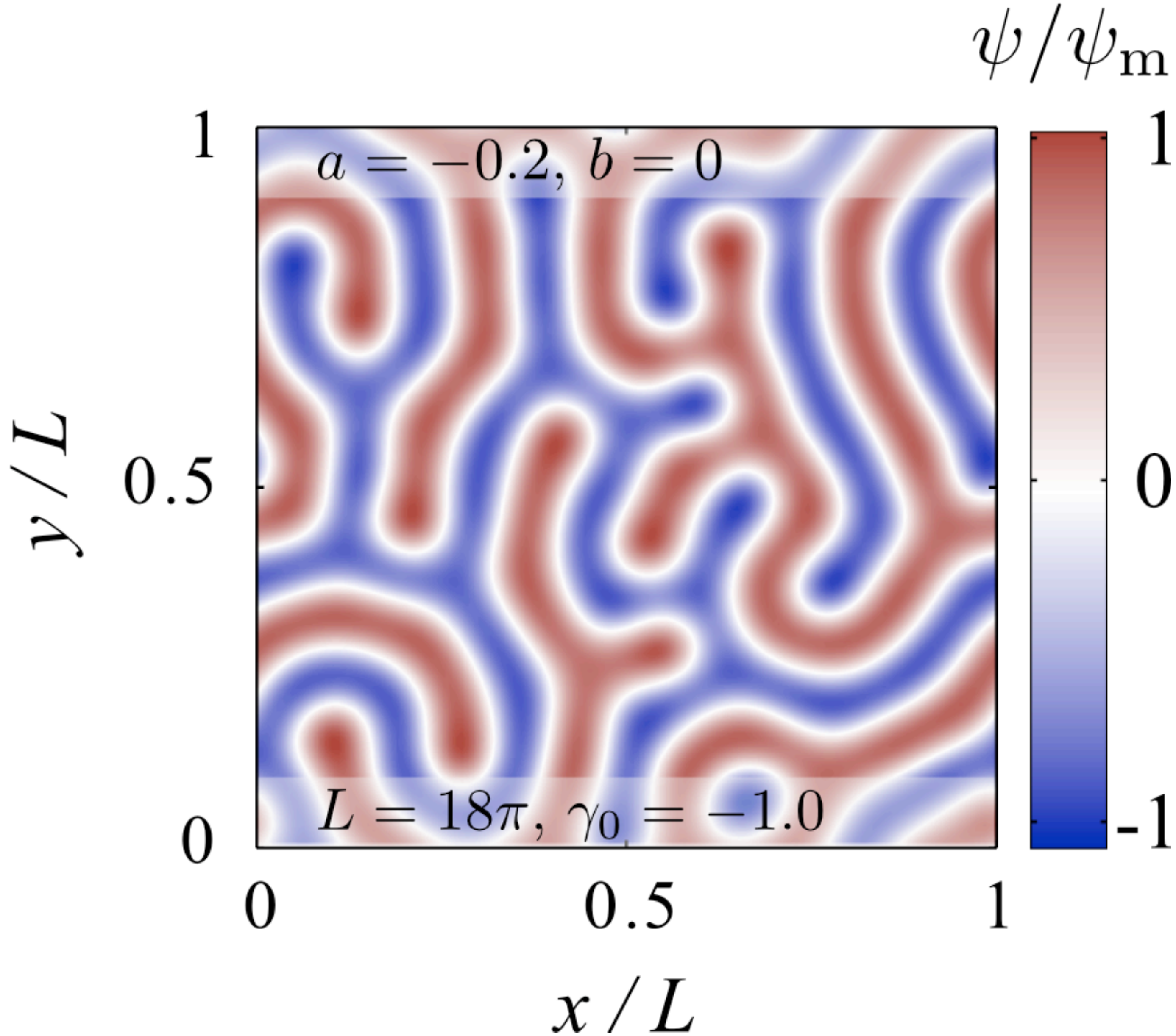

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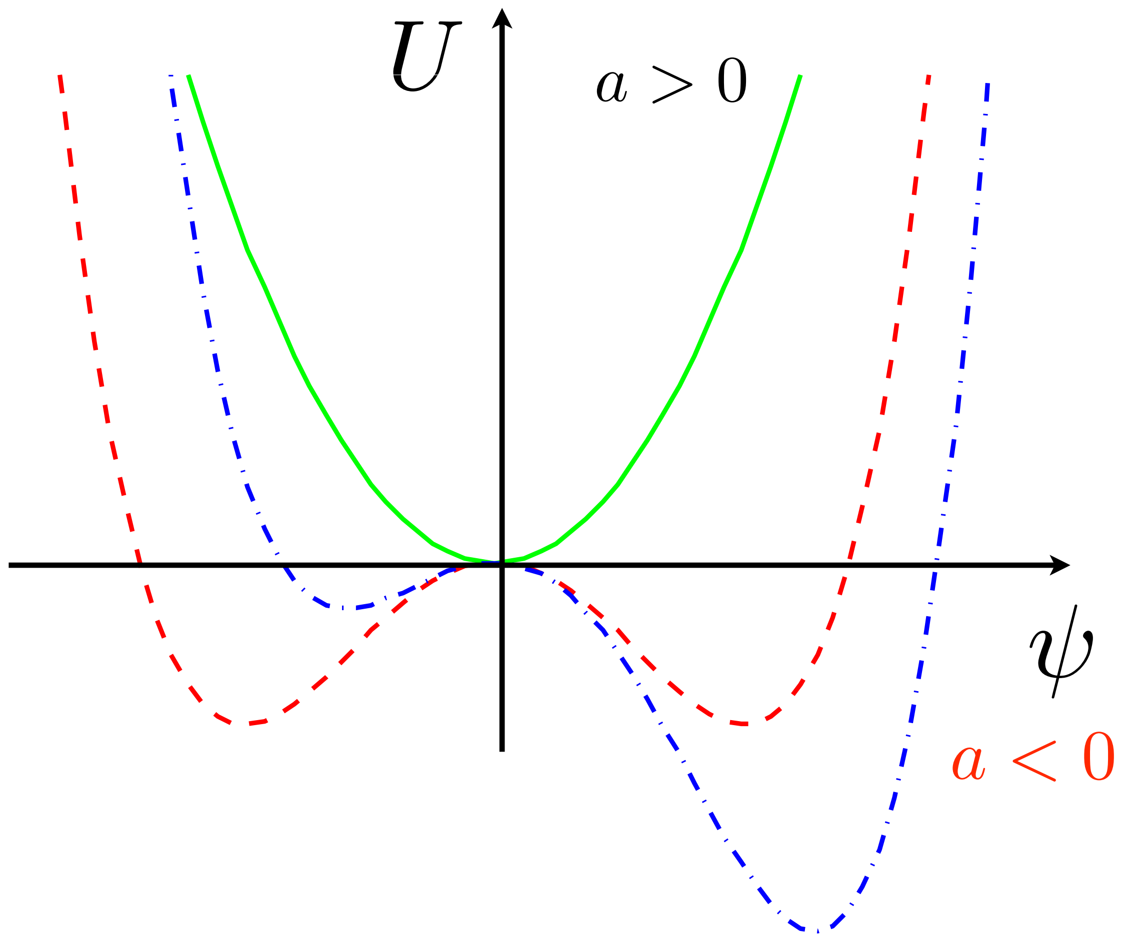

broken
reflection-symmetry

$$\psi \not\mapsto -\psi$$

2d Swift-Hohenberg model

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

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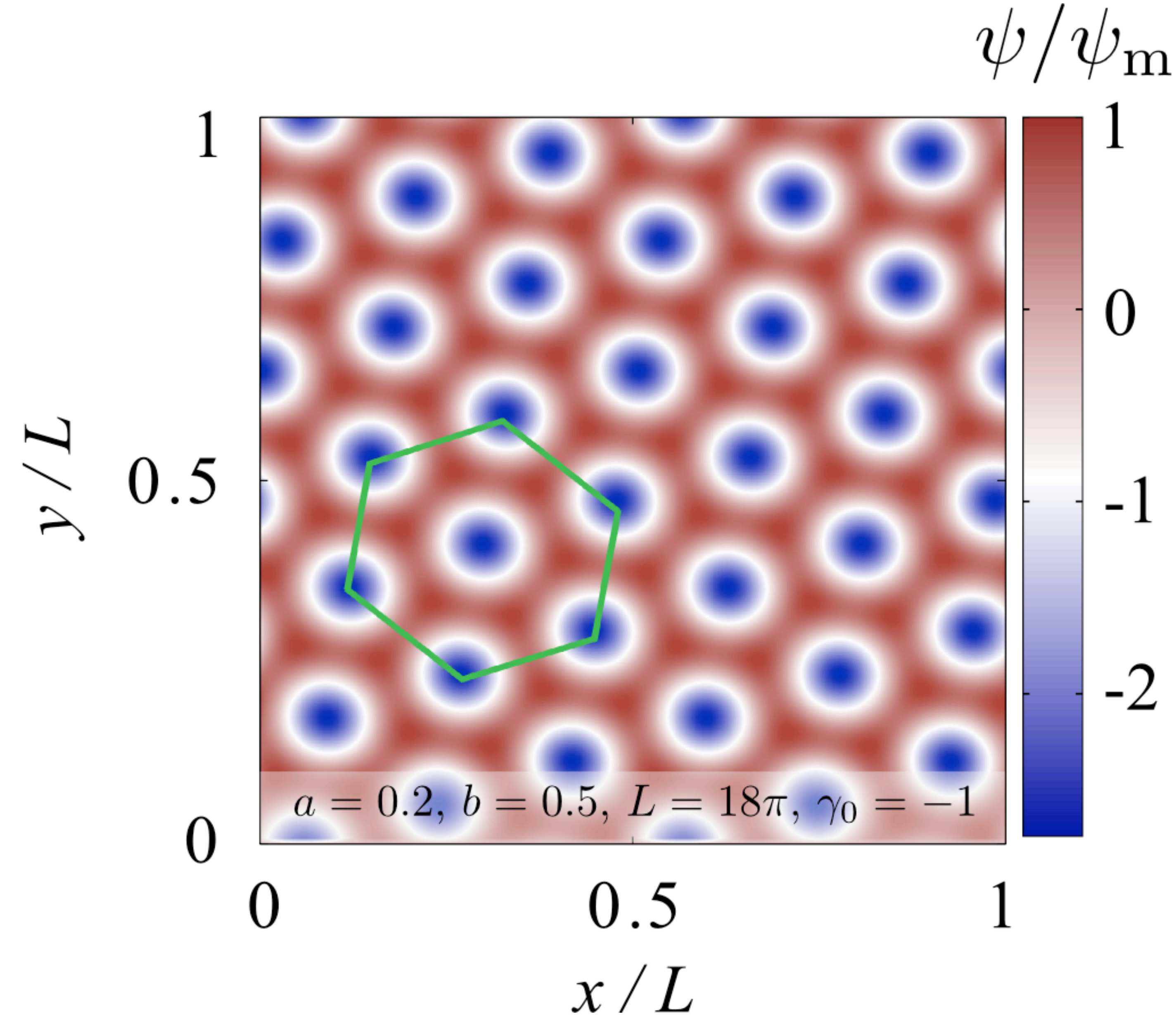
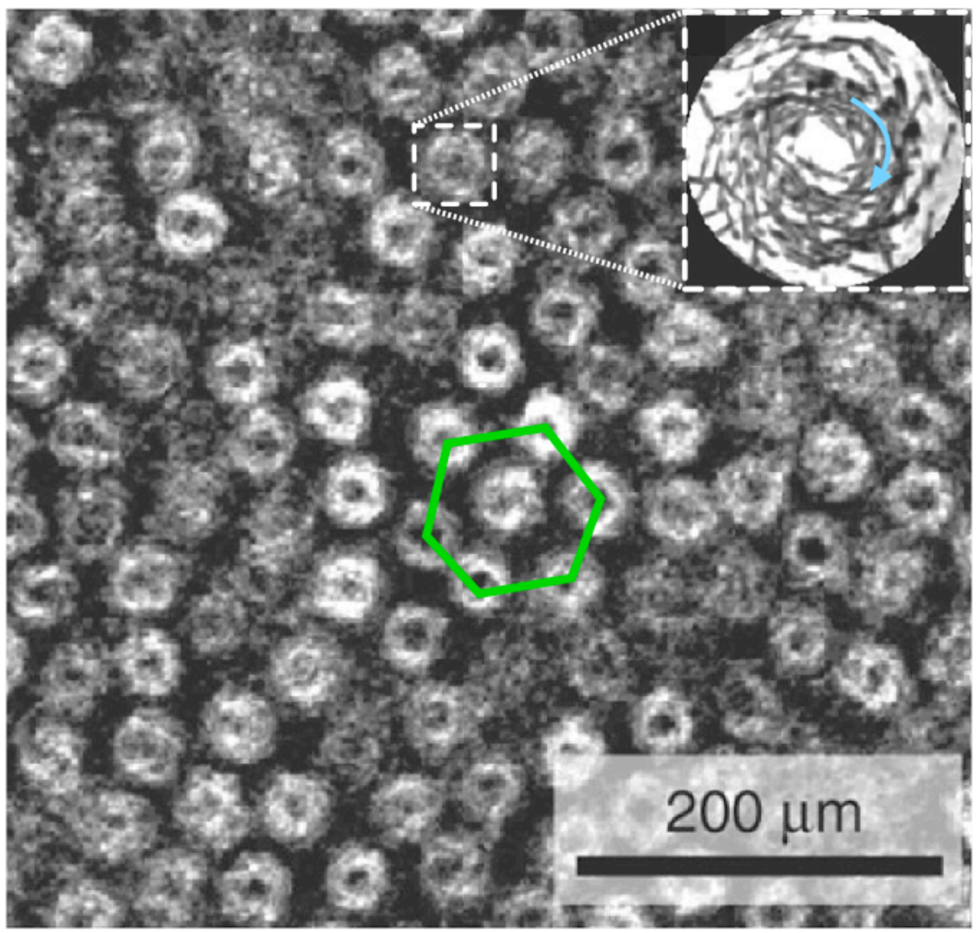


2d Swift-Hohenberg model

arxiv: 1208.4464

Sea urchin sperm cells
near surface
(high concentration)

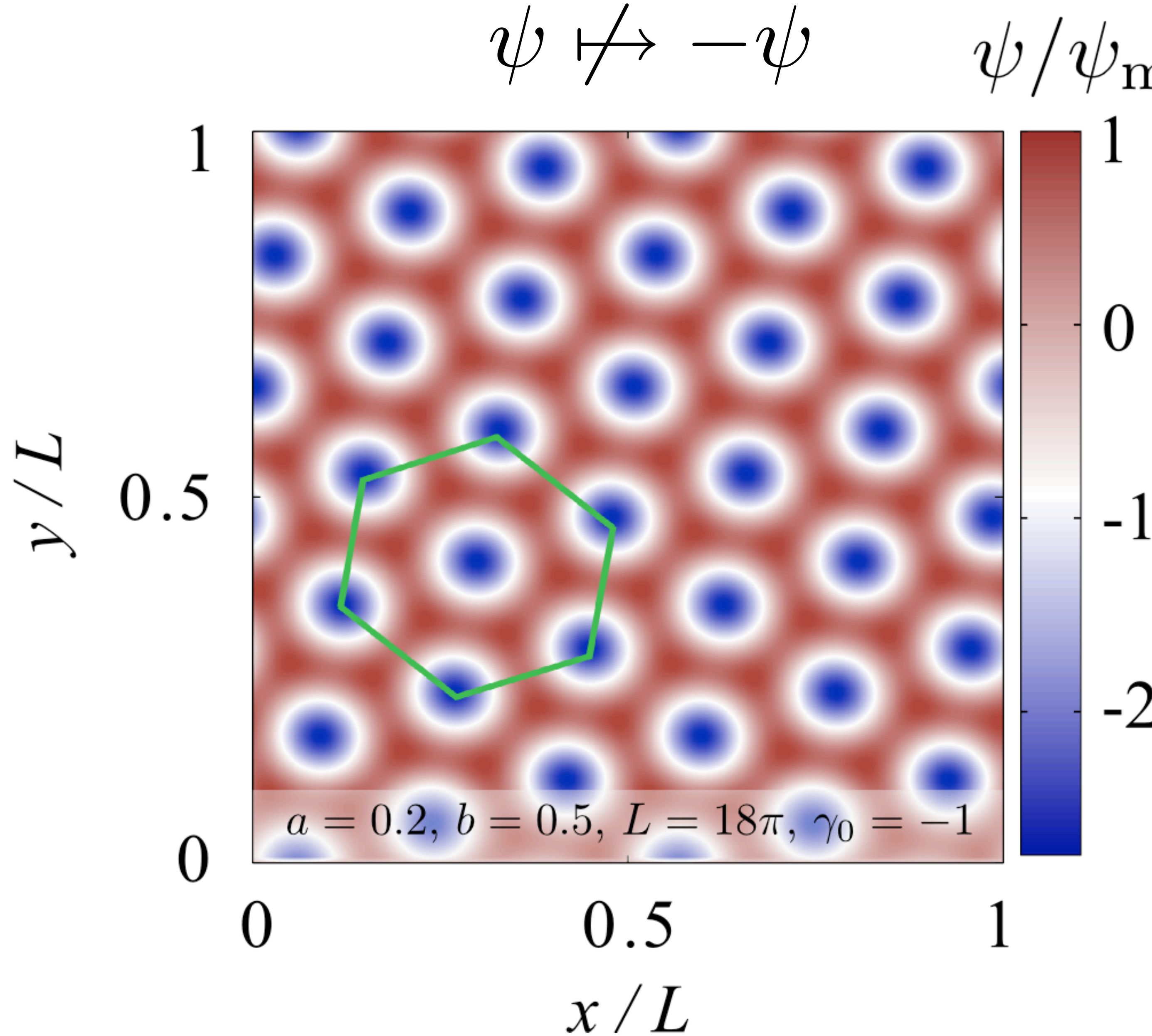


Riedel et al (2007) Science

broken
reflection-symmetry

$$b \neq 0$$

$$\psi \not\mapsto -\psi$$



Mathematical description

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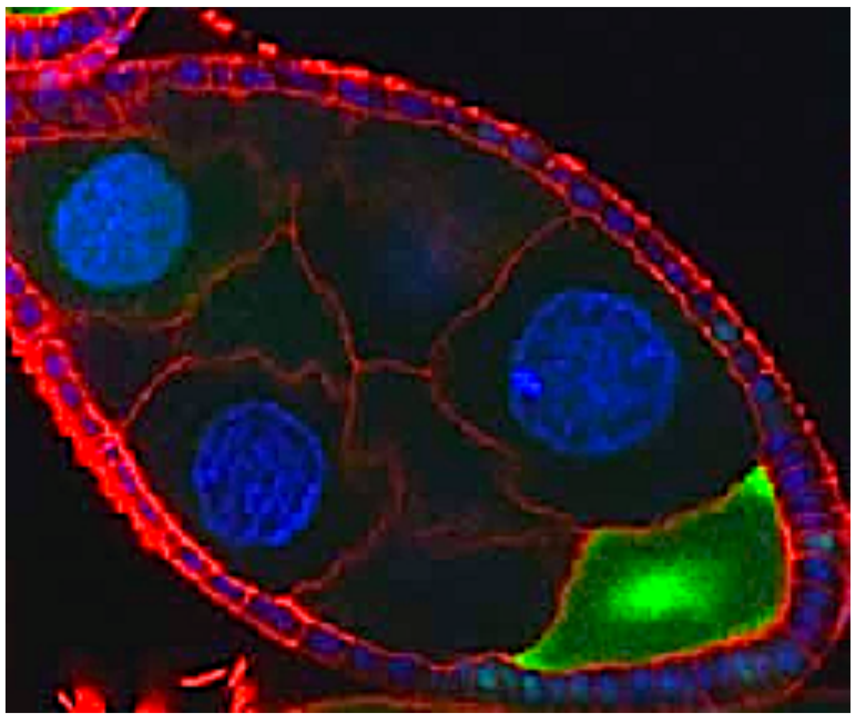
dunkel@math.mit.edu



?

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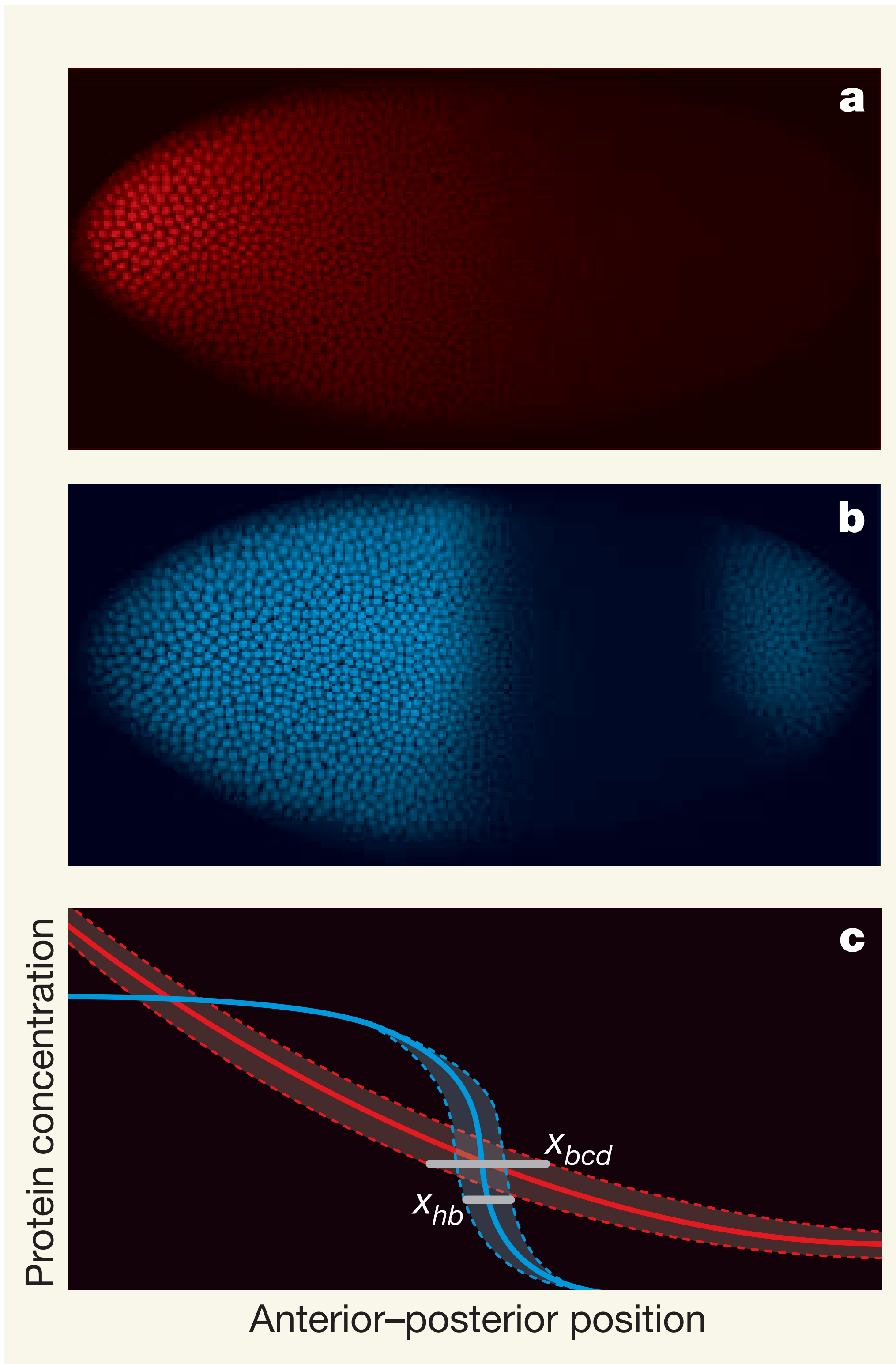
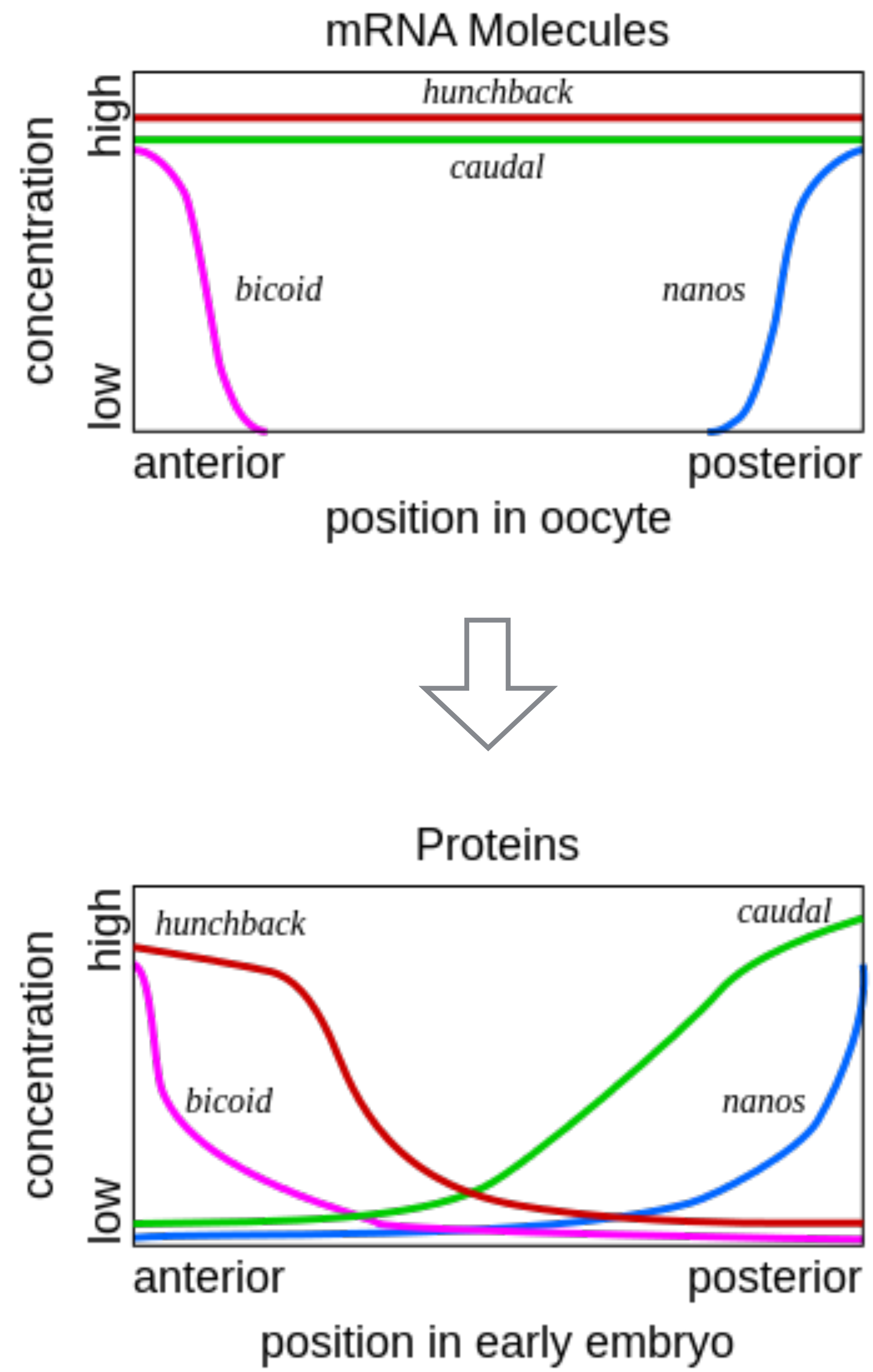
Maternal input

From: Reinitz

Bicoid

Hunchback

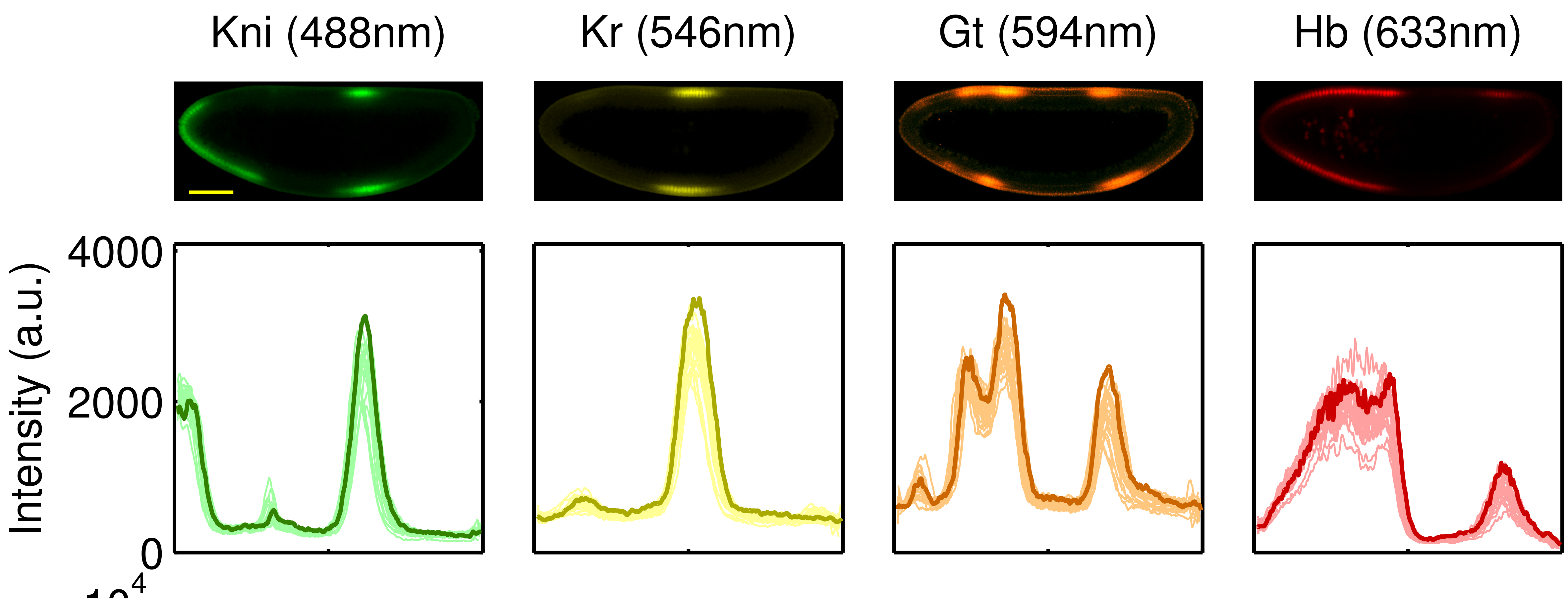




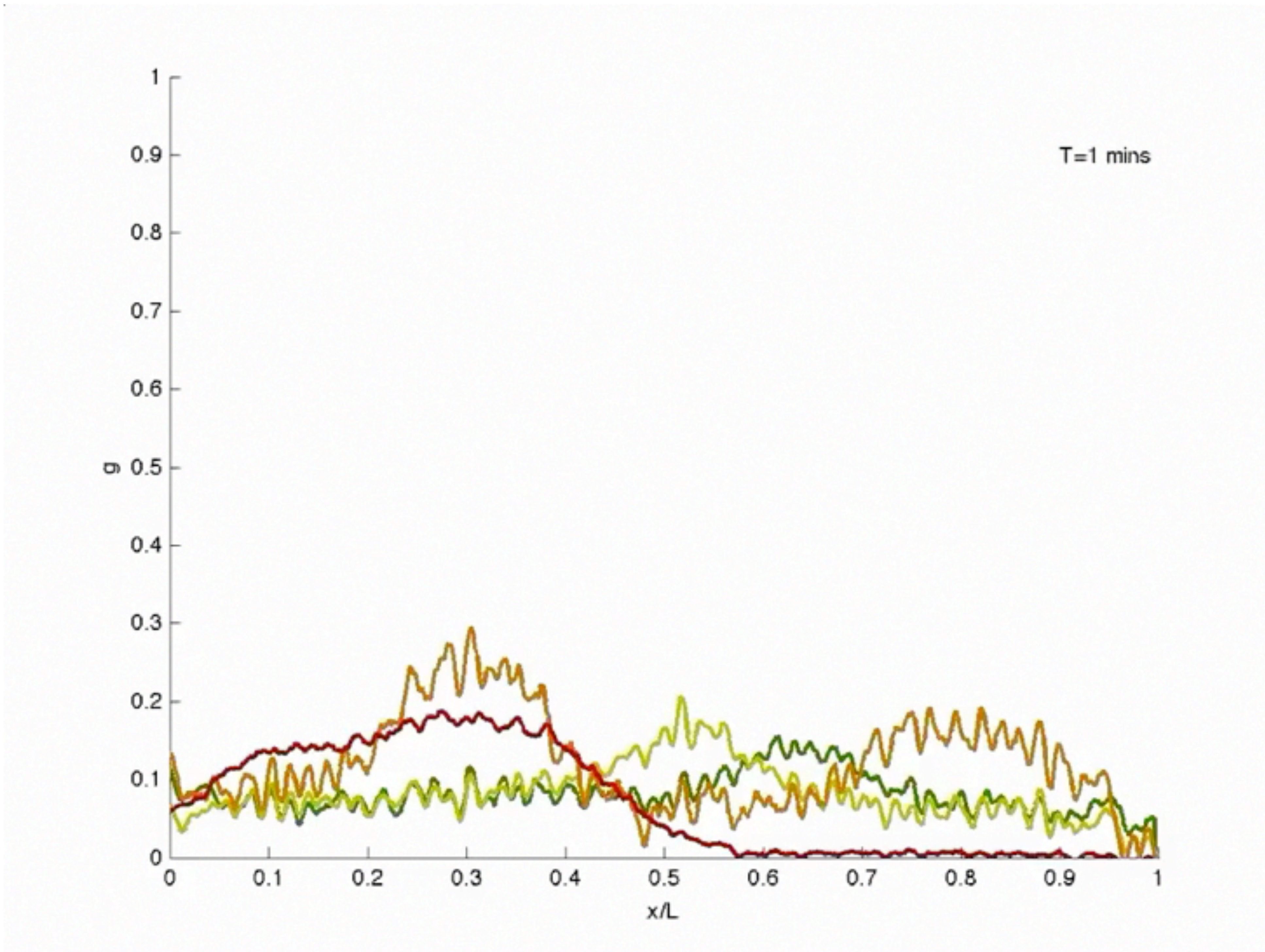
NEWS & VIEWS

NATURE|Vol 448|26 July 2007

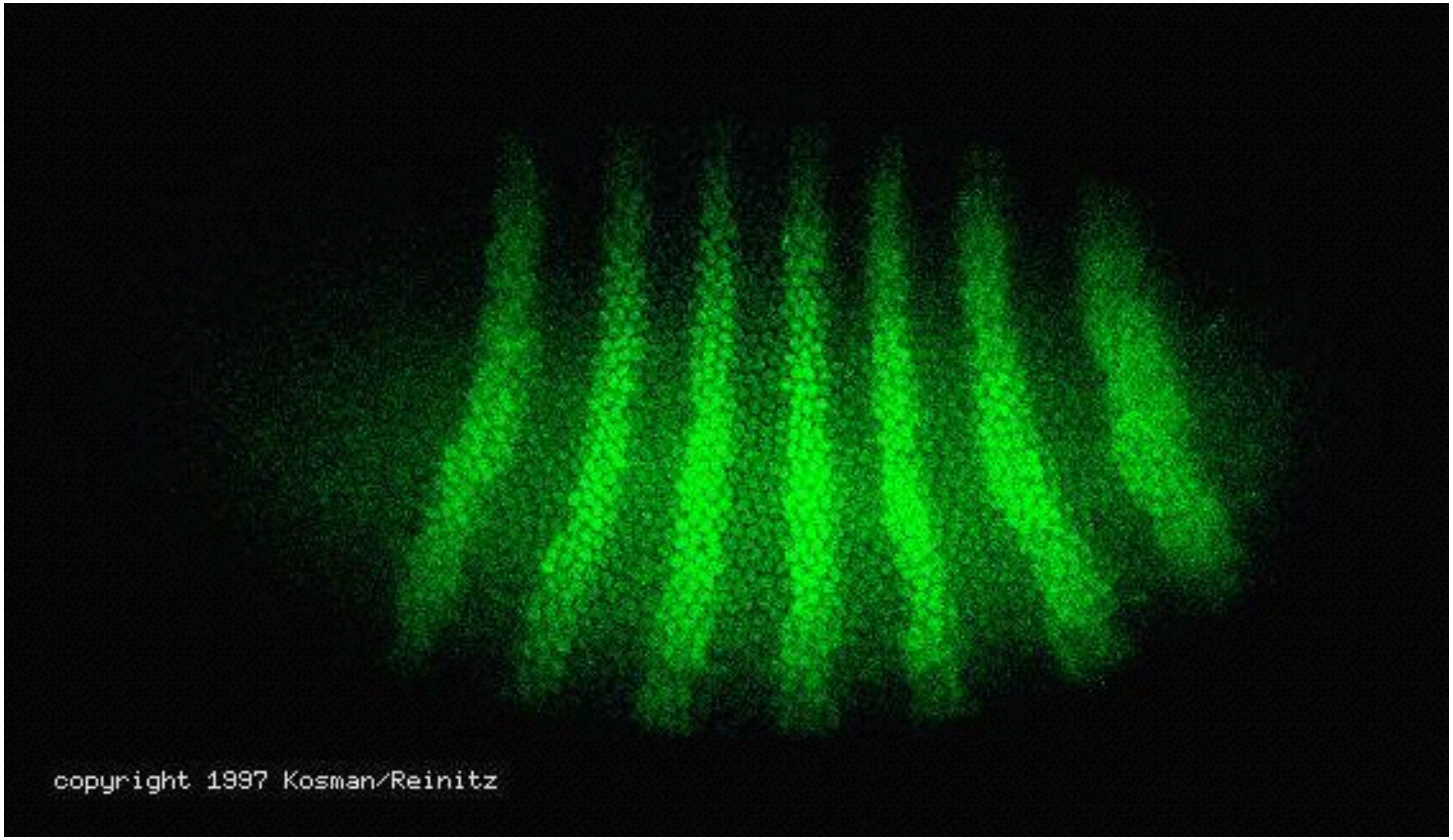
Intensity profiles



Dubois et al (2013) Mol Sys Biol



Gregor Lab, Princeton



http://www.iephb.nw.ru/hoxpro/ftz.html

Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, Vol. 237, No. 641. (Aug. 14, 1952), pp. 37-72.

1912-1954

THE CHEMICAL BASIS OF MORPHOGENESIS

By A. M. TURING, F.R.S. *University of Manchester*

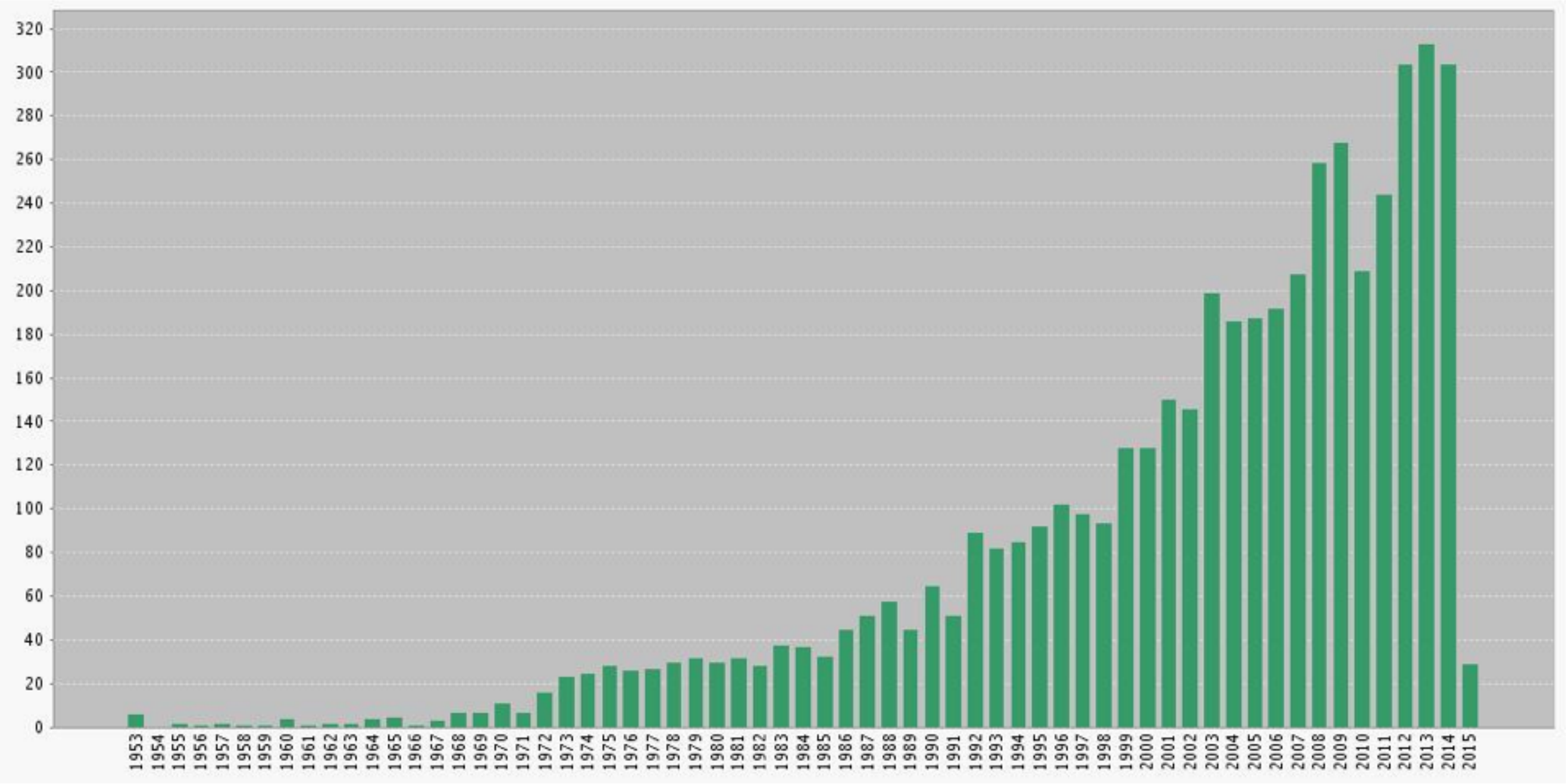
(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

It is suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis. Such a system, although it may originally be quite homogeneous, may later develop a pattern or structure due to an instability of the homogeneous equilibrium, which is triggered off by random disturbances. Such reaction-diffusion systems are considered in some detail in the case of an isolated ring of cells, a mathematically convenient, though biologically unusual system. The investigation is chiefly concerned with the onset of instability. It is found that there are six essentially different forms which this may take. In the most interesting form stationary waves appear on the ring. It is suggested that this might account, for instance, for the tentacle patterns on *Hydra* and for whorled leaves. A system of reactions and diffusion on a sphere is also considered. Such a system appears to account for gastrulation. Another reaction system in two dimensions gives rise to patterns reminiscent of dappling. It is also suggested that stationary waves in two dimensions could account for the phenomena of phyllotaxis.

The purpose of this paper is to discuss a possible mechanism by which the genes of a zygote may determine the anatomical structure of the resulting organism. The theory does not make any new hypotheses; it merely suggests that certain well-known physical laws are sufficient to account for many of the facts. The full understanding of the paper requires a good knowledge of mathematics, some biology, and some elementary chemistry. Since readers cannot be expected to be experts in all of these subjects, a number of elementary facts are explained, which can be found in text-books, but whose omission would make the paper difficult reading.

Google: 8721 citations

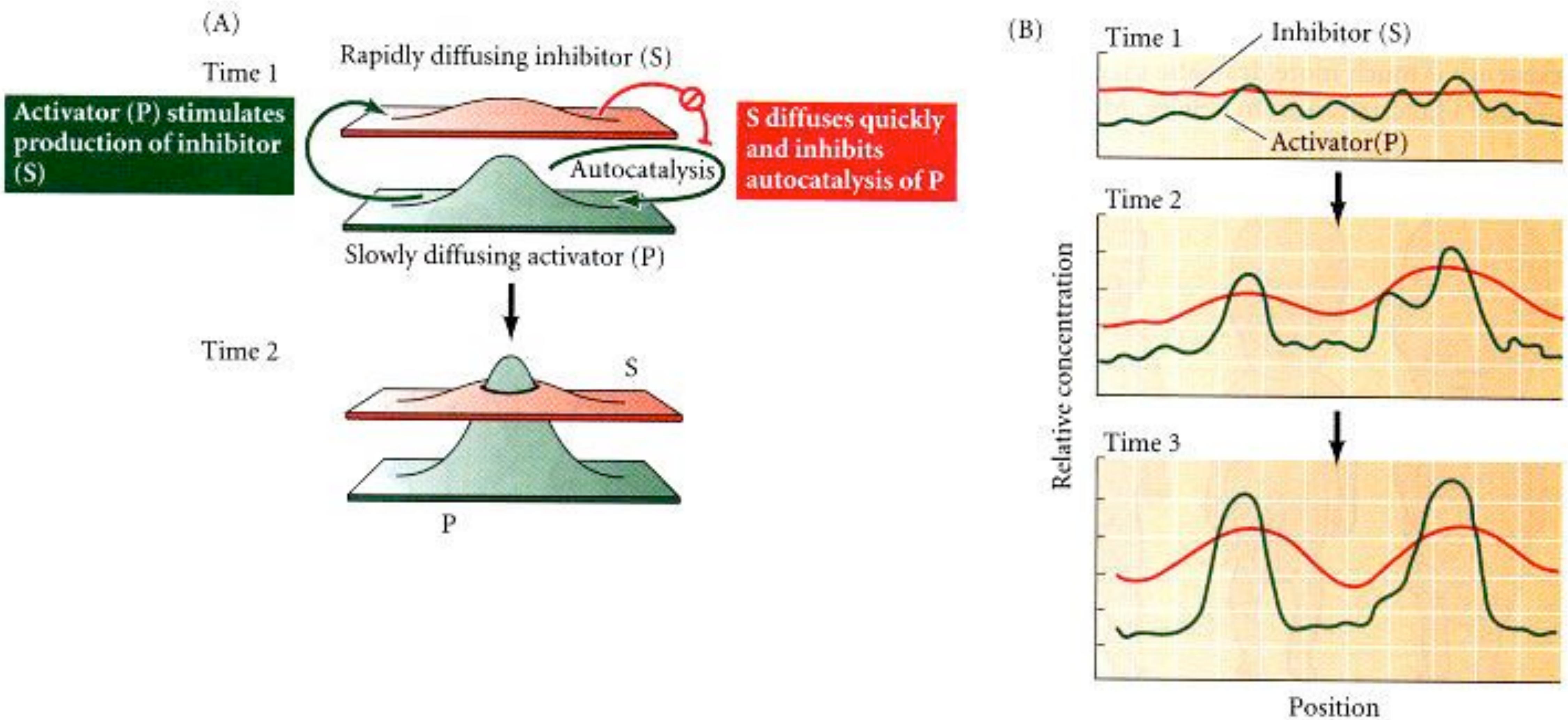
Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, Vol. 237, No. 641. (Aug. 14, 1952), pp. 37-72.



Web of Science >4800 citations

dunkel@math.mit.edu

Turing model



dunkel@math.mit.edu

Turing model

$$\frac{\partial u}{\partial t} = F(u,v) - d_u v + D_u \Delta u$$

$$\frac{\partial v}{\partial t} = G(u,v) - d_v v + D_v \Delta v$$

Rate of concentration change

Production

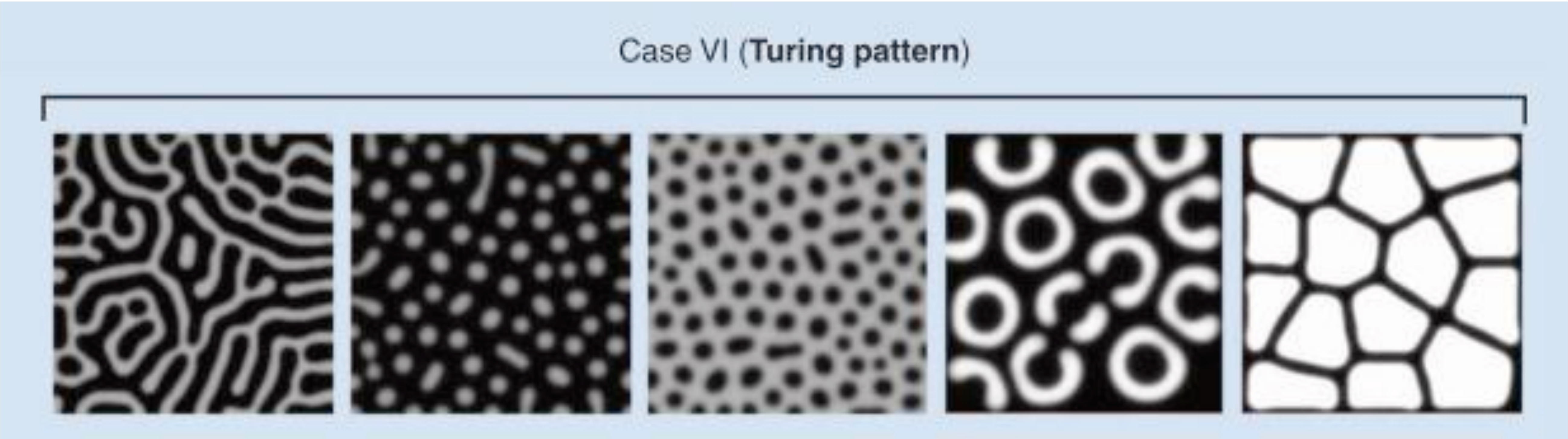
Degradation

Diffusion

Reaction

A. M. Turing. **The chemical basis of morphogenesis.** *Phil. Trans. Royal Soc. London. B* **327**, 37–72 (1952)

Turing pattern



Kondo S, & Miura T (2010). Reaction-diffusion model as a framework for understanding biological pattern formation. *Science, 329* (5999), 1616-20

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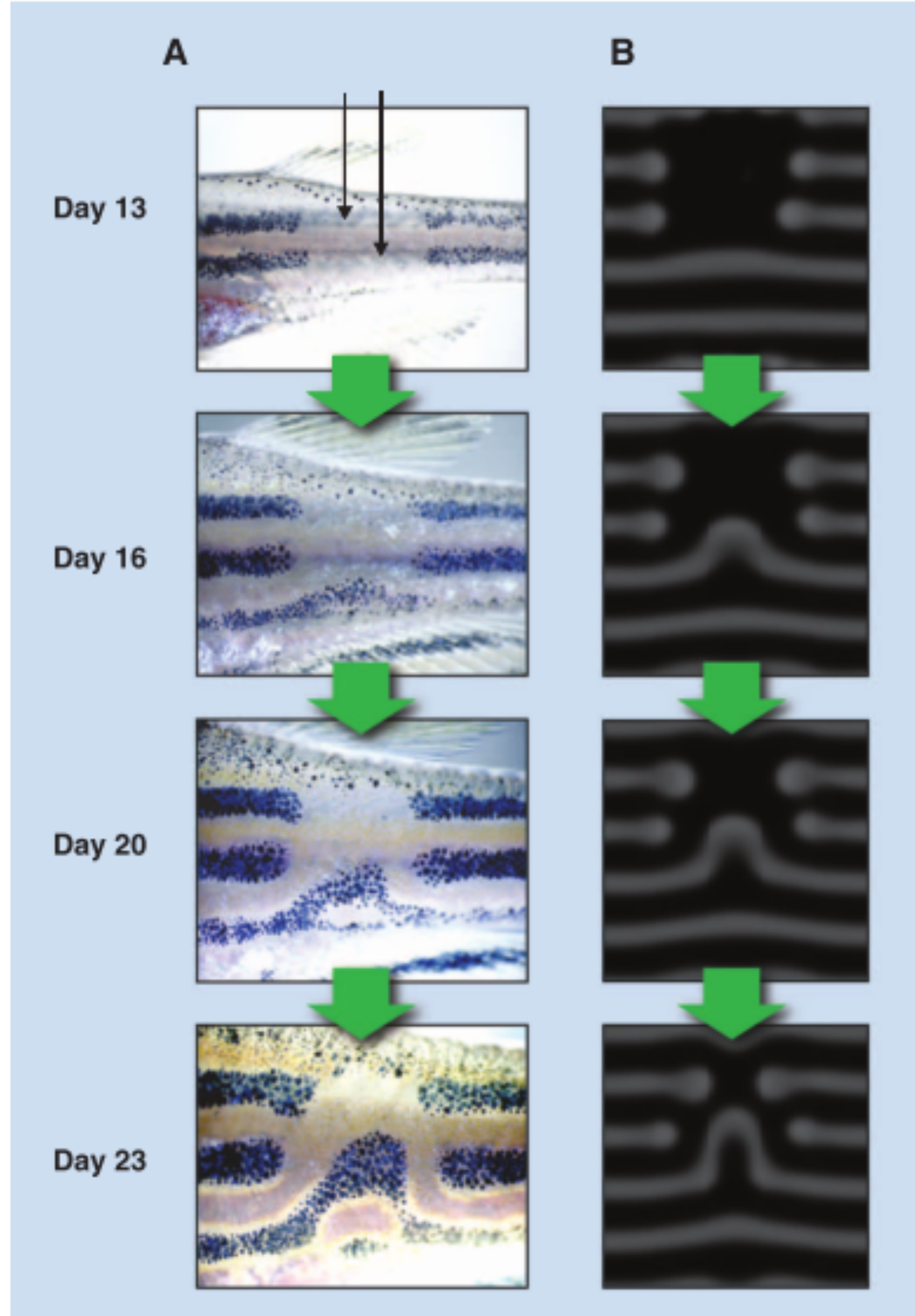
The matching of zebrafish stripe formation and a Turing model



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