Course intro

18.354

18.354J Nonlinear Dynamics II: Continuum Systems

Spring 2015 – Course Info

Lectures:	TR 1-2:30 in E17-136
Instructor:	Jörn Dunkel
Contact:	`dunkel@mit.edu` 253-7826 (office phone)
Office Hours:	R 2:30-3:30 (E17-412)
Course website:	`math.mit.edu/~dunkel/Teach/18.354/`
Teaching assistant:	TBD, Office: TBD Email: `tbd`

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GRADING

- 35%: Problem sets
- 30%: Mid-term exam
- 5%: Project proposal
- 30%: Final project presentation + report

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TEXTBOOKS

Although there are no textbooks which follow the precise spirit of this course, there are literally hundreds of textbooks where the topics we will cover are discussed. For most lectures, typed notes can be downloaded from the course webpage. Additional material will be handed out in class. One book that will be useful frequently is: D. J. Acheson, *Elementary Fluid Dynamics*, Oxford University Press (1990).

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HOMEWORK - PROBLEM SETS

Homework will be assigned roughly every two-three weeks. Each homework set will contain analytical and computational problems, and even the odd experiment. Assignments must be handed in by 1pm (start of class) on the due date. *First unexcused late homework score will be multiplied by 3/4. No subsequent unexcused late homework is accepted.* You are welcome to discuss solution strategies and even solutions, but please write up the solution on your own. Be sure to support your answer by listing any relevant Theorems or by explaining important steps. Be as clear and concise as possible. I strongly encourage the computational problems to be written in MATLAB.

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MID-TERM EXAM

There will be a mid-term exam, which will take place about 3/4's of the way through the course. The exam will be a take home, and will be in place of a homework set. There will be no final exam.

FINAL PROJECT

The ideas we will be discussing have applications to many fields, many of which we will not cover. To give you a chance to explore an area of interest to you, the course will require a final project, in which you explore in depth something of interest to you and within the course's scope. Final projects will be presented in class during the final two classes.

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IMPORTANT DATES

• Tues Feb 24 - Problem Set 1: DUE
• Thur Mar 12 - Problem Set 2: DUE • Thur Mar 12 - Proposal (1 page) for final project.
DUE
• Thur Apr 2 - Problem Set 3: DUE
• Tues Apr 14 - Problem Set 4: DUE
• Thur Apr 16 - Take-home Midterm exam. POSTED
• Thur Apr 23 - Take-home Midterm exam. DUE
• May 5/7 - Final project presentations.
• May 7 - Final project report. DUE

Note: The exact due dates for the P-sets may be subject to change

1.	T	Feb	3	Introduction, overview & mathematical basics	
2.	R	Feb	5	Dimensional analysis & scalings	
3.	T	Feb	10	Hamiltonian dynamics & Kepler's Laws	
4.	R	Feb	12	Random walkers	
—	T	Feb	17	MIT MONDAY (PRESIDENTS DAY)	
5.	R	Feb	19	Diffusion equation: Fourier method	
6.	T	Feb	24	Diffusion equation: Green's function method	**PS1 due**
7.	R	Feb	26	Linear stability analysis & pattern formation	
8.	T	Mar	3	Calculus of variations	
9.	R	Mar	5	Surface tension	
10.	T	Mar	10	Elasticity	
11.	R	Mar	12	Deformation of a thin beam	**PS2 & proposal due**
12.	T	Mar	17	Towards hydrodynamics	
13.	R	Mar	19	Navier-Stokes equations I	
—	TR	Mar	23-27	SPRING VACATION	

14.	R	Apr	2	Stokes limit & Oseen tensor	**PS3 due**
15.	T	Apr	7	Navier-Stokes equations II	
16.	R	Apr	9	Singular perturbations	
17.	T	Apr	14	Rotating flows & Taylor columns	**PS4 due**
18.	R	Apr	16	2D hydrodynamics & conformal maps	**Mid-term posted**
—	T	Apr	20,21	MIT HOLIDAY (PATRIOTS DAY)	
19.	R	Apr	23	Hydrodynamic instabilities (overview)	**Mid-term due**
20.	T	Apr	28	Solitons	
21.	R	Apr	30	Active Matter	
22.	T	May	5	Final projects: student presentations	
23.	R	May	7	Final projects: student presentations	**Project report due**
24.	T	May	12	Bouncing droplets	
25.	R	May	14	Topological defects & summary	

Q: What is Physical Applied Maths?

A:  PAM is like cooking...

Often the ingredients (*physical principles*) are already known but not the way (*mathematics/equations/couplings*) to turn them into a nice dinner

With some creativity, many new dishes (*novel phenomena*) can be created (*discovered/understood*)

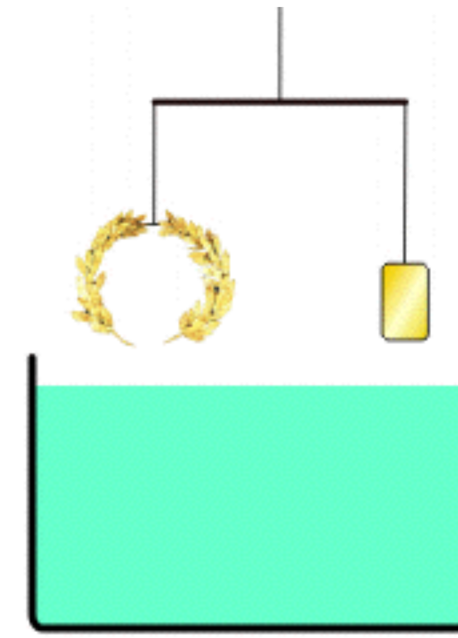
Why study Applied Maths?

- intellectual challenge
- obtain general understanding of physical phenomena and the world around us
- be able to make prediction about physical processes
- development of general tools to be applied to other fields

Historical backdrop

Some famous thoughts on Applied Maths

'*Eureka, Eureka*'
(Archimedes)



c. 287 BC - c. 212 BC

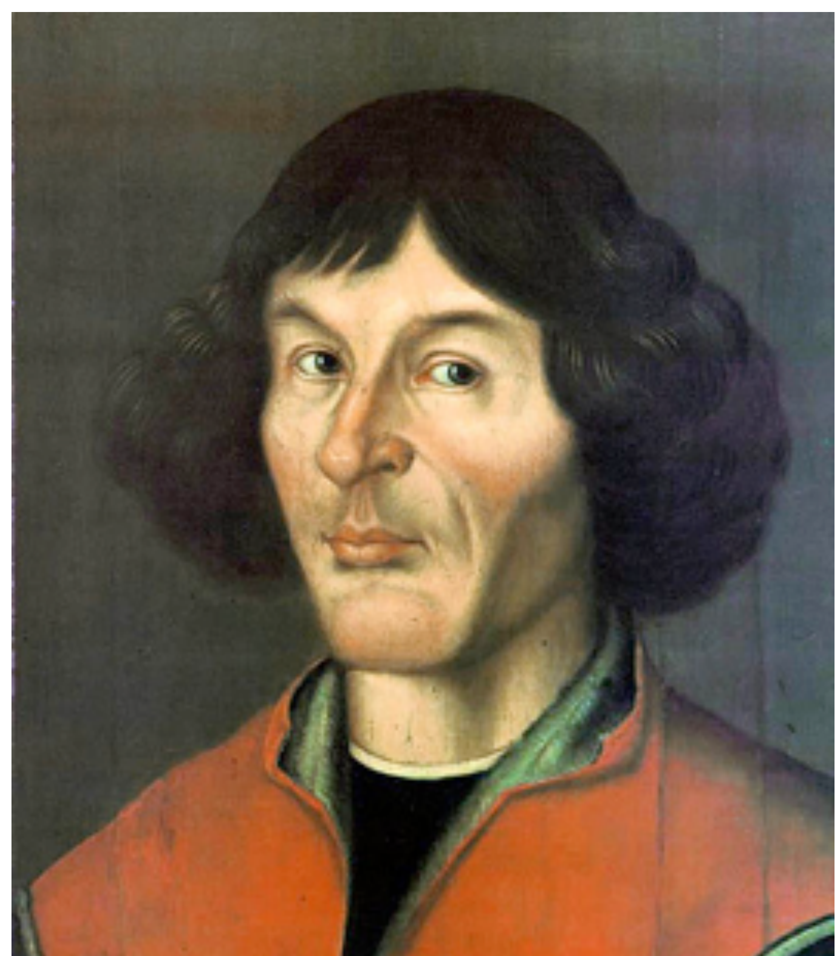
Some famous thoughts on Applied Maths

'*Mathematic is written for mathematicians*'
(Nicolaus Copernicus)


NICOLAI CO
PERNICI TORINENSIS
DE REVOLVTIONIBVS ORBIum coelestium, Libri VI.

Norimbergæ apud Ioh. Petreium,
Anno M. D. XLIII.

1543

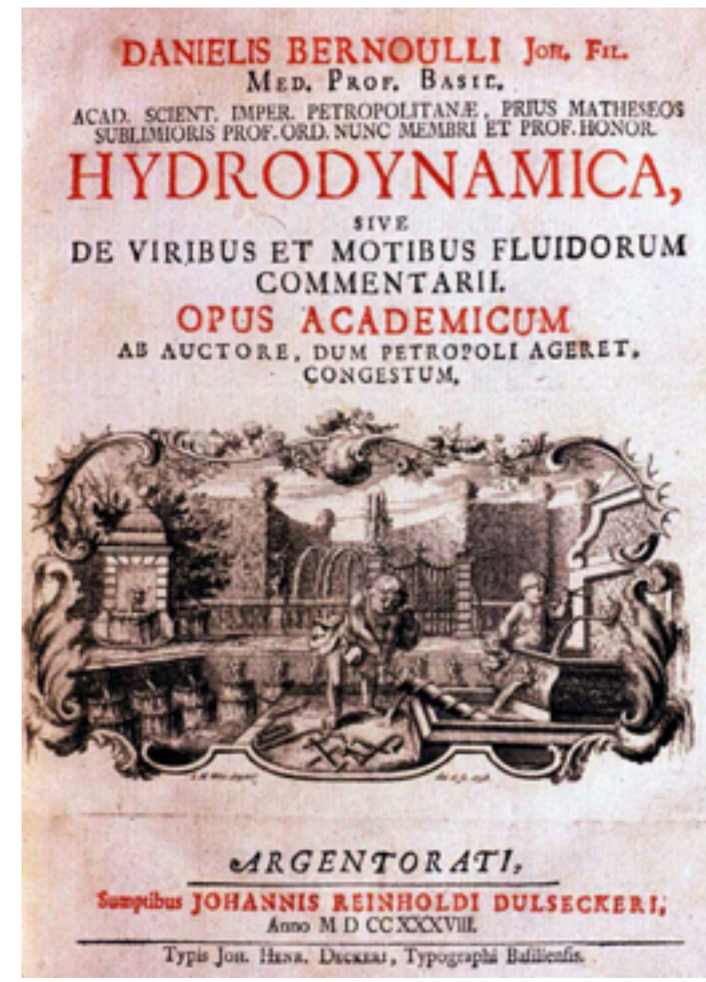


19 February 1473 – 24 May 1543

Some famous thoughts on Applied Maths?

'*It would be better for the true physics if there were no mathematicians on earth*'
(Daniel Bernoulli)

8 February 1700 – 17 March 1782



Some famous thoughts on Applied Maths

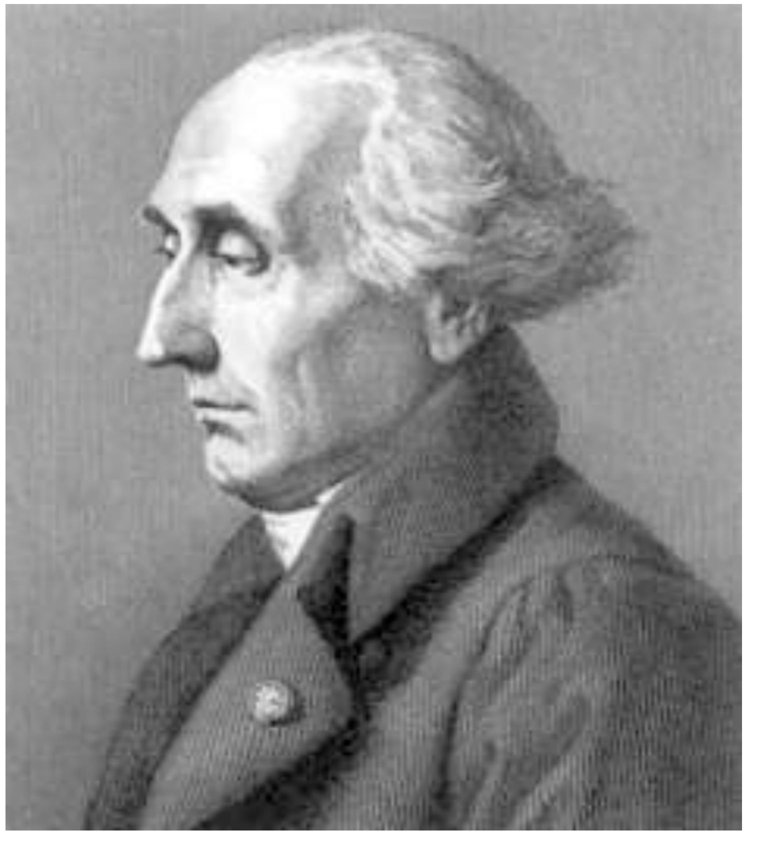
'*Now I will have less distraction*'
(Leonhard Euler, upon losing the use of his right eye)

15 April 1707 – 18 September 1783

Some famous thoughts on Applied Maths

'I do not know'

(Joseph-Louis Lagrange, summarizing his life's work)



25 January 1736 - 10 April 1813

Some famous thoughts on Applied Maths

'*Nature laughs at the difficulties of integration*'
(Pierre-Simon Laplace)

23 March 1749 – 5 March 1827

Some famous thoughts on Applied Maths?

'*Mathematicians are born, not made*'
(Henri Poincaré)

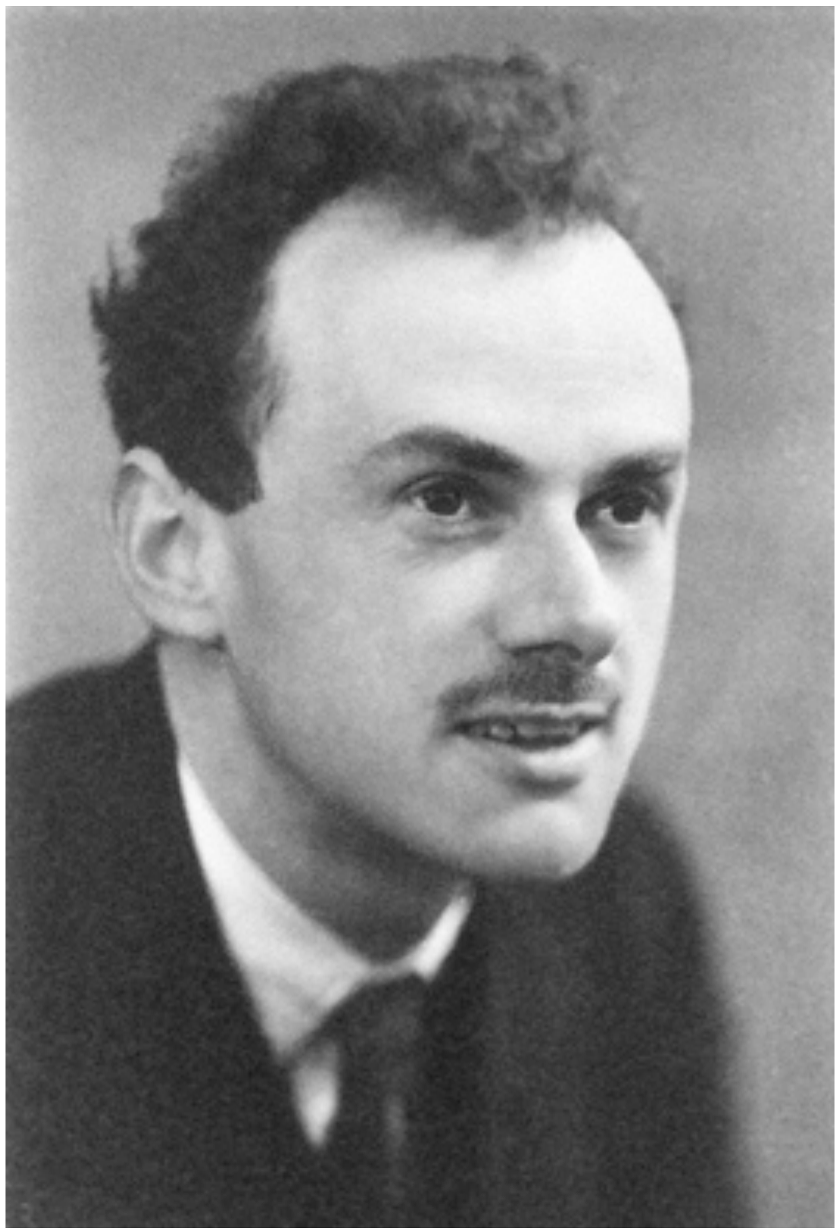
29 April 1854 – 17 July 1912

Some famous thoughts on Applied Maths?

'*Prediction is very difficult, especially about the future*'
(Niels Bohr)

7 October 1885 – 18 November 1962

Some famous thoughts on Applied Maths?



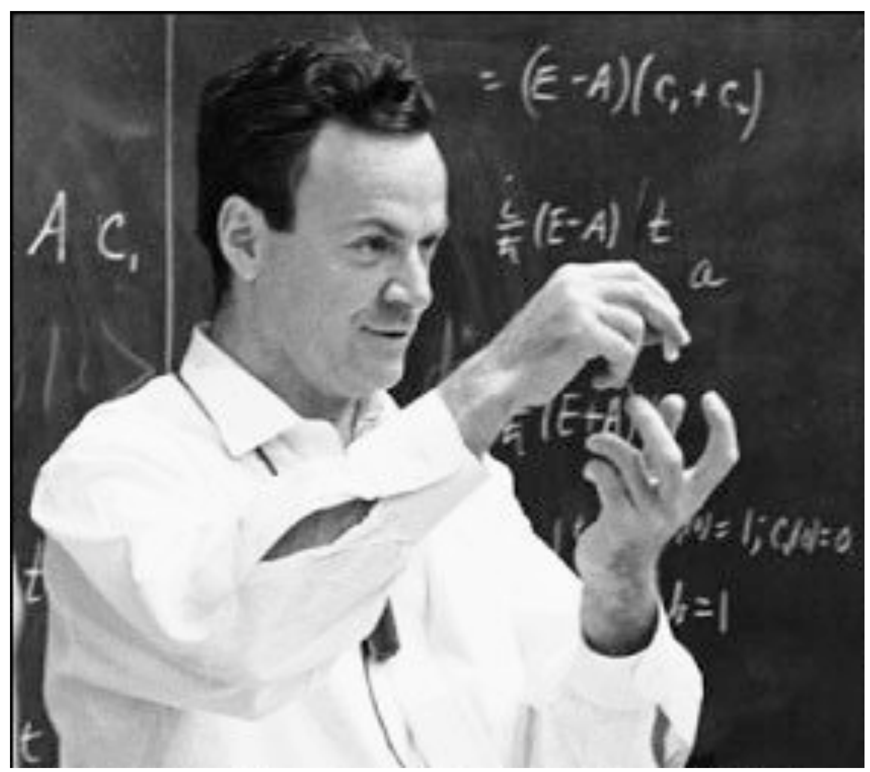
'*It is more important to have beauty in one's equations than to have them fit experiment*' and '*This result is too beautiful to be false*'
(Paul Dirac)

8 August 1902 – 20 October 1984

Some famous thoughts on Applied Maths?

'*To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature*'

(Richard Feynman)



May 11, 1918 – February 15, 1988

BSc 1939

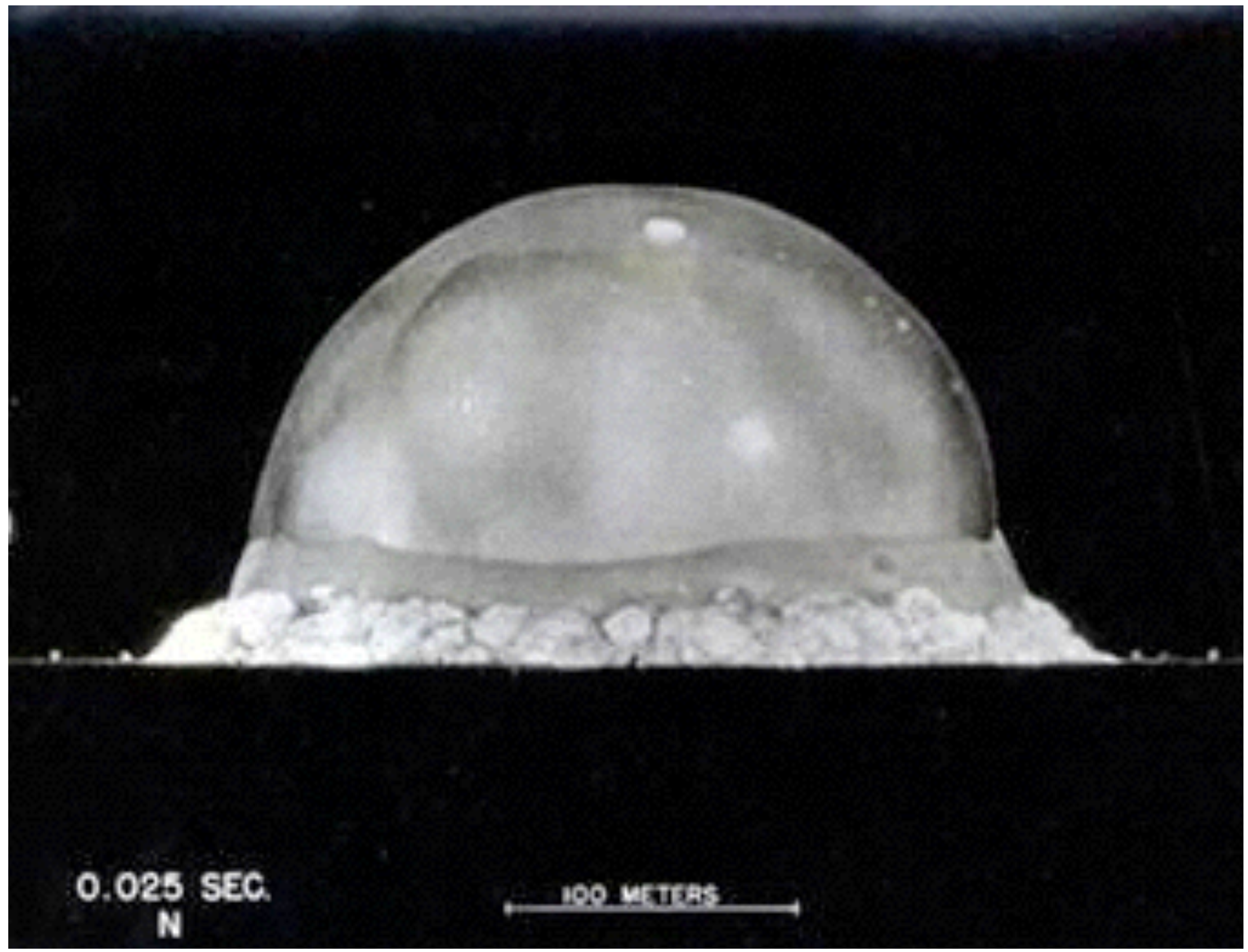
Course overview

18.354

- ODEs
- linear/nonlinear PDEs for scalar & vector fields
- perturbation theory
- calculus of variations
- Fourier transformations
- complex numbers, conformal maps

Dimensional analysis

G.I. Taylor
1886-1975

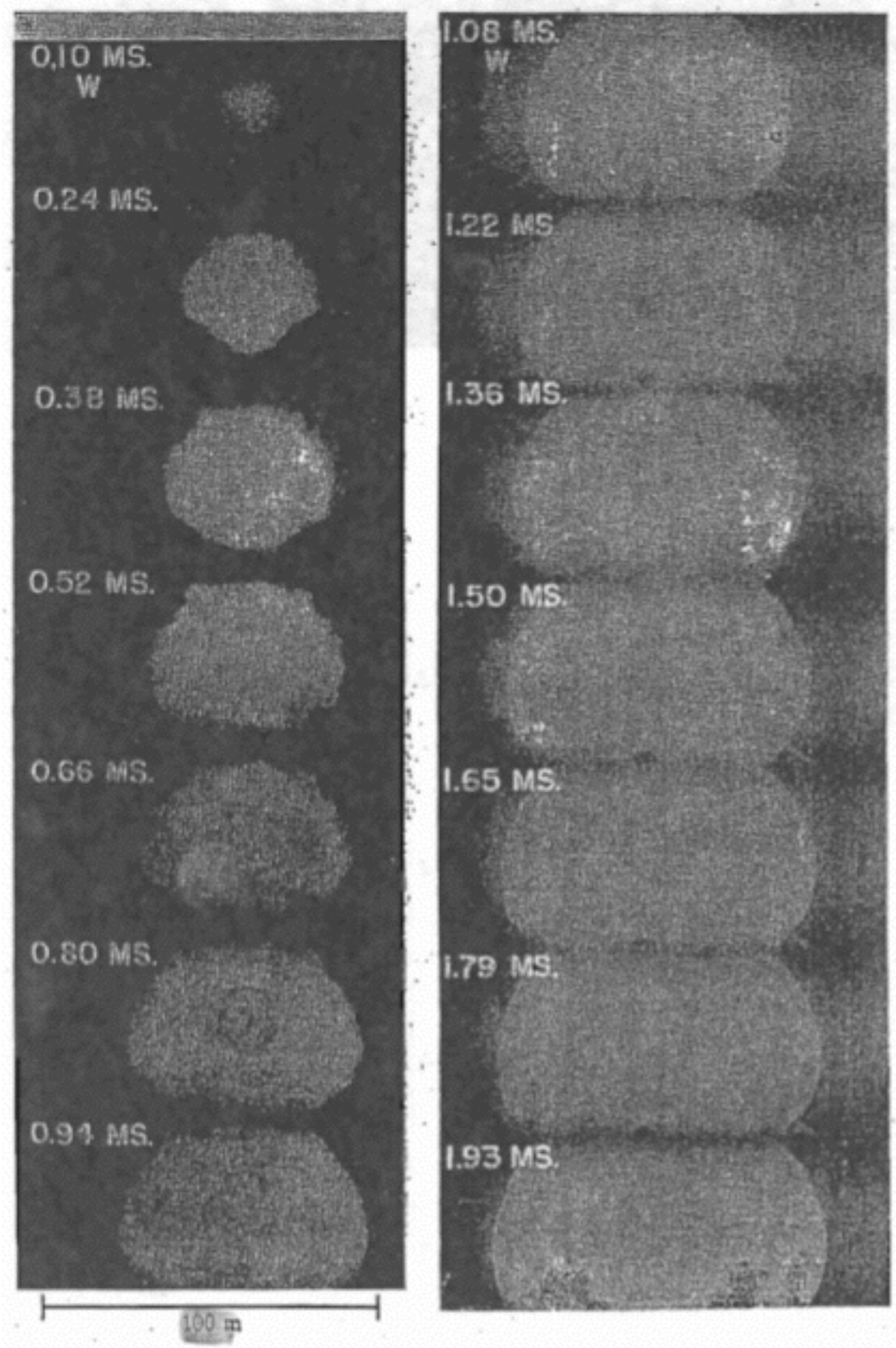


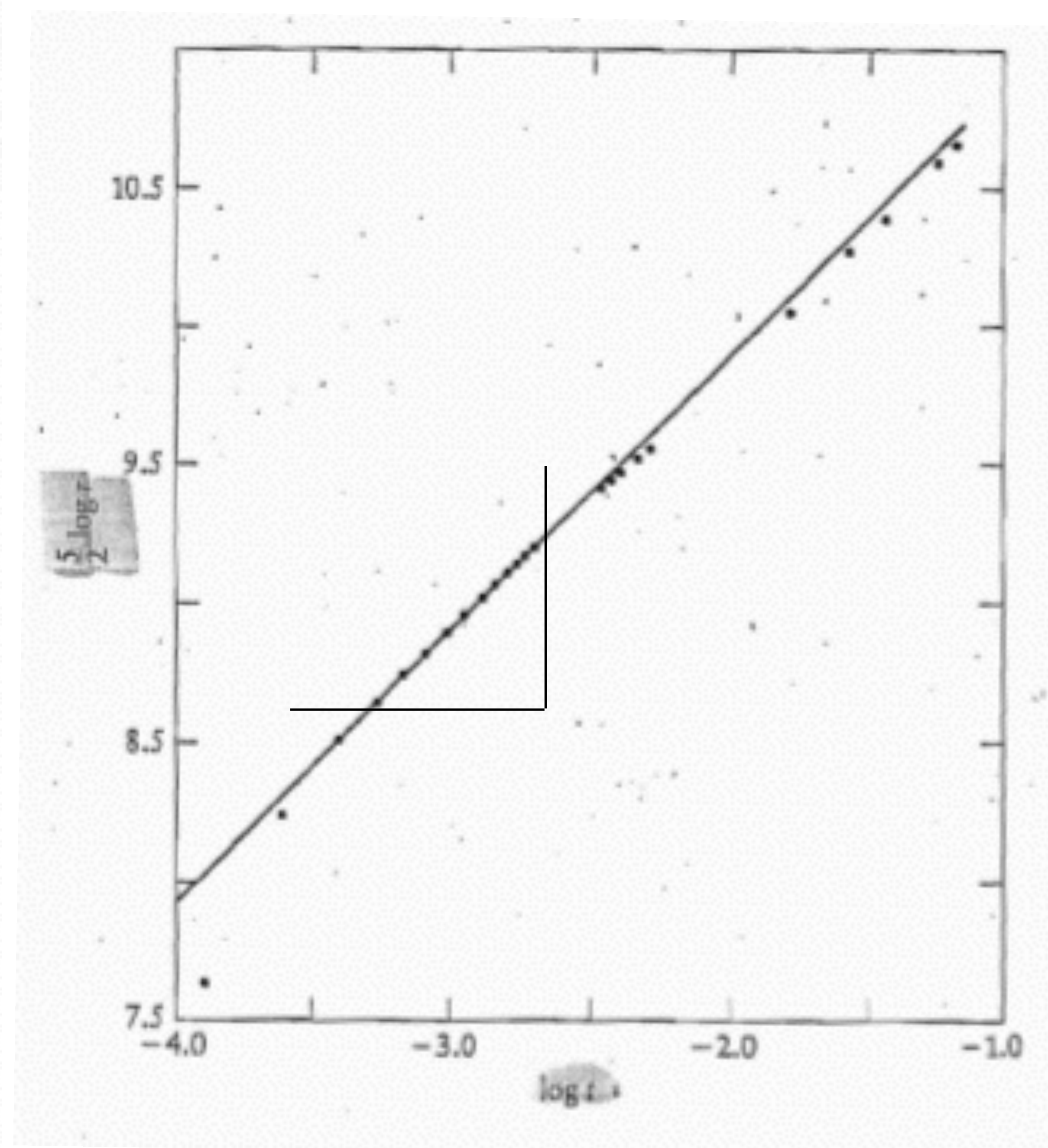
Trinity nuclear test, July 1945
Life Magazine, August 20, 1945

The formation of a blast wave by a very intense explosion.
II. The atomic explosion of 1945

By Sir Geoffrey Taylor, F.R.S.

(*Received* 10 *November* 1949)





$$\ln R = \ln\left[c\left(\frac{E}{\rho}\right)^{1/5}\right] + \frac{2}{5}\ln t$$

Hamiltonian dynamics & Kepler's problem

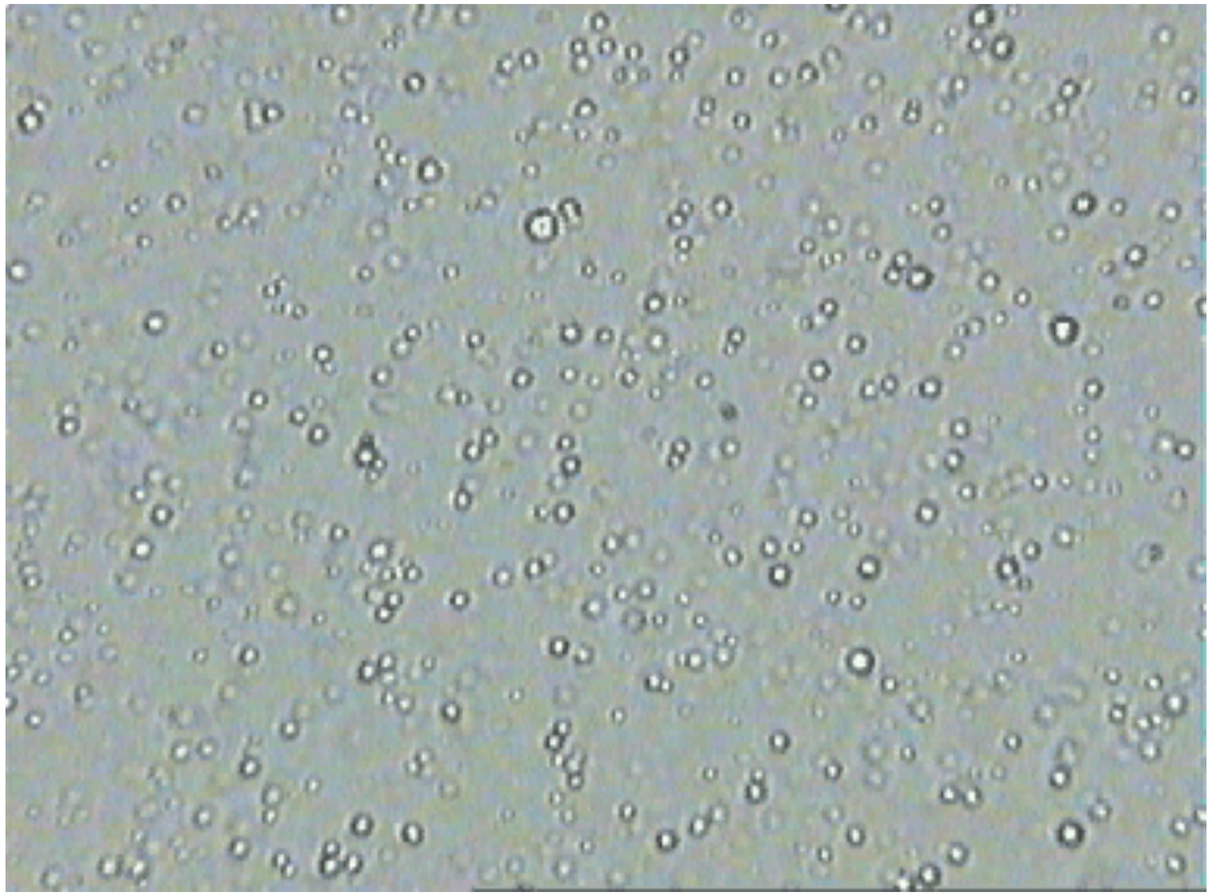
$$H = \sum_i \frac{p_i^2}{2m_i} + U(x_1, \ldots, x_N)$$

$$\mathbf{L} = \mathbf{r} \times m \frac{d\mathbf{r}}{dt}$$

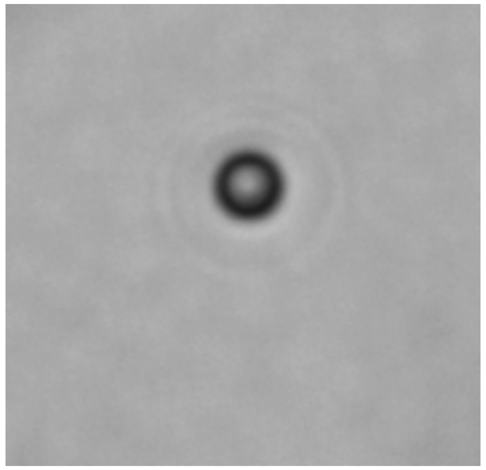
Brownian motion

Random walks & diffusion

$$\frac{\partial n}{\partial t} = -\frac{\partial J_x}{\partial x} = D\frac{\partial^2 n}{\partial x^2},$$



David Walker

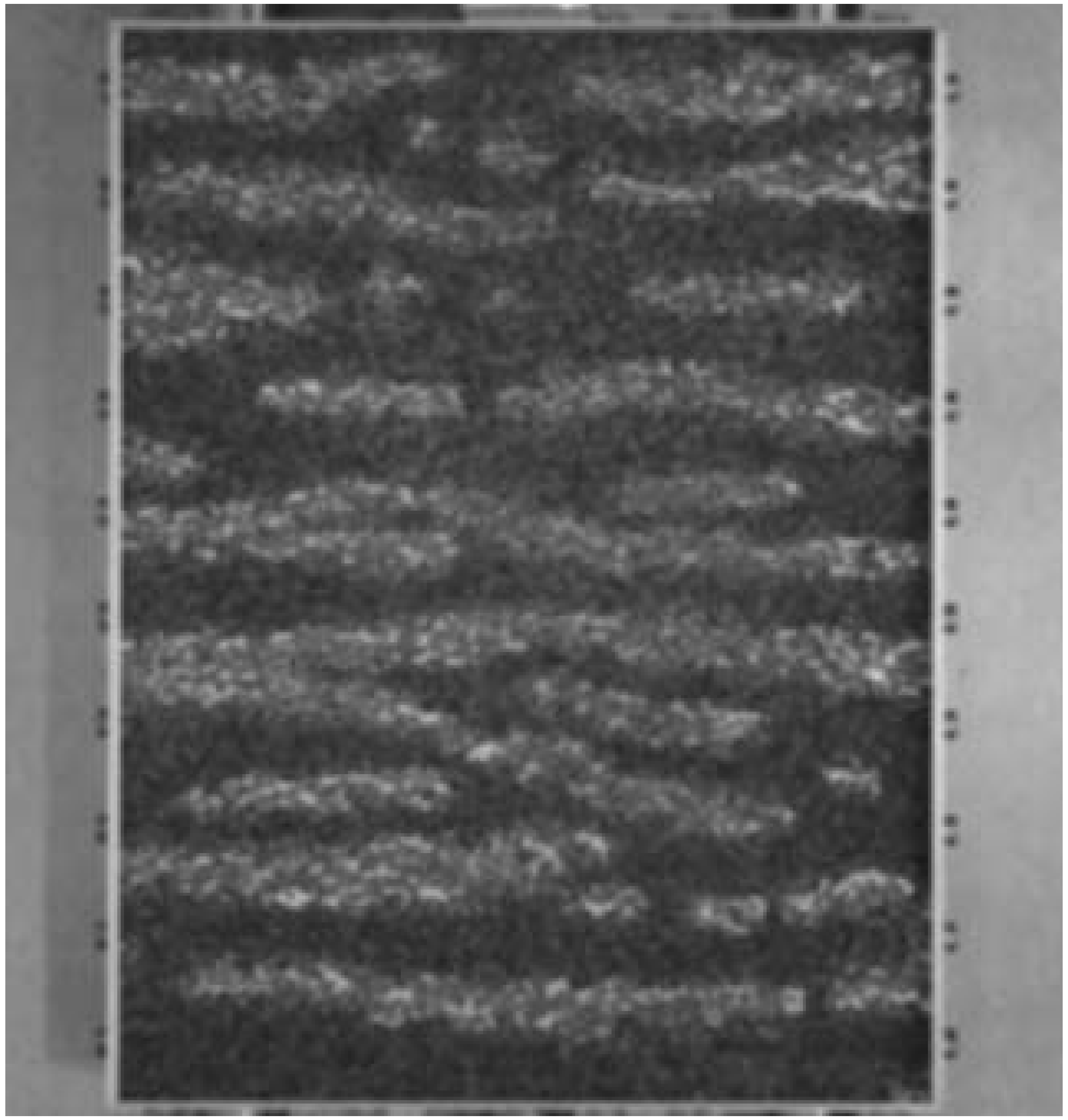


Mark Haw

Pattern formation

dunkel@math.mit.edu

Zebra vs. granular media



dunkel@math.mit.edu

arxiv: 1208.4464

2d Swift-Hohenberg model

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

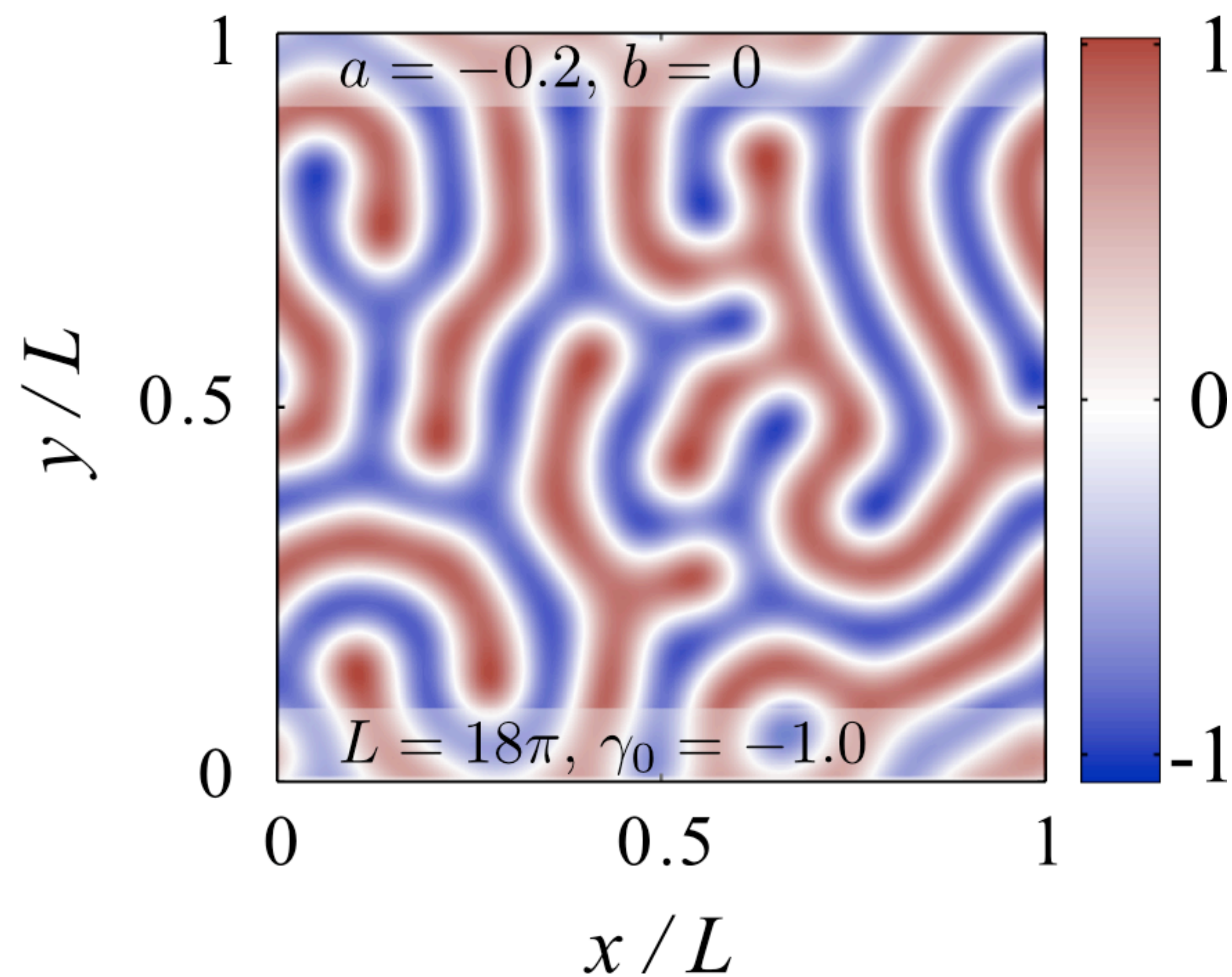
$$U(\psi) = \frac{a}{2}\psi^2 + \frac{b}{3}\psi^3 + \frac{c}{4}\psi^4$$

$$\psi(t, \boldsymbol{x}) = \nabla \times \boldsymbol{v}$$

reflection-symmetry

$$b = 0$$

$$\psi \mapsto -\psi$$



2d Swift-Hohenberg model

arxiv: 1208.4464

broken reflection-symmetry

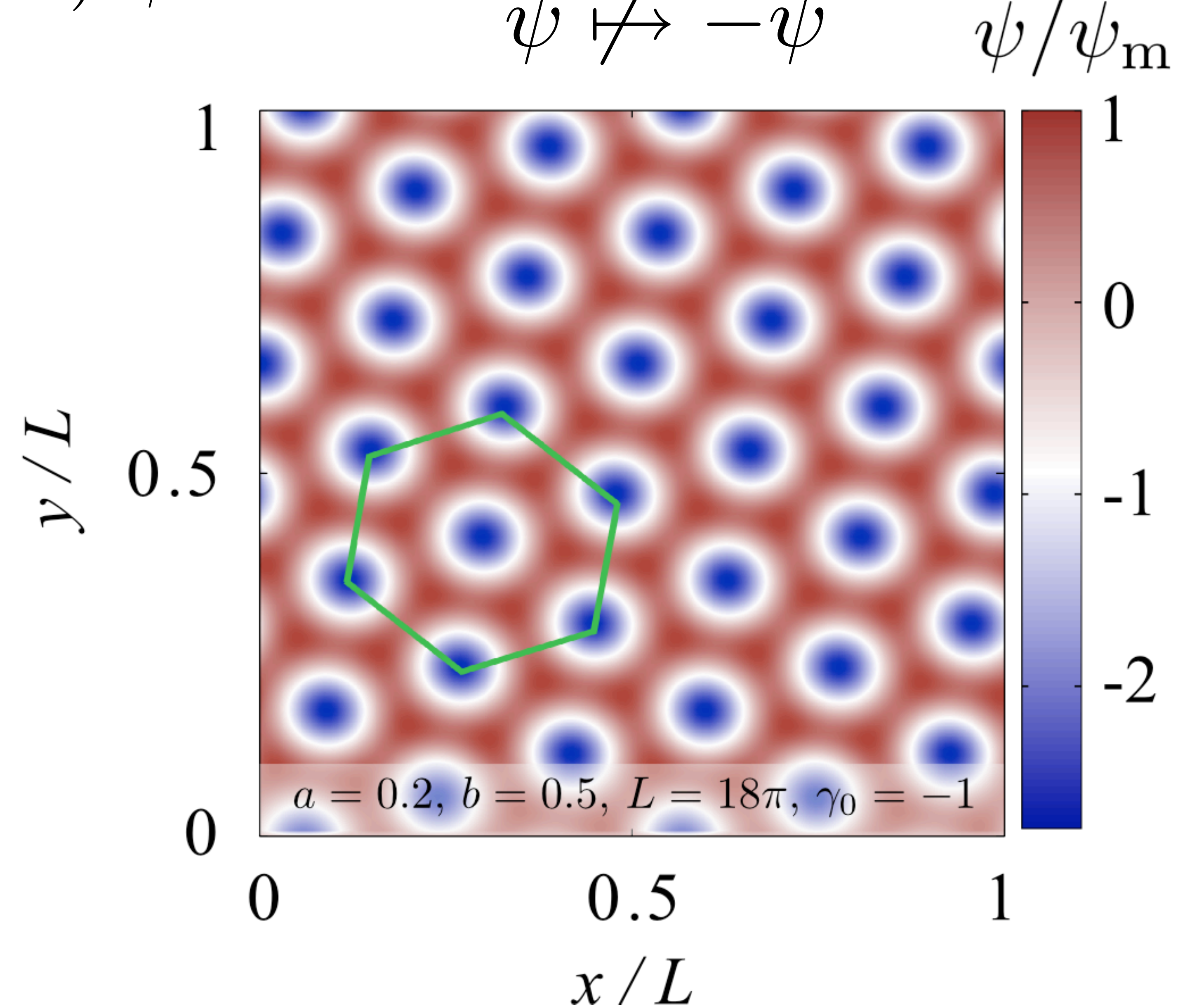
$$b \neq 0$$

$$\psi \not\mapsto -\psi$$

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

$$U(\psi) = \frac{a}{2}\psi^2 + \frac{b}{3}\psi^3 + \frac{c}{4}\psi^4$$

$$\psi(t, \boldsymbol{x}) = \nabla \times \boldsymbol{v}$$

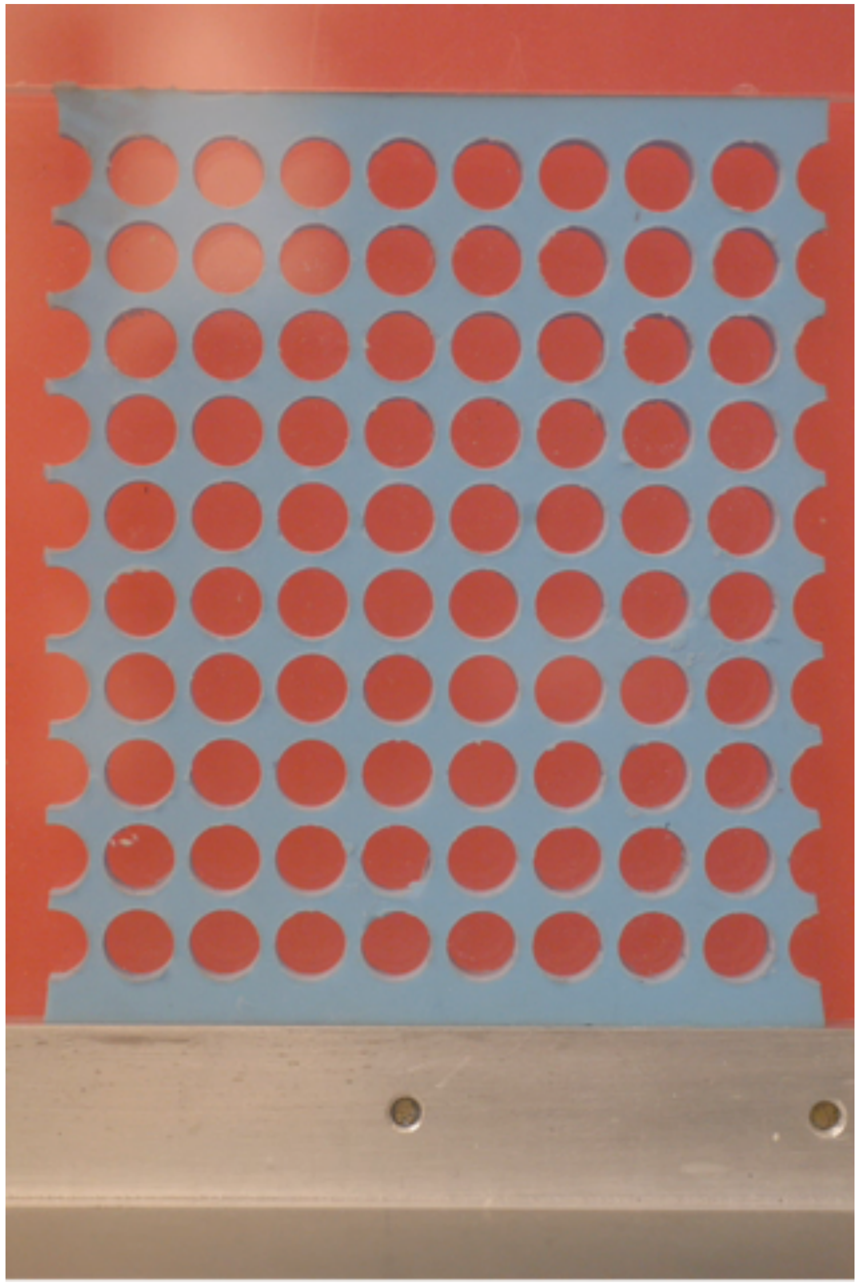


Calculus of variations

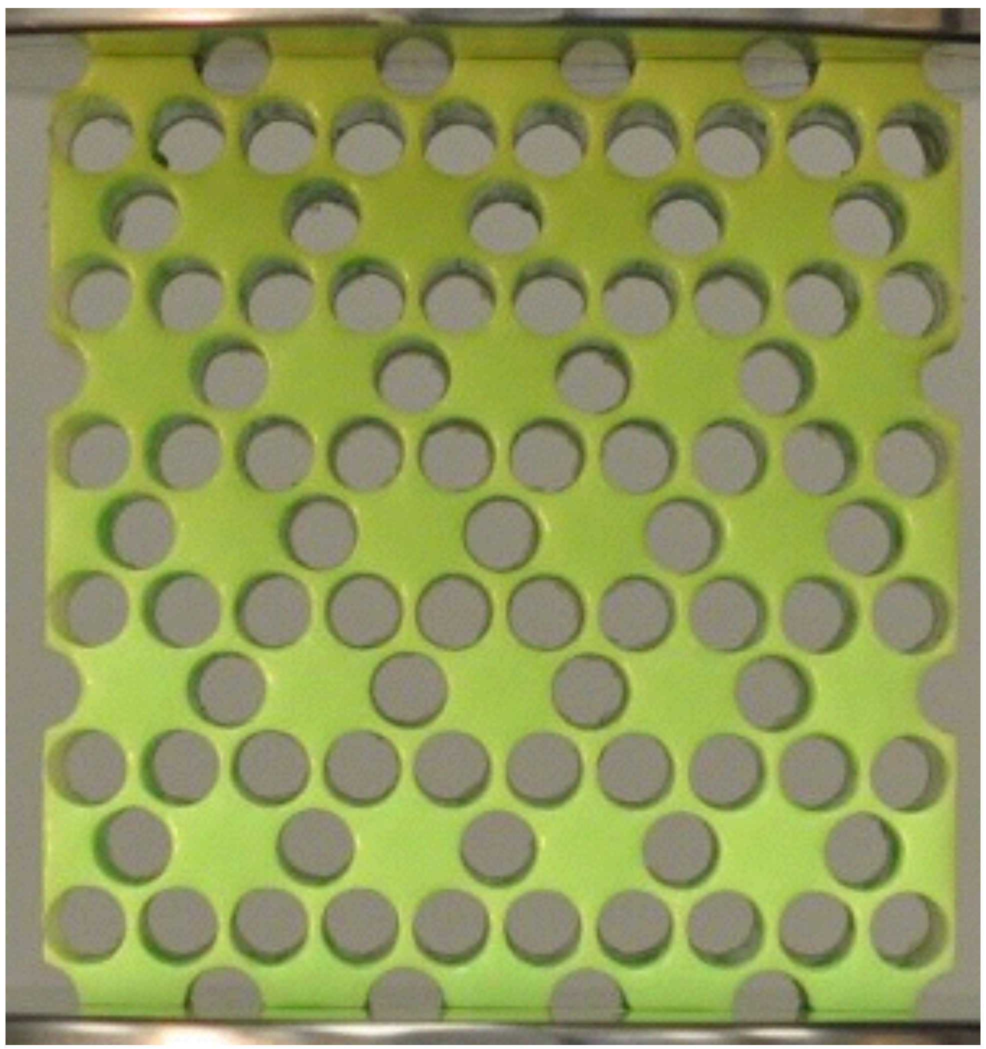
$$\begin{aligned}
\frac{\delta I[Y]}{\delta Y} &= \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left\{ I[f(x) + \epsilon\delta(x-y)] - I[f(x)] \right\} \\
&= \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial Y} \delta(x-y) + \frac{\partial f}{\partial Y'} \delta'(x-y) \right] dx \\
&= \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial Y} - \frac{d}{dx} \frac{\partial f}{\partial Y'} \right] \delta(x-y) dx.
\end{aligned}$$

$$0 = \frac{\partial f}{\partial Y} - \frac{d}{dx} \frac{\partial f}{\partial Y'}$$

Elasticity



Elasticity



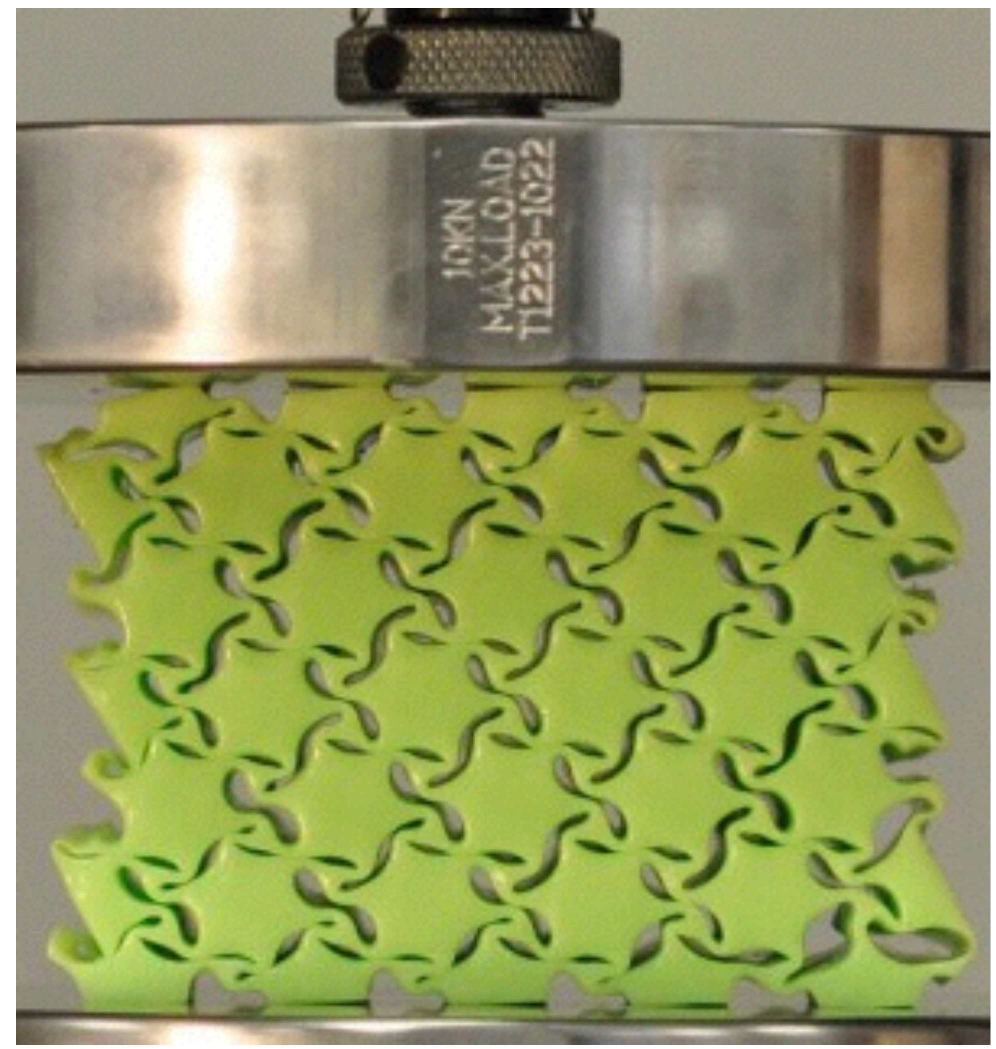
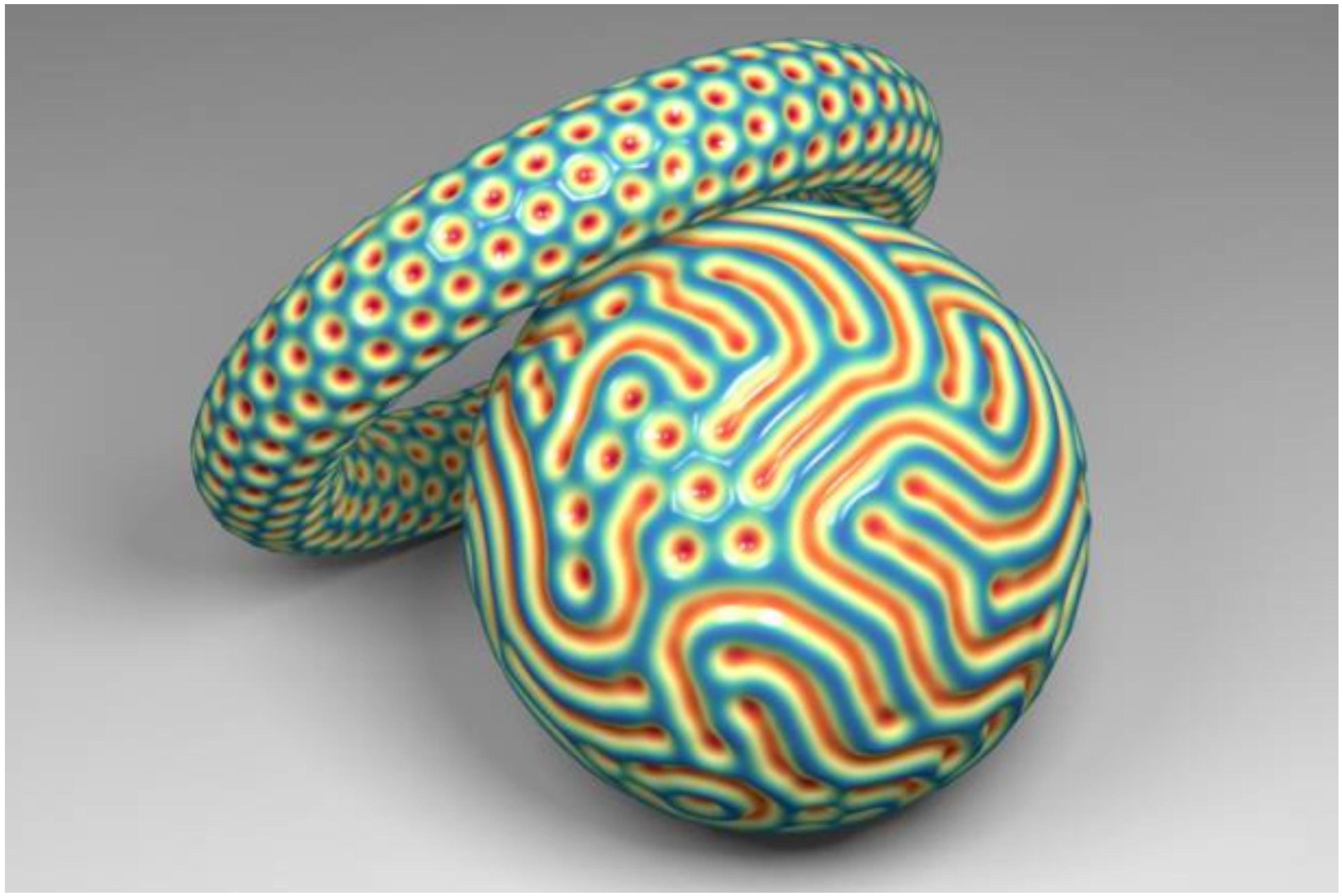


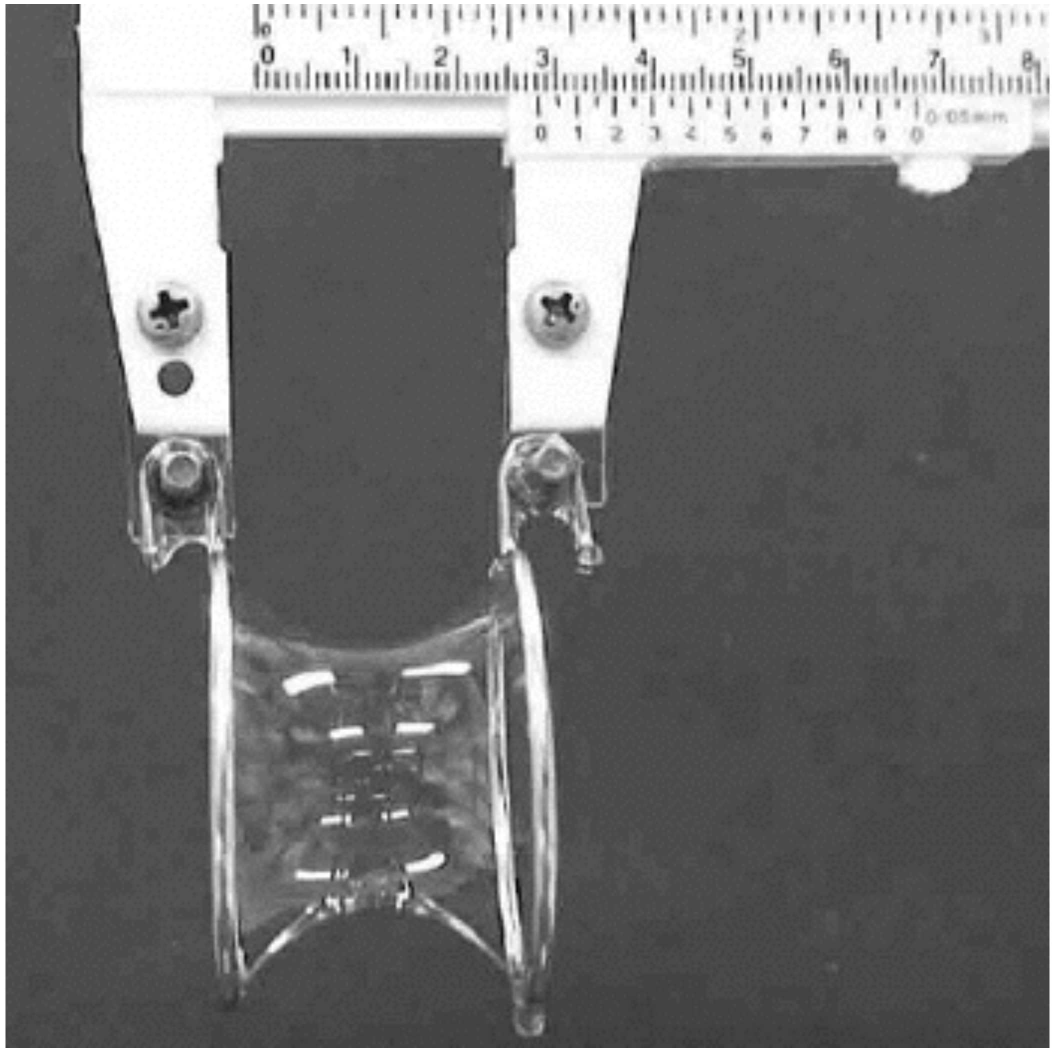
photo: Andrej Kosmrlj

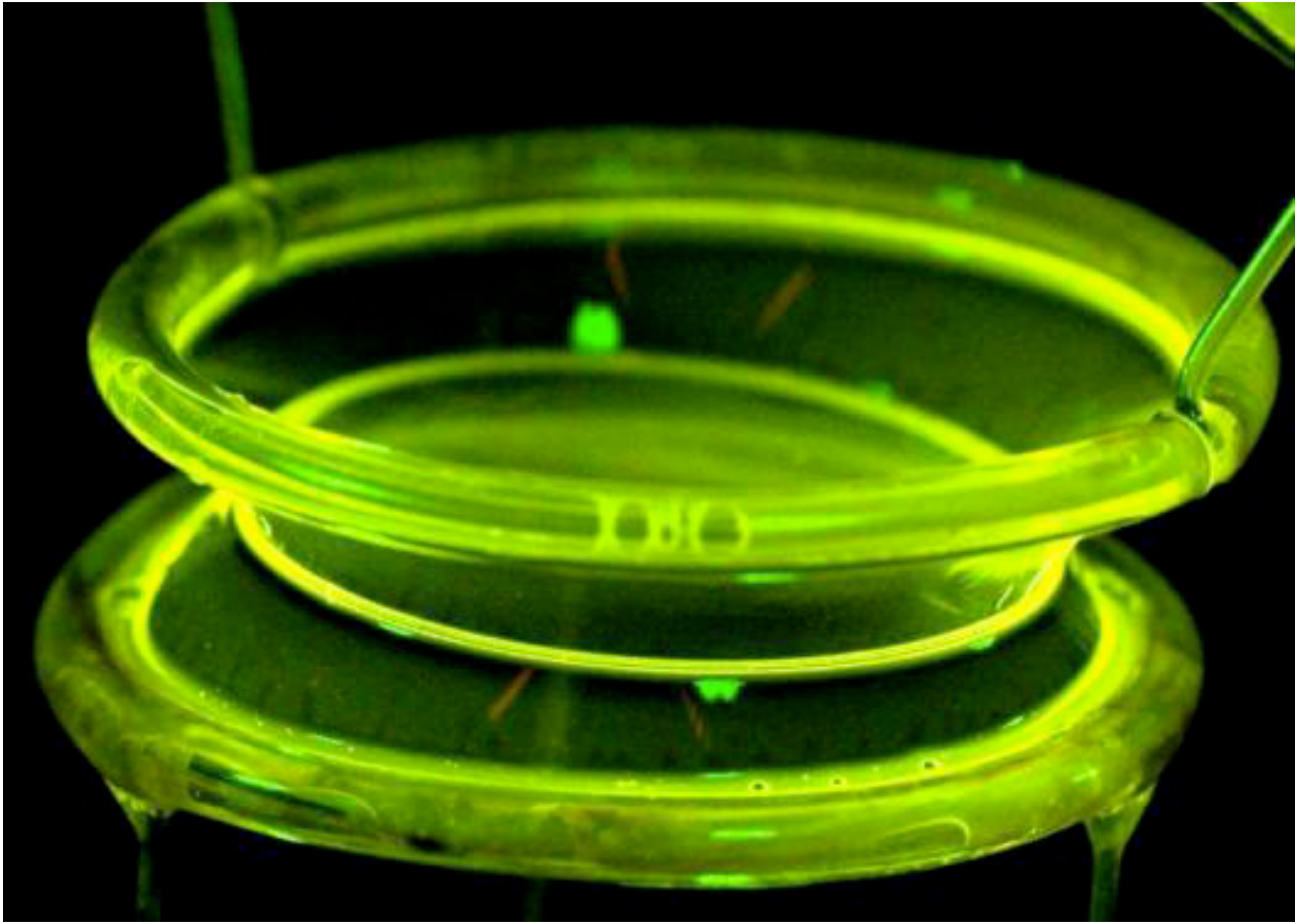
http://newsoffice.mit.edu/2015/predicting-wrinkles-fingerprints-curved-surfaces-0202



collaboration with Reis lab (MechE)

Surface tension





Goldstein lab, Cambridge

Large drop in microgravity

Hydrodynamics

$$\int_V \frac{\partial \rho}{\partial t} dV = -\int_S \rho \mathbf{u} \cdot \mathbf{n} dS = -\int_V \nabla \cdot (\rho \mathbf{u}) dV.$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0.$$

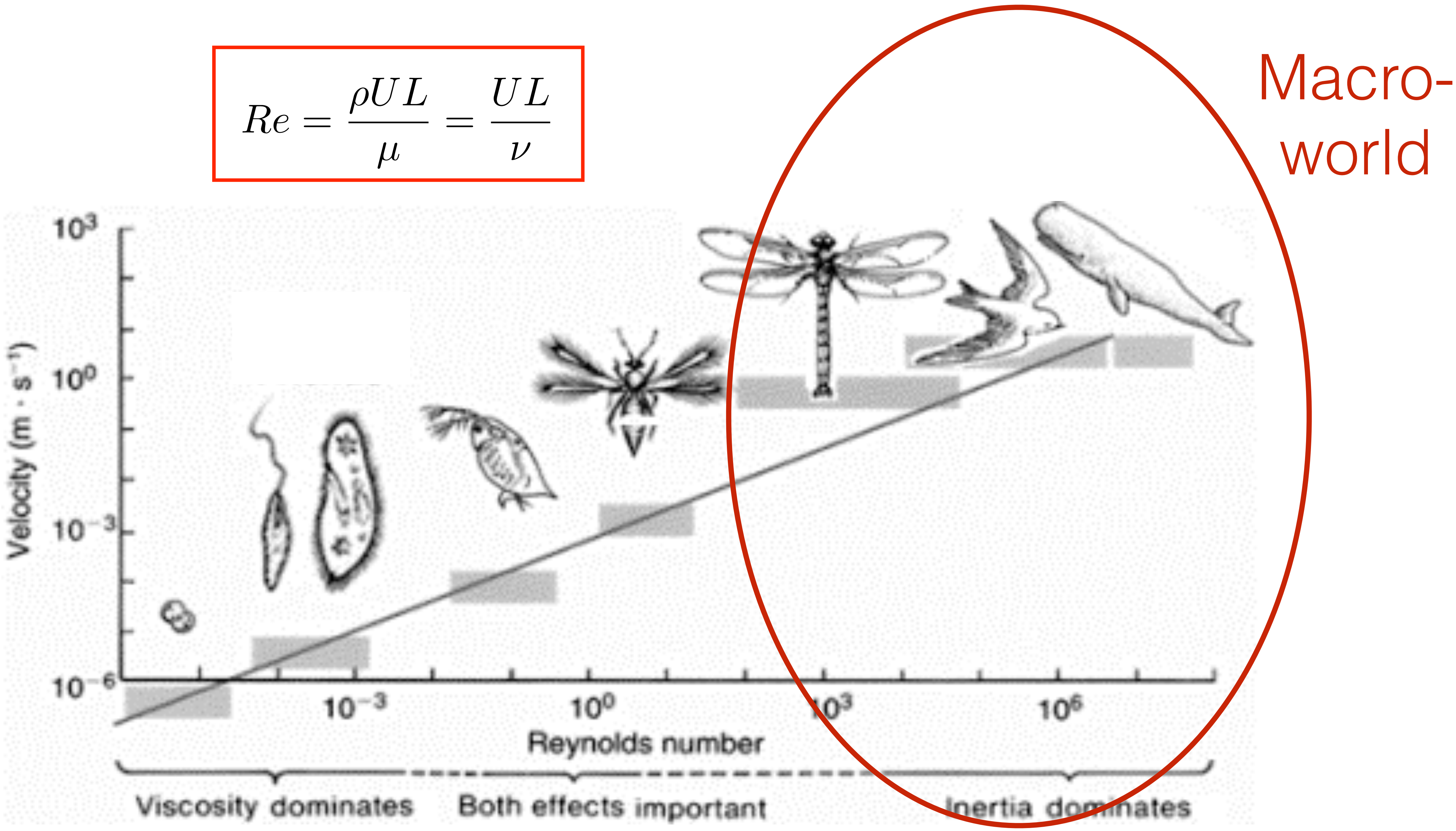
$$\int_{V(t)} \rho \frac{D\mathbf{u}}{Dt} dV = \int_{V(t)} (-\nabla p + \rho \mathbf{g}) dV$$

$$\frac{D\mathbf{u}}{Dt} = \frac{-\nabla p}{\rho} + \mathbf{g}.$$

Typical Reynolds numbers

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} - \tfrac{2}{3}\mu\nabla(\nabla\cdot\mathbf{u}) + \rho\mathbf{g}$$

$$Re = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$$



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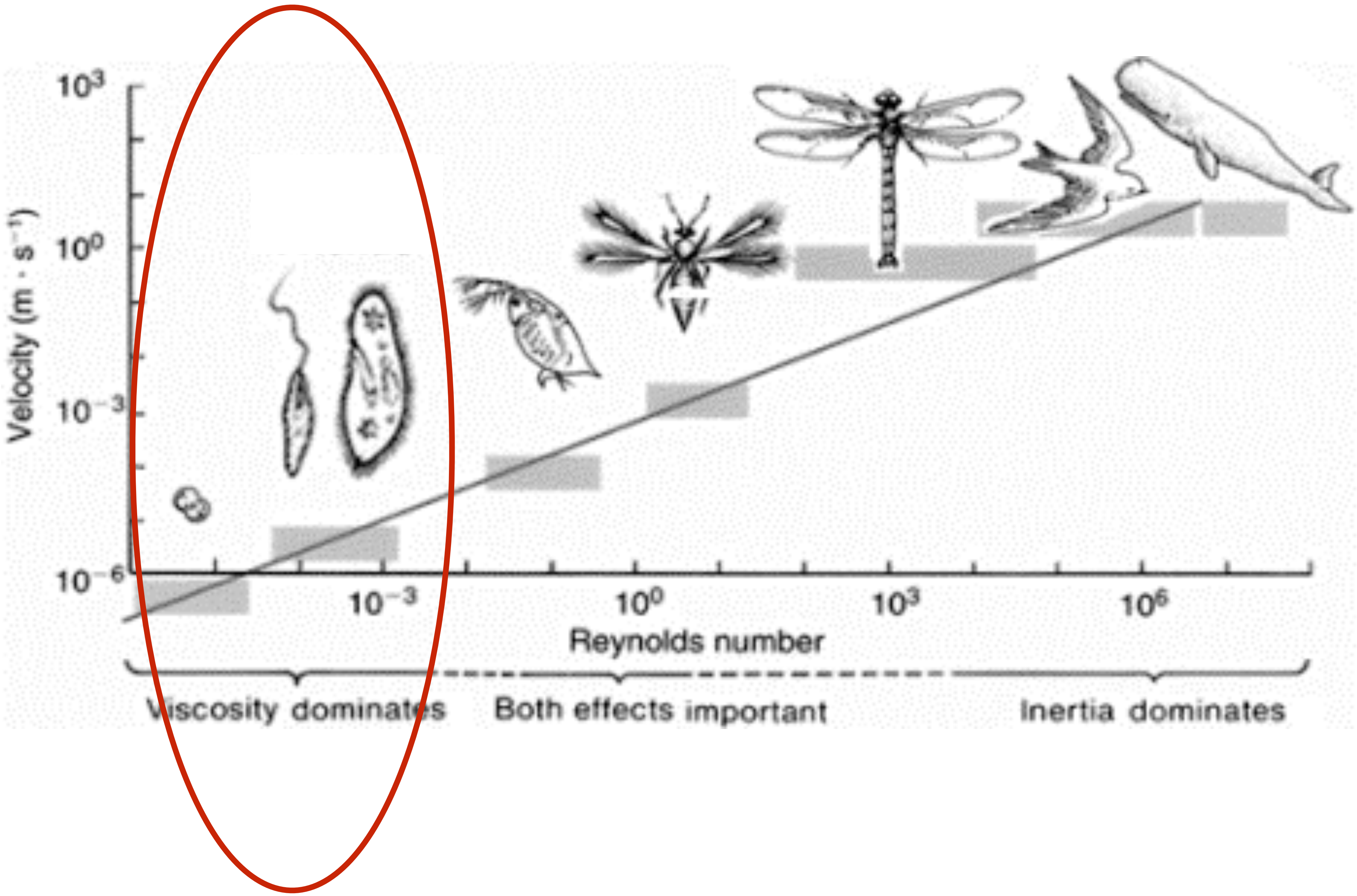
meters

What happens at low Reynolds numbers ?

$$Re = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$



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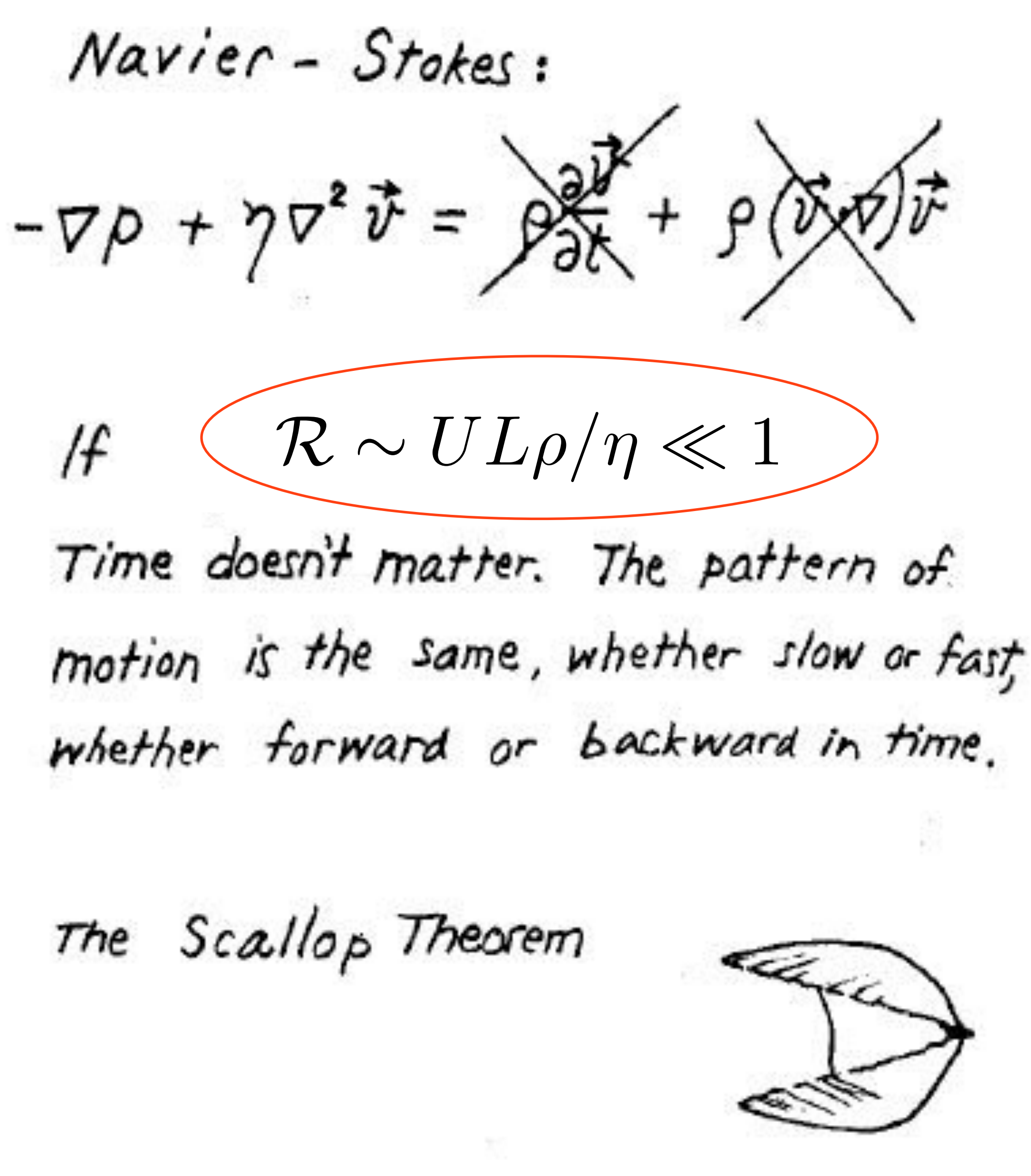
Low-Re (laminar) flow

Swimming at low Reynolds number


Navier - Stokes:

$$-\nabla p + \eta \nabla^2 \vec{v} = \rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v}$$

If $\mathcal{R} \sim UL\rho/\eta \ll 1$

Time doesn't matter. The pattern of motion is the same, whether slow or fast, whether forward or backward in time.

The Scallop Theorem

American Journal of Physics, Vol. 45, No. 1, January 1977


Geoffrey Ingram Taylor


James Lighthill

$$\begin{aligned} 0 &= \mu \nabla^2 \boldsymbol{u} - \nabla p + \boldsymbol{f}, \\ 0 &= \nabla \cdot \boldsymbol{u}. \end{aligned}$$

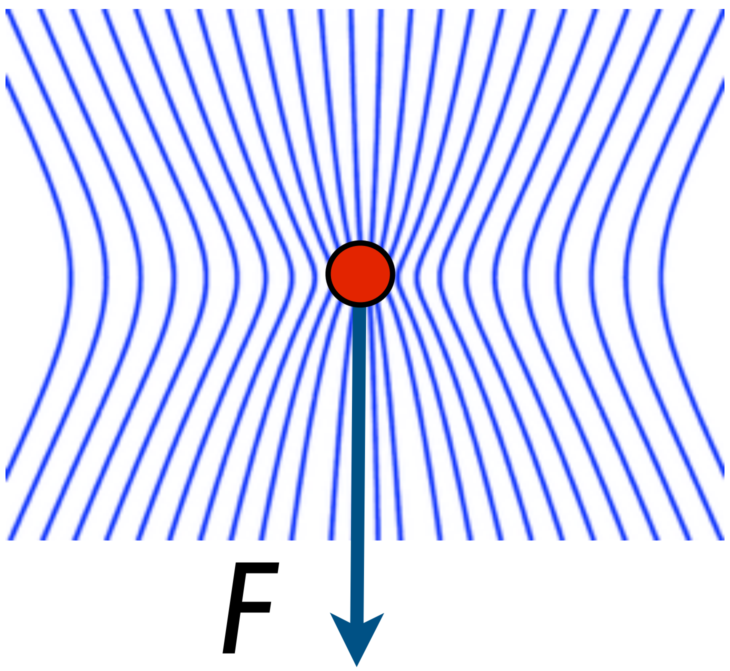
+ time-dependent BCs


Edward Purcell

Shapere & Wilczek 1987 PRL

Superposition of singularities

stokeslet

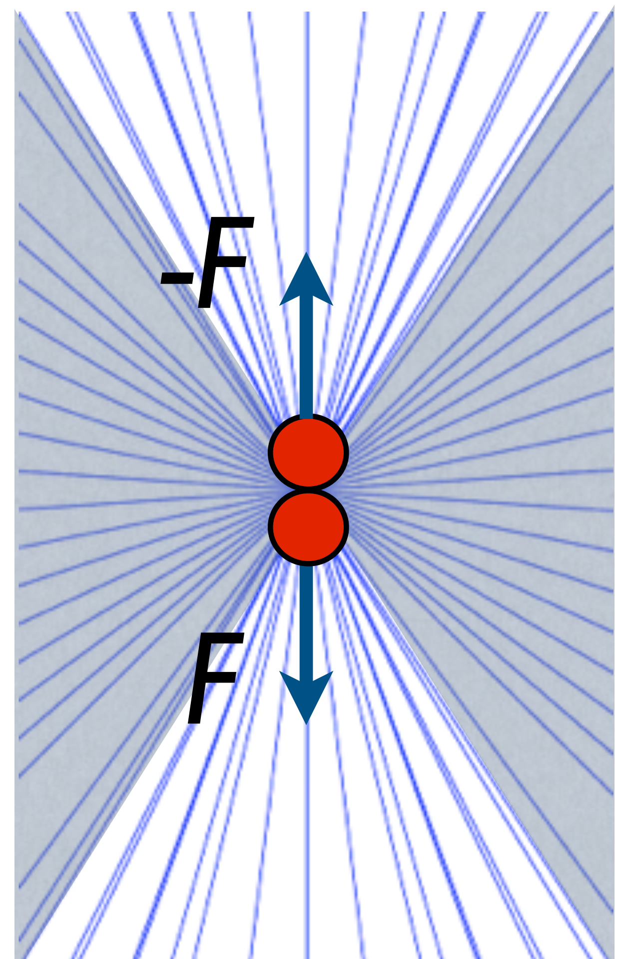


$$p(\boldsymbol{r}) = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{F}}{4\pi r^2} + p_0$$

$$v_i(\boldsymbol{r}) = \frac{(8\pi\mu)^{-1}}{r}[\delta_{ij} + \hat{r}_i\hat{r}_j]F_j$$

flow ~ r^{-1}

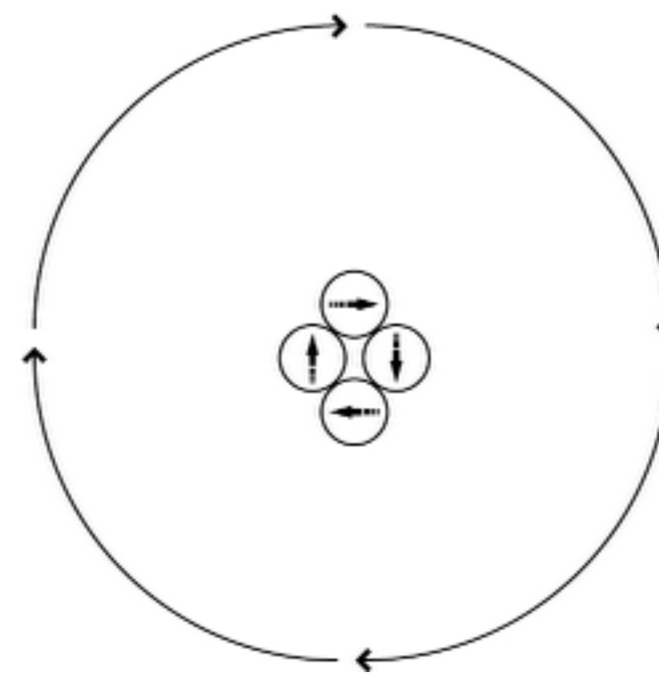
2x stokeslet =
symmetric dipole



r^{-2}

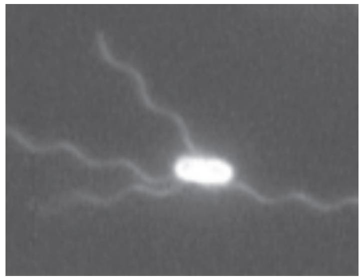
'pusher'

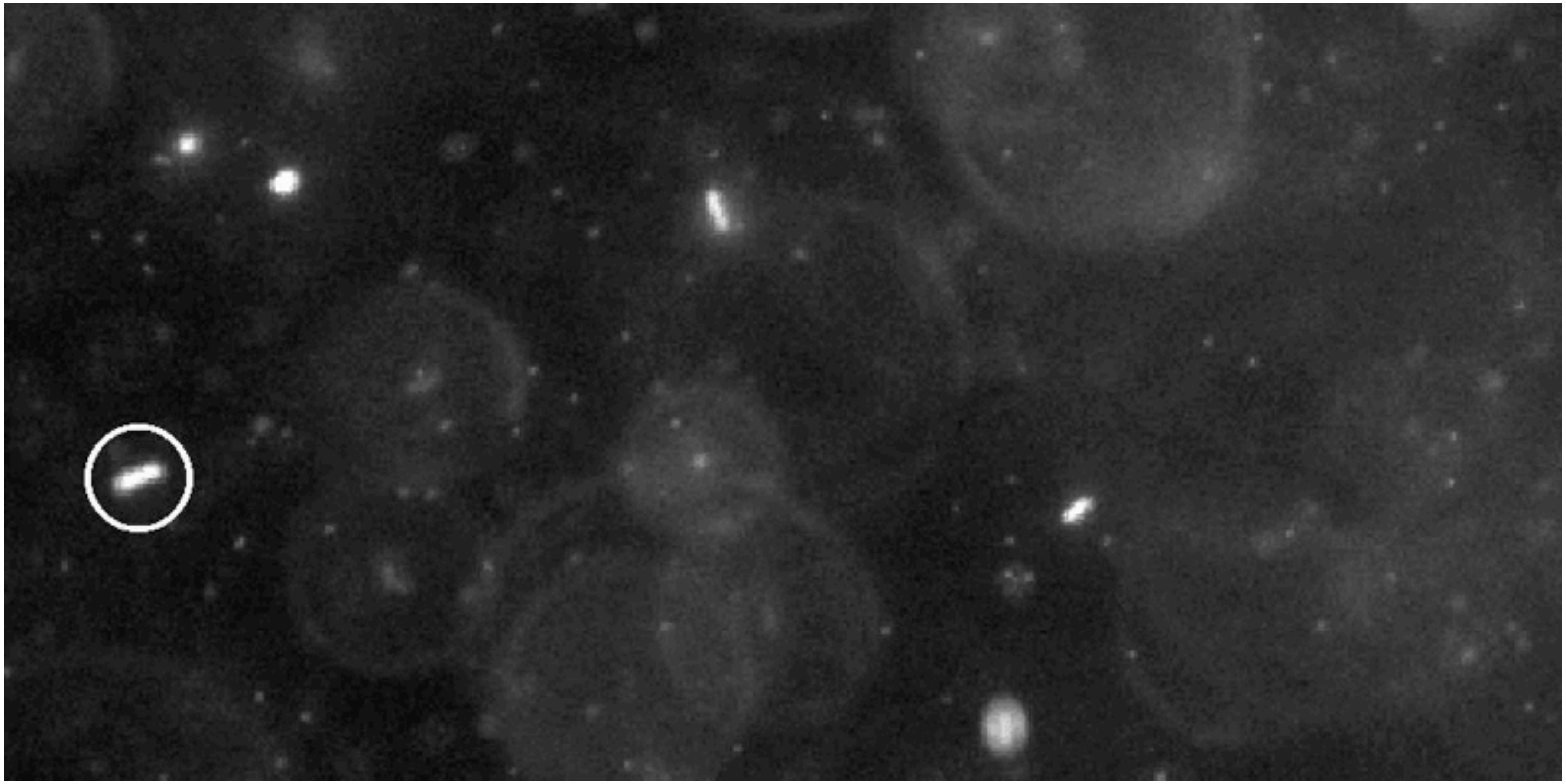
rotlet



r^{-2}

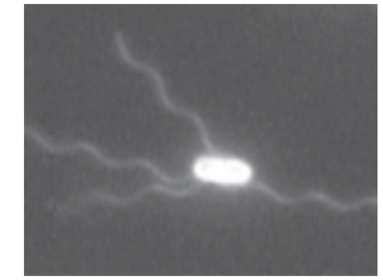
E.coli (non-tumbling HCB 437)

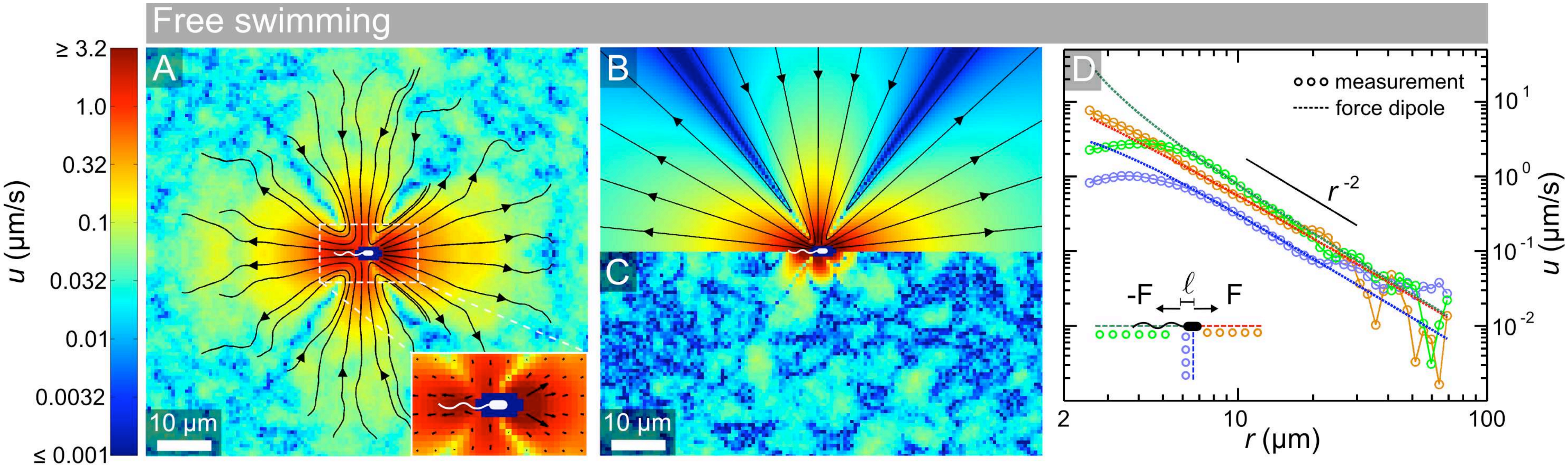




Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

E.coli (non-tumbling HCB 437)





$$\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$V_0 = 22 \pm 5 \ \mu\mathrm{m/s}$$

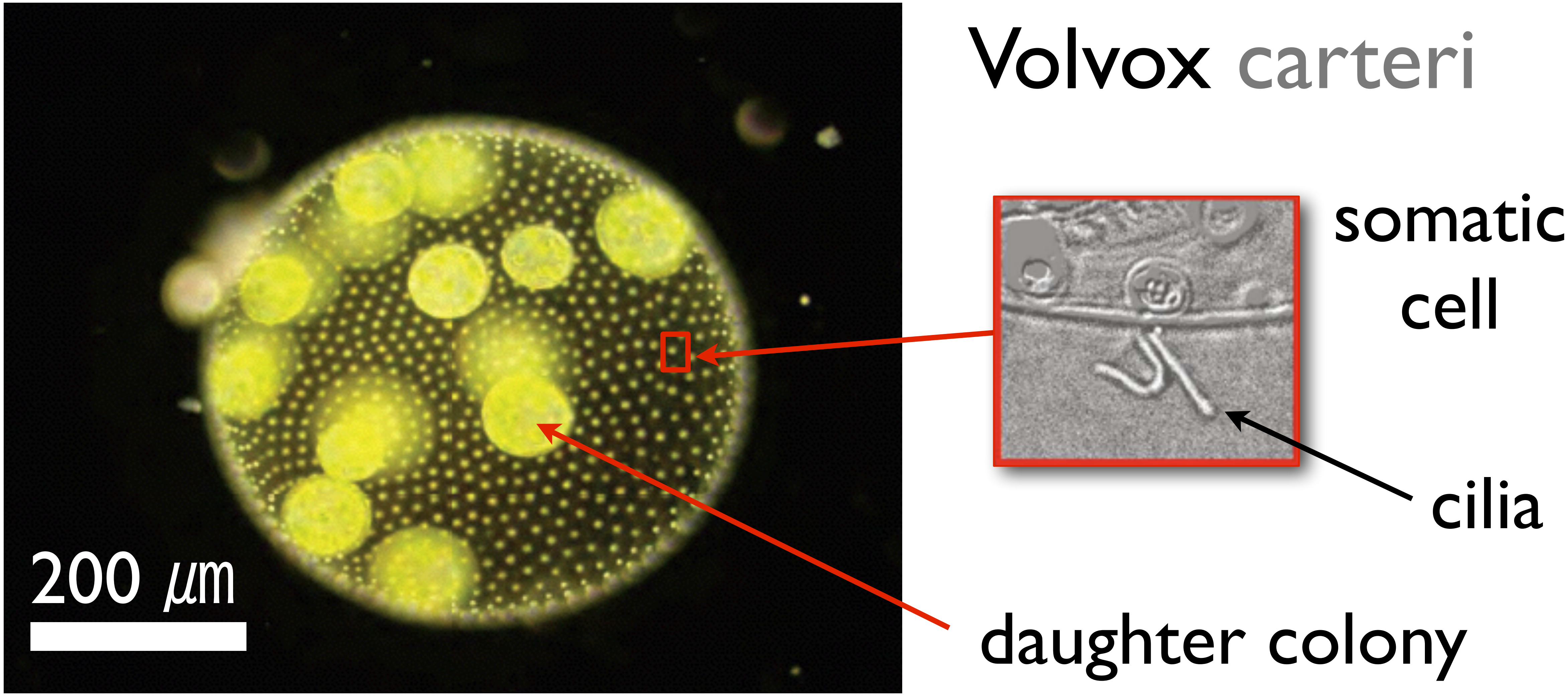
$$\ell = 1.9 \ \mu\mathrm{m}$$

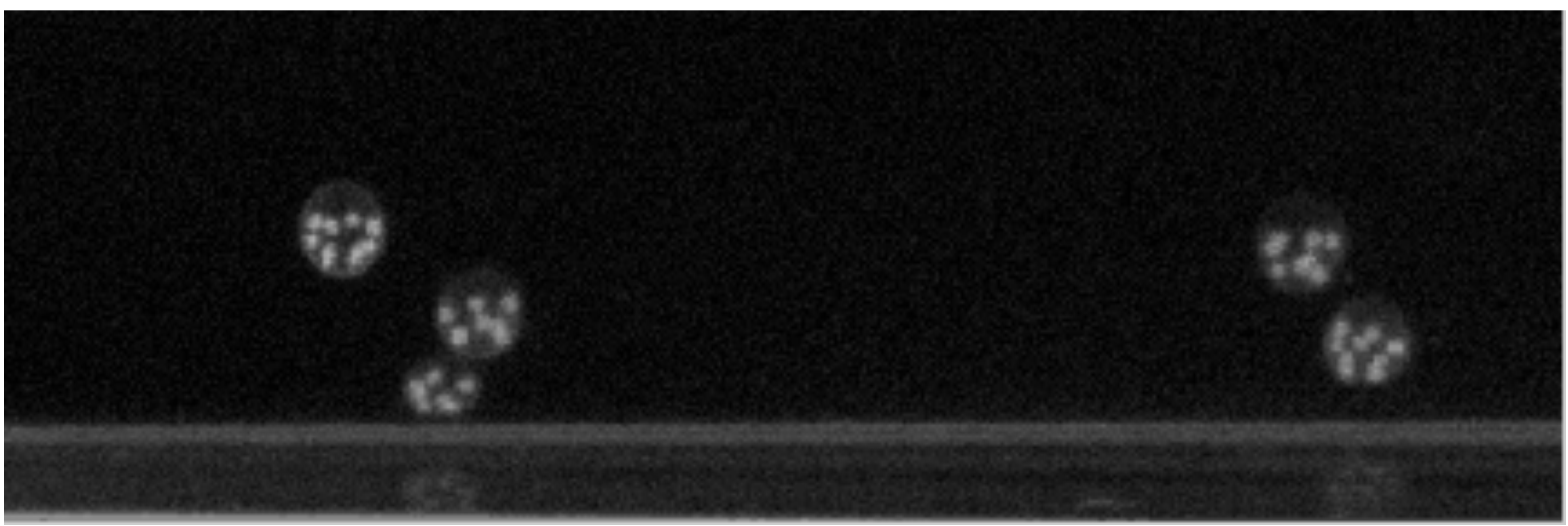
$$F = 0.42 \ \mathrm{pN}$$

weak 'pusher' dipole

Drescher, Dunkel, Ganguly, Cisneros, Goldstein (2011) PNAS

Volvox carteri





Drescher et al (2010) PRL

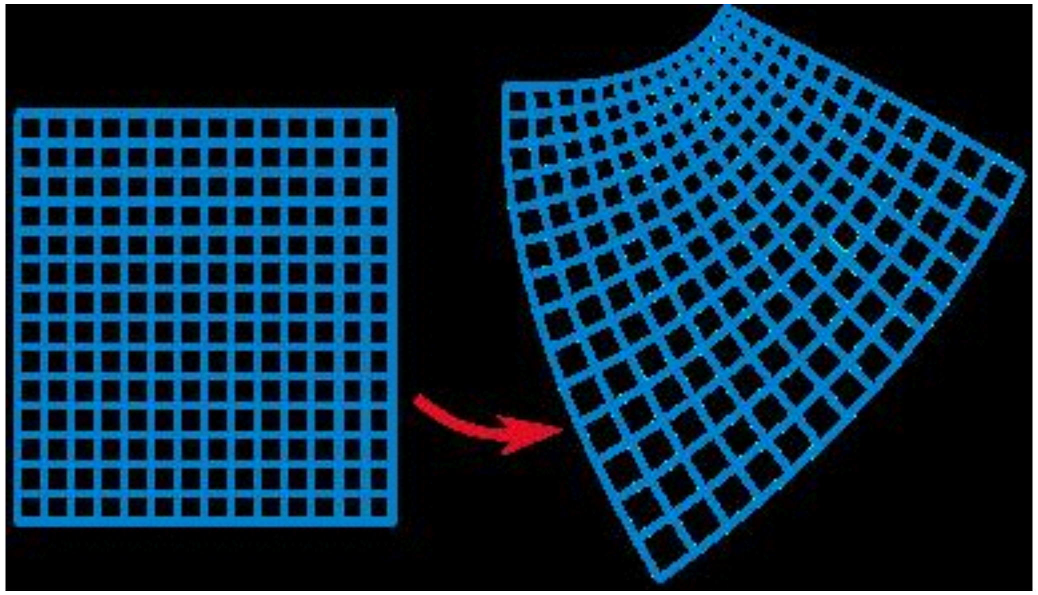
dunkel@math.mit.edu

Singular perturbations

$$\epsilon \frac{d^2u}{dx^2} + \frac{du}{dx} = 1.$$

Conformal mappings

$$\frac{dw}{dz} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} = u - iv.$$



$$W(Z) = u_0\left(Ze^{-i\alpha} + \frac{R^2}{Z}e^{i\alpha}\right) - \frac{i\Gamma}{2\pi}\ln Z.$$

Rotating flows

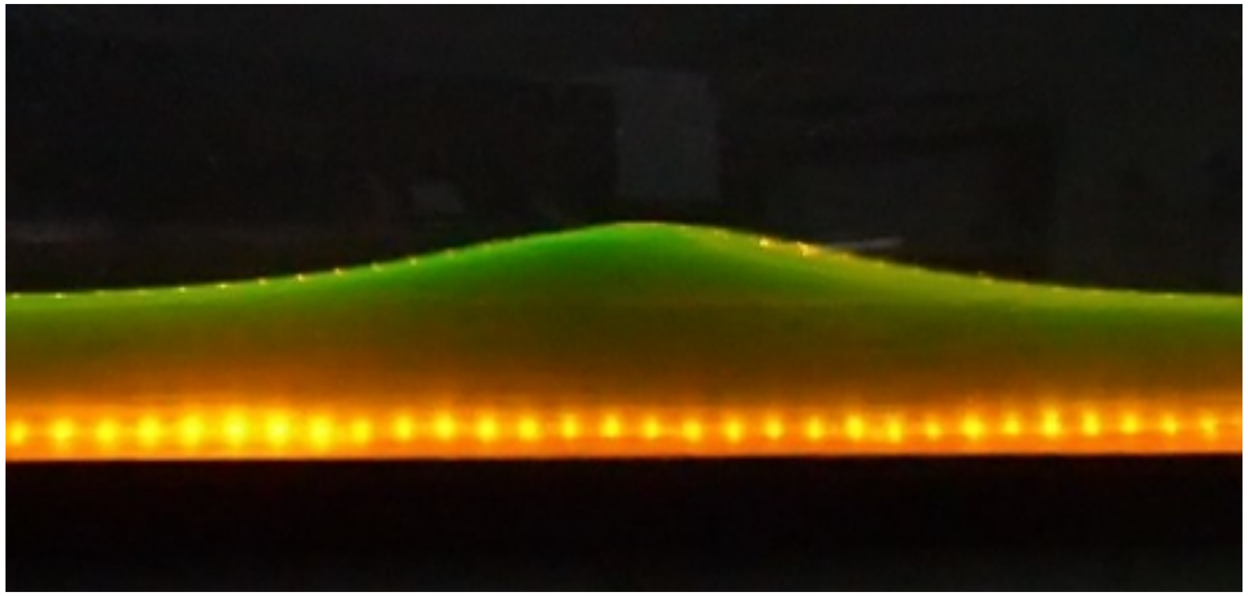
$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \Omega \times (\Omega \times \boldsymbol{r}) &= -\frac{1}{\rho} \nabla p_\Omega + \nu \nabla^2 \boldsymbol{u} - 2\Omega \times \boldsymbol{u}, \\
\nabla \cdot \boldsymbol{u} &= 0.
\end{aligned}
$$

Taylor columns, etc

http://ocw.mit.edu/

Taylor - column

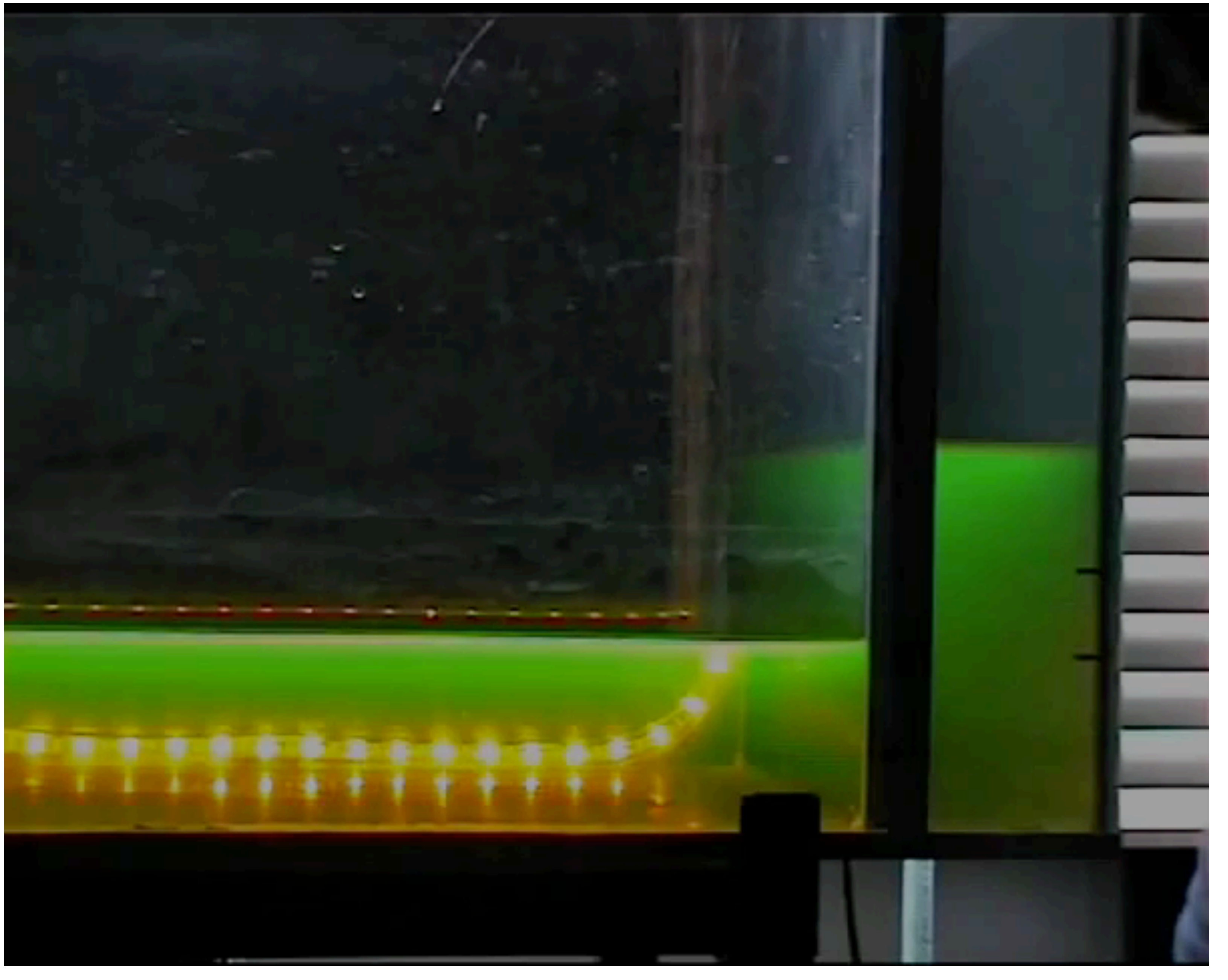
Solitons



KdV equation

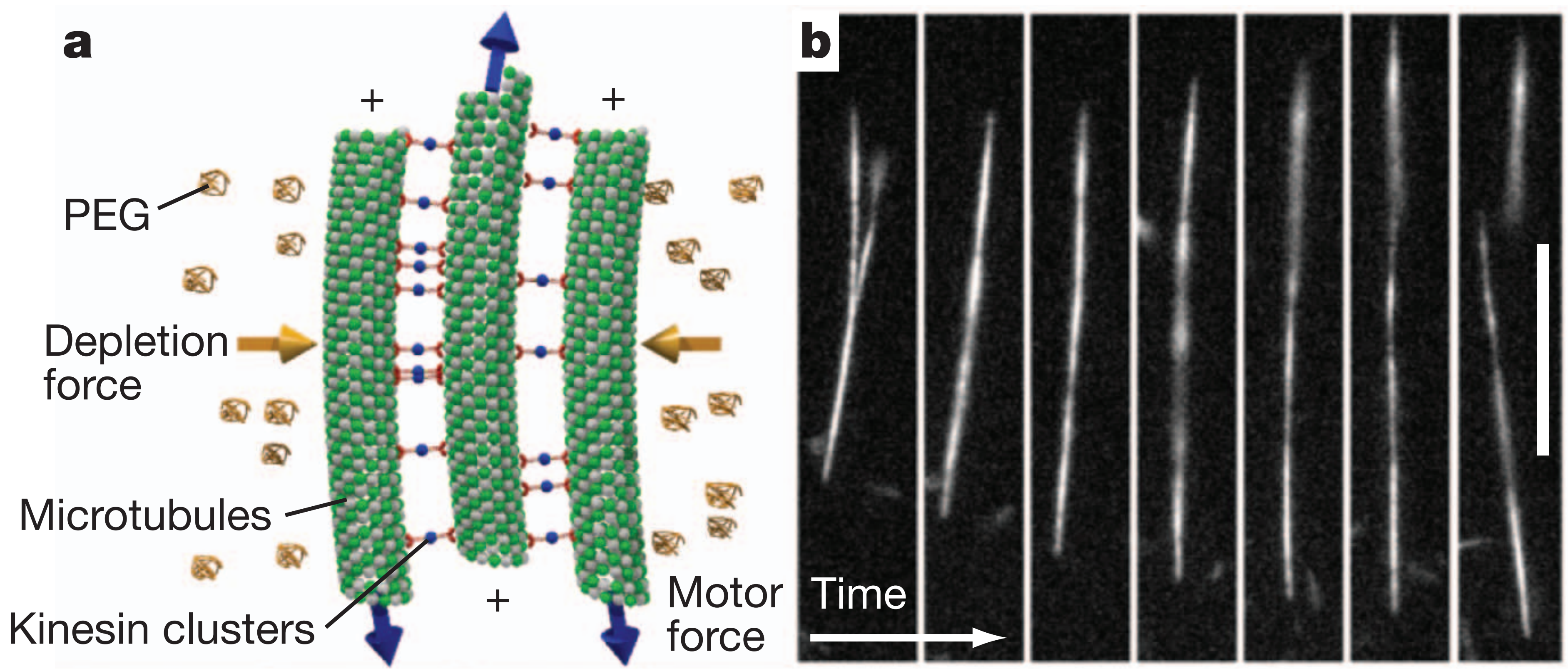
$$\partial_t \phi + \partial_x^2 \phi + 6\phi\partial_x \phi = 0$$

Solitons



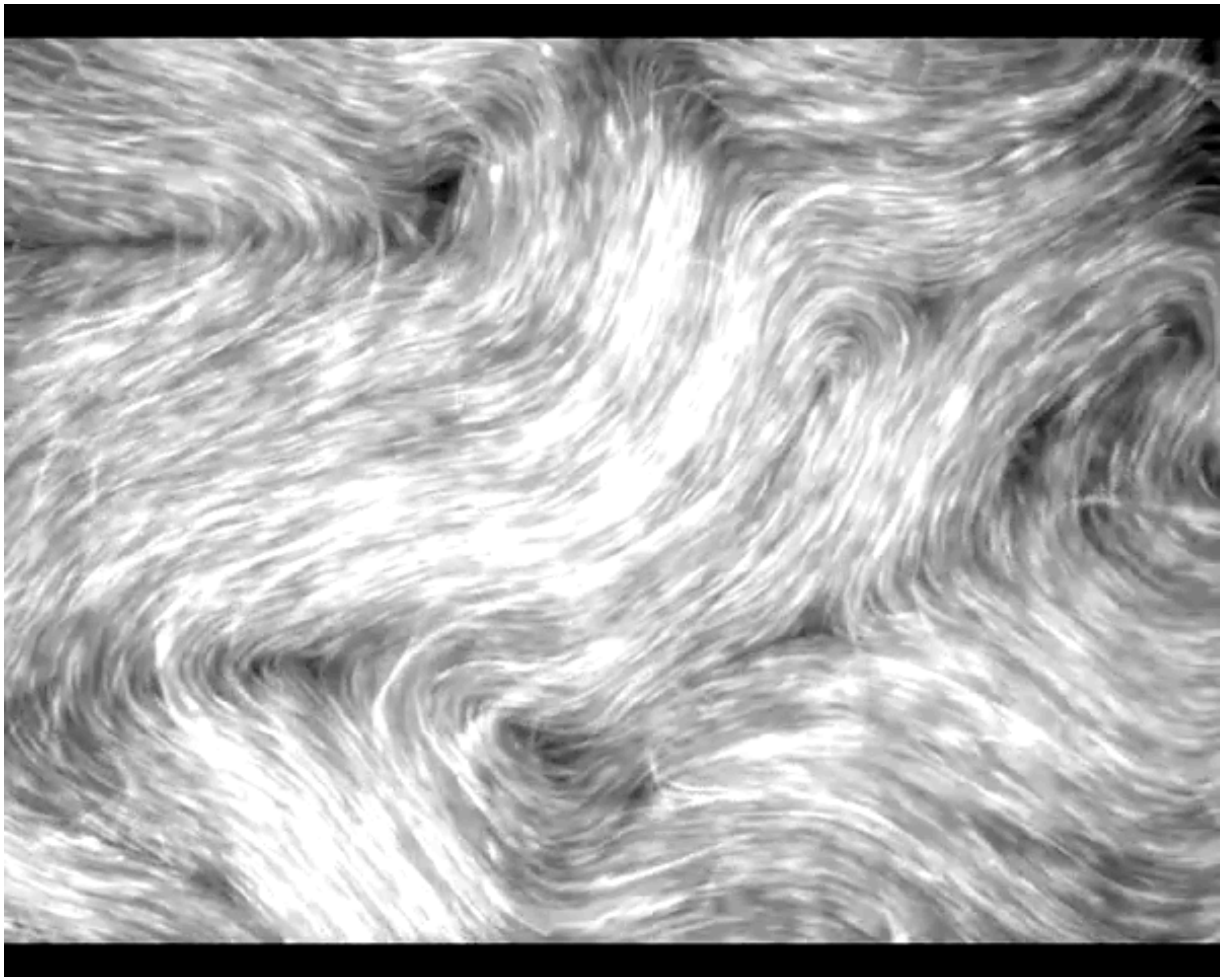
credit: Christophe Finot

Active matter



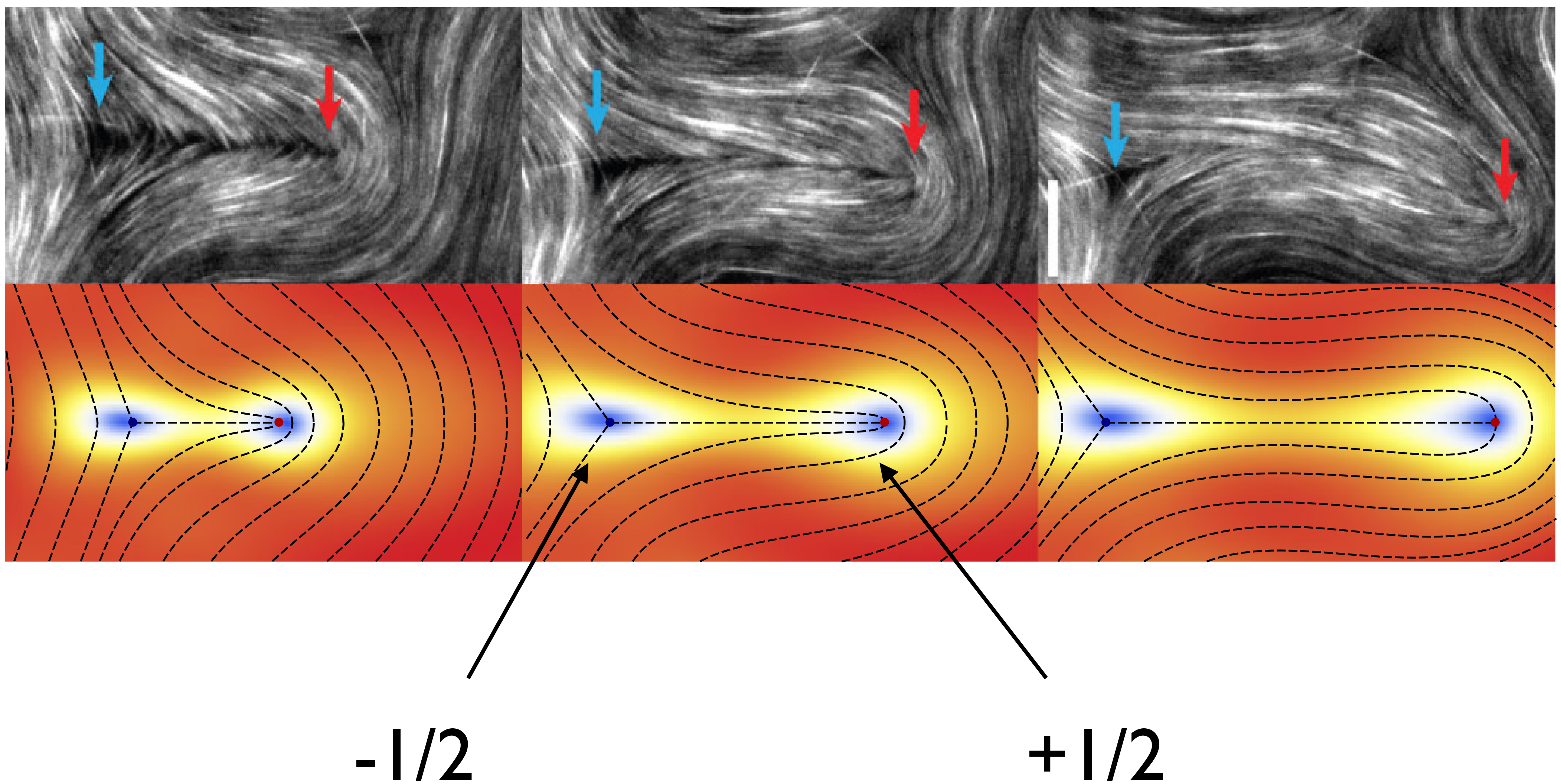
Dogic lab (Brandeis) Nature 2012

Active matter



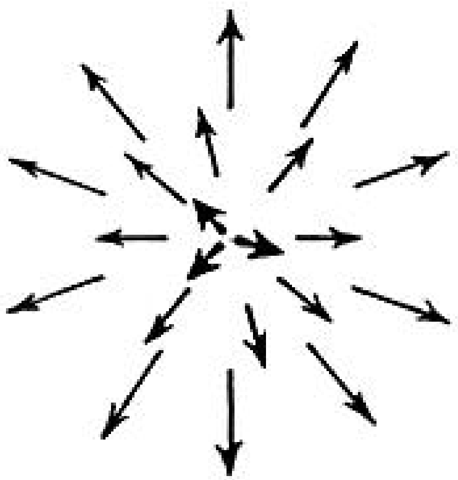
Dogic lab (Brandeis) Nature 2012

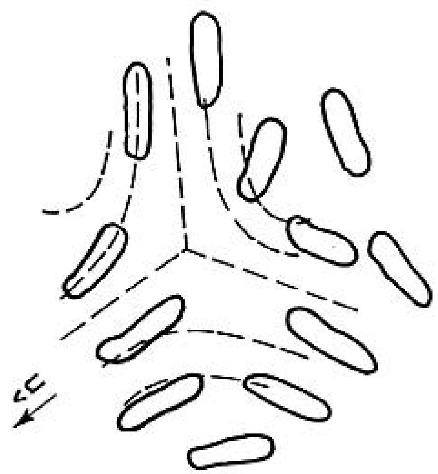
Active nematics

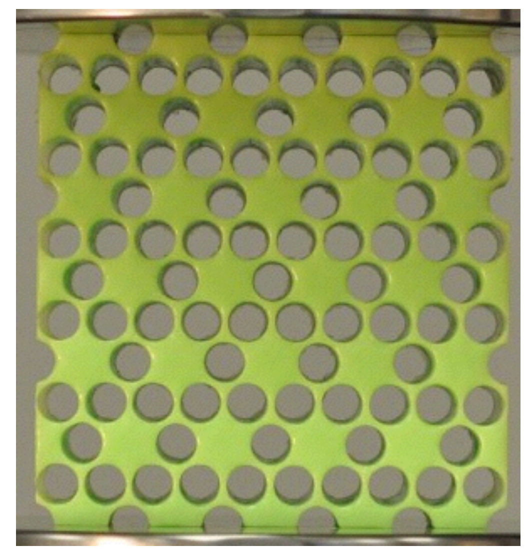


Giomi et al PRL 2012

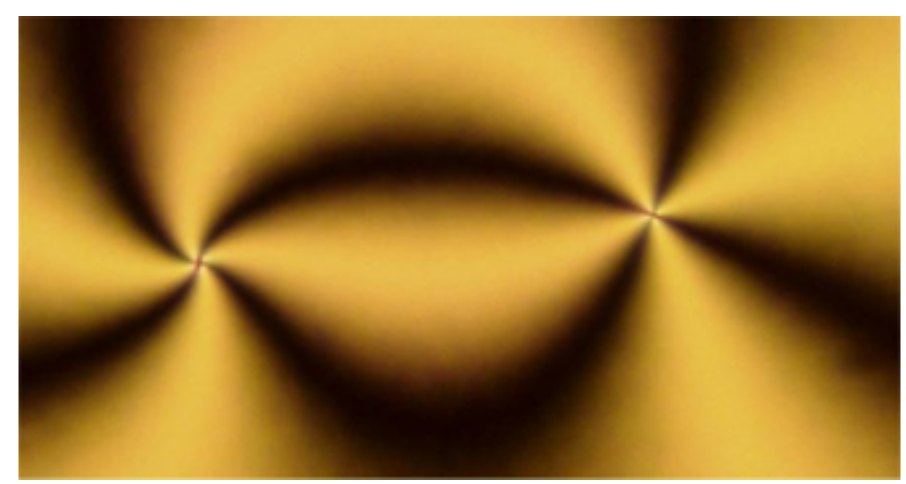
Topological defects are discontinuities in order-parameter fields







- optical effects
- work hardening, etc



"umbilic defects" in a nematic liquid crystal

"Quantum" HD

Couder lab (Paris) Bush group