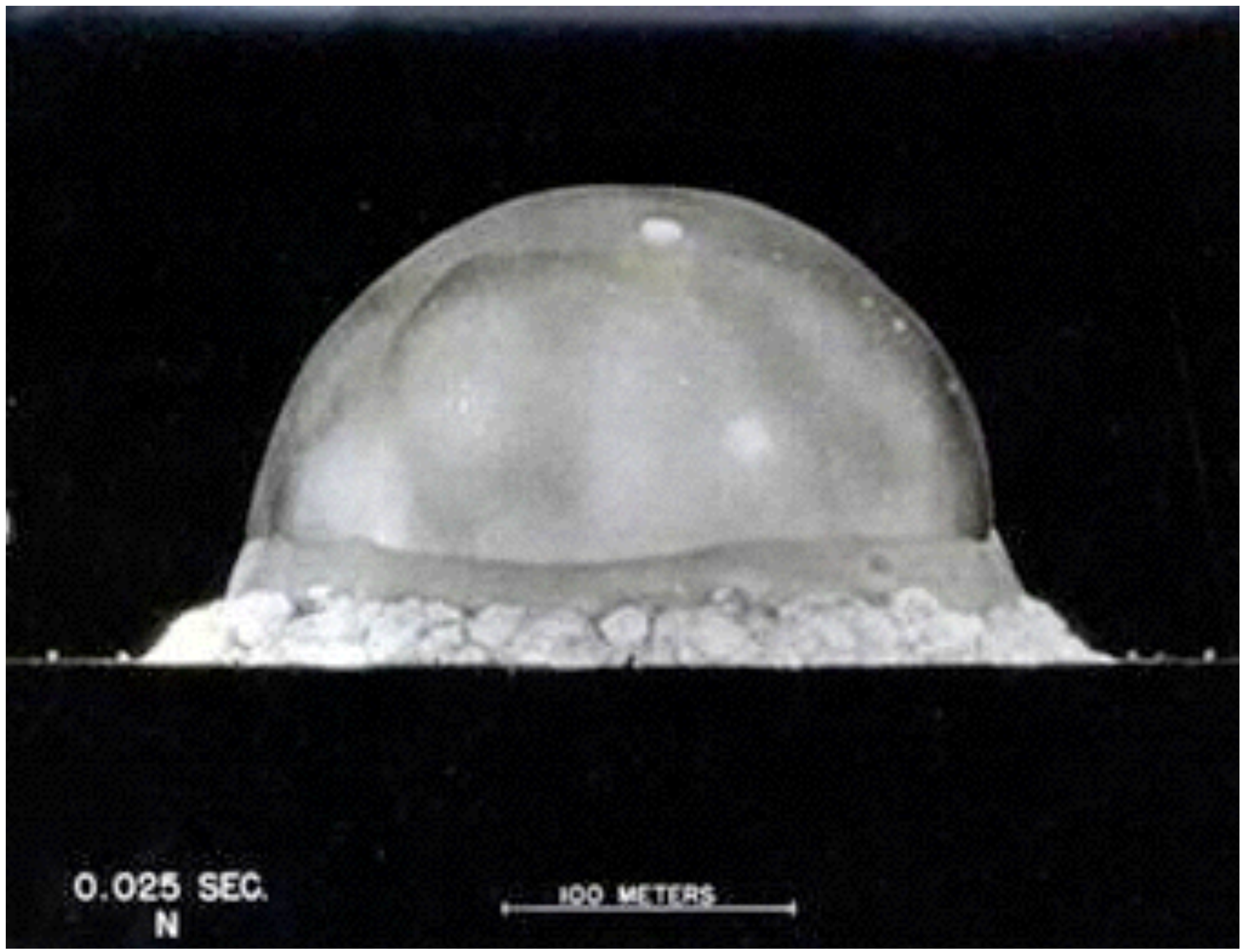
Course summary

18.354

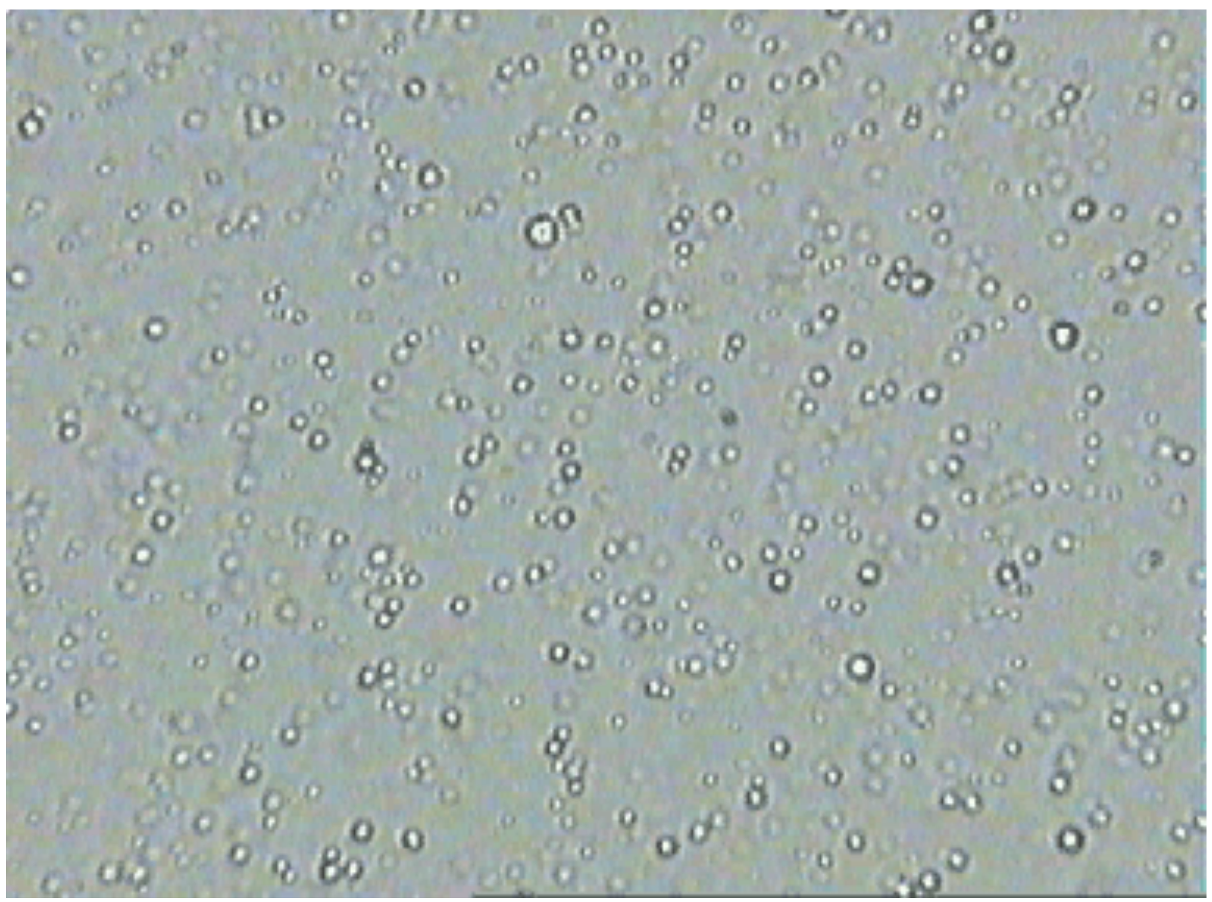
Dimensional analysis



Kepler's problem

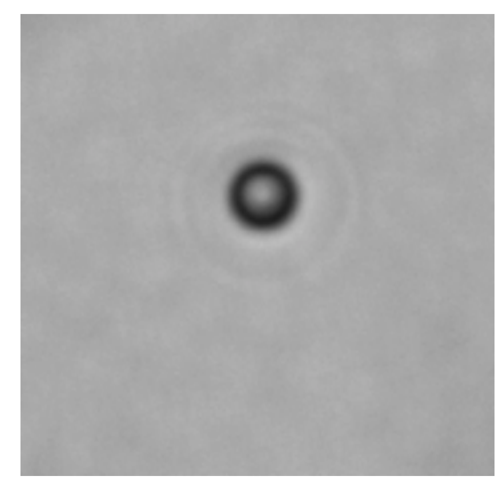
$$\mathbf{L} = \mathbf{r} \times m\frac{d\mathbf{r}}{dt}$$

Random walks & diffusion



David Walker

$$\frac{\partial n}{\partial t} = -\frac{\partial J_x}{\partial x} = D\frac{\partial^2 n}{\partial x^2},$$

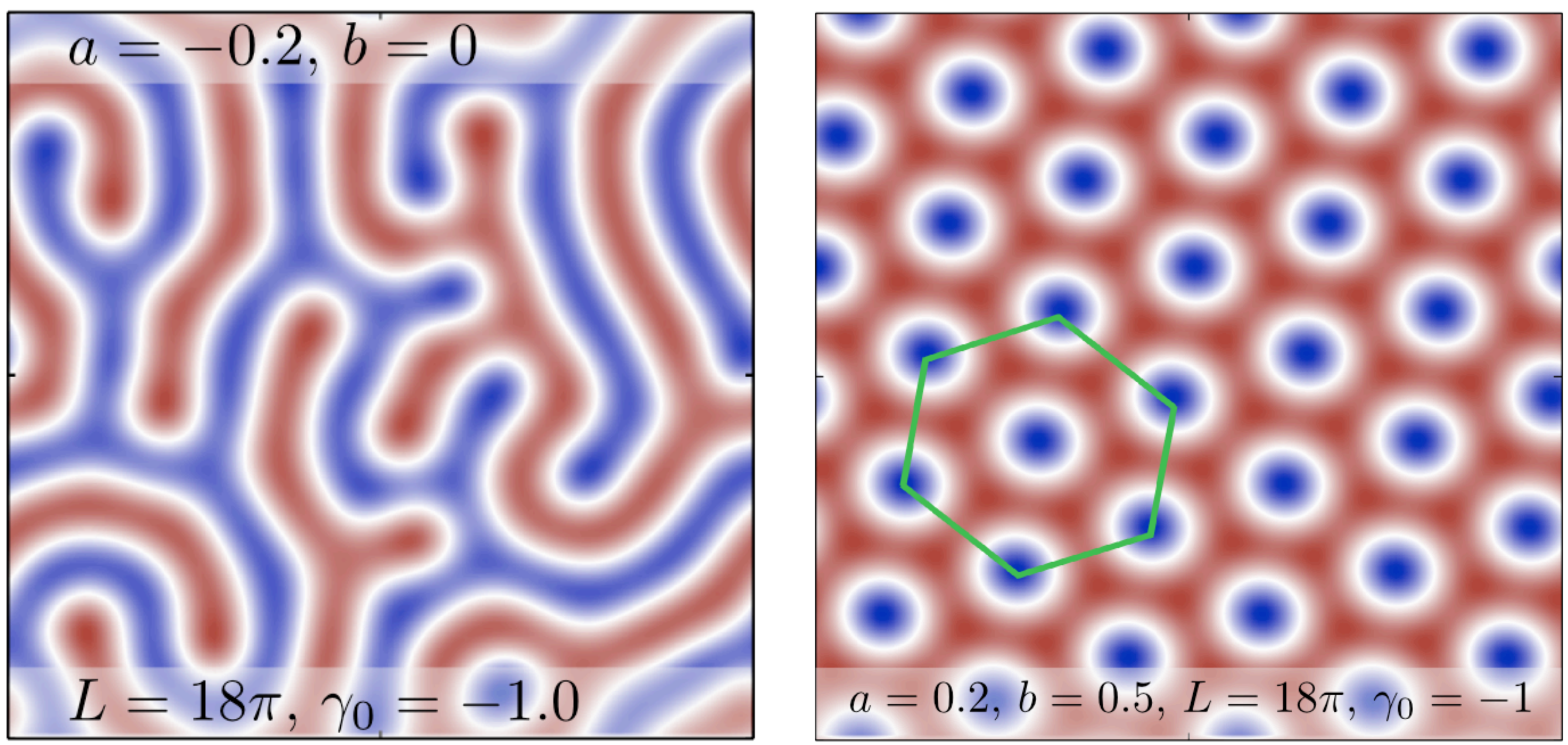


Mark Haw

(In)stability analysis & pattern formation

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

$$U(\psi) = \frac{a}{2}\psi^2 + \frac{b}{3}\psi^3 + \frac{c}{4}\psi^4$$

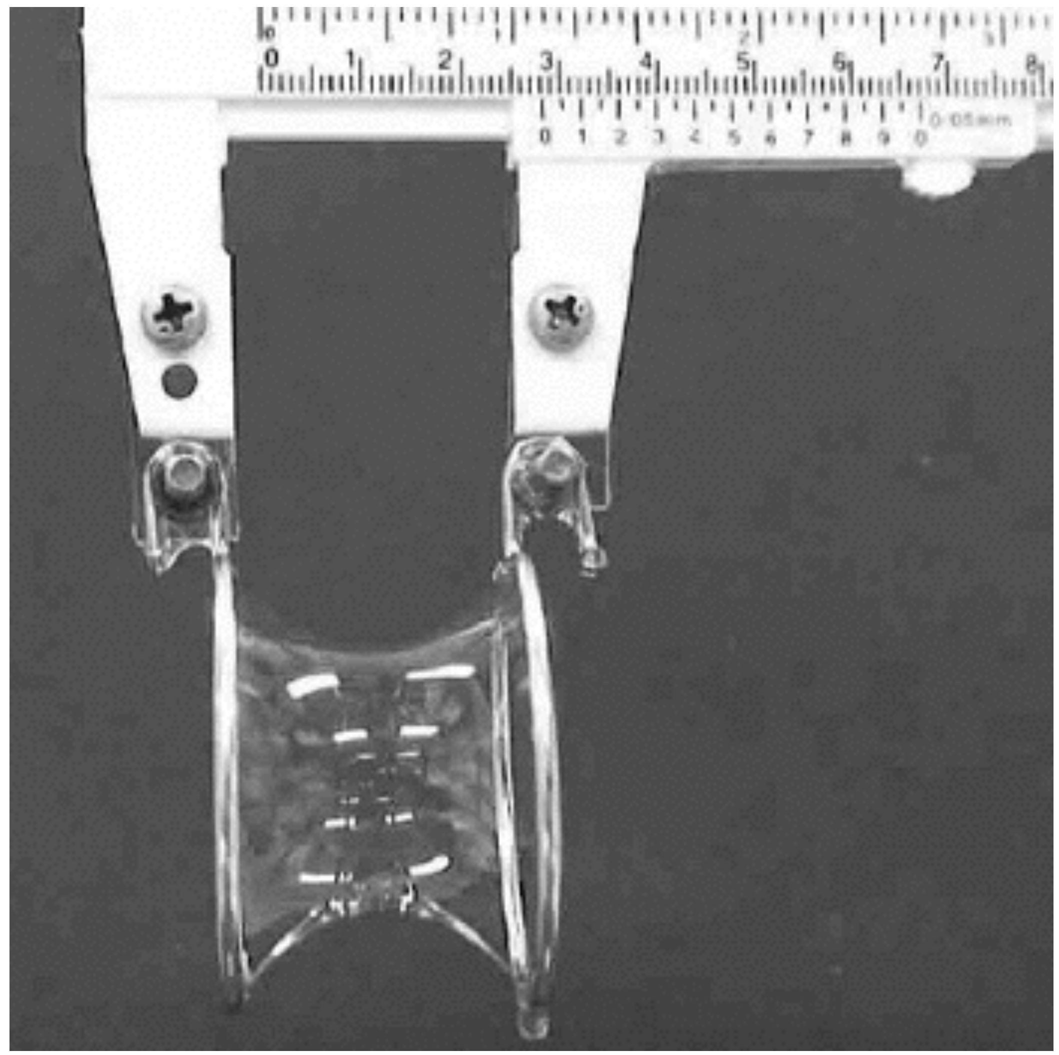


Calculus of variations

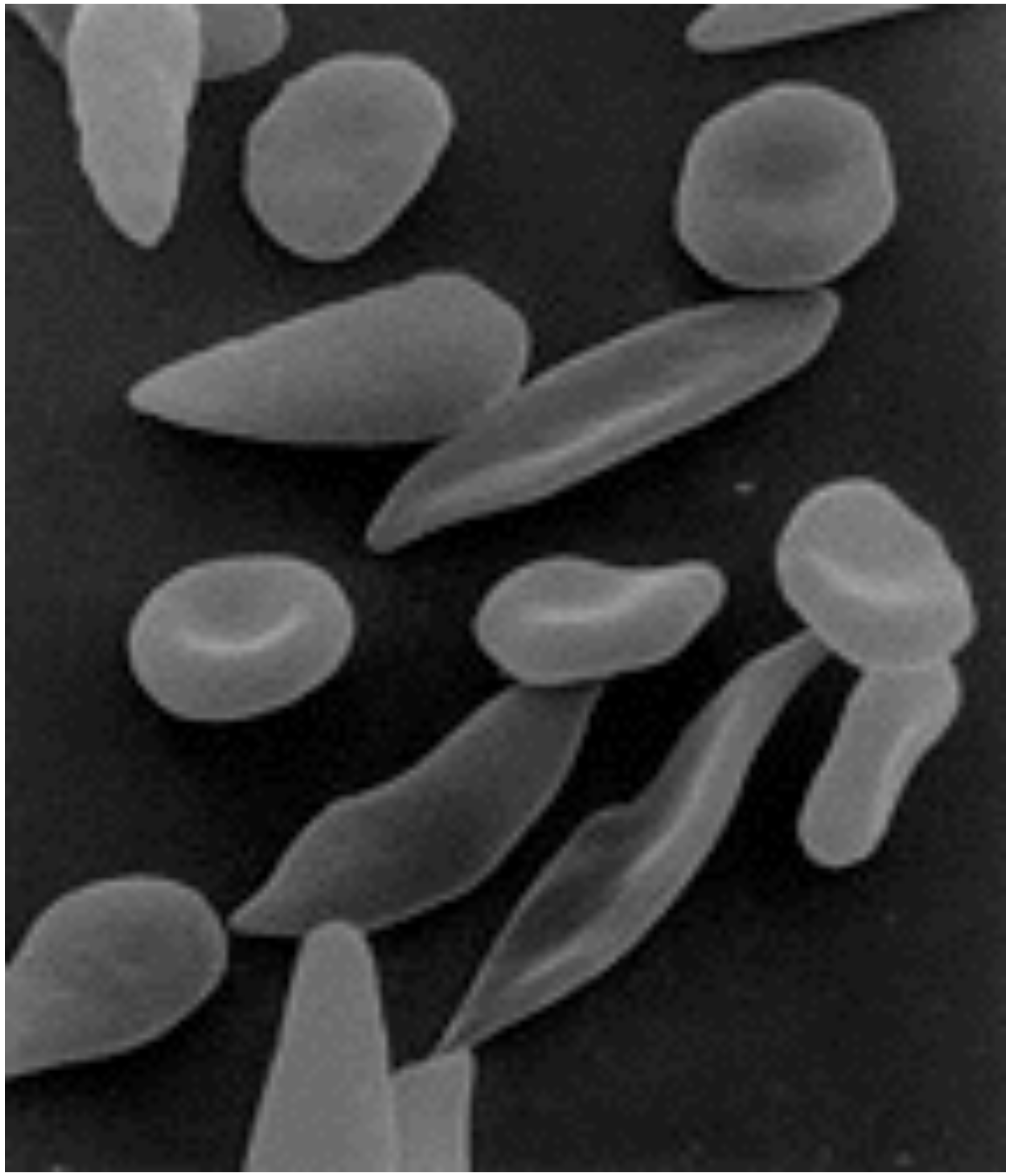
$$\begin{aligned}\frac{\delta I[Y]}{\delta Y} &= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\left\{I[f(x)+\epsilon\delta(x-y)]-I[f(x)]\right\}\\ &= \int_{x_1}^{x_2}\left[\frac{\partial f}{\partial Y}\delta(x-y)+\frac{\partial f}{\partial Y'}\delta'(x-y)\right]dx\\ &= \int_{x_1}^{x_2}\left[\frac{\partial f}{\partial Y}-\frac{d}{dx}\frac{\partial f}{\partial Y'}\right]\delta(x-y)dx.\end{aligned}$$

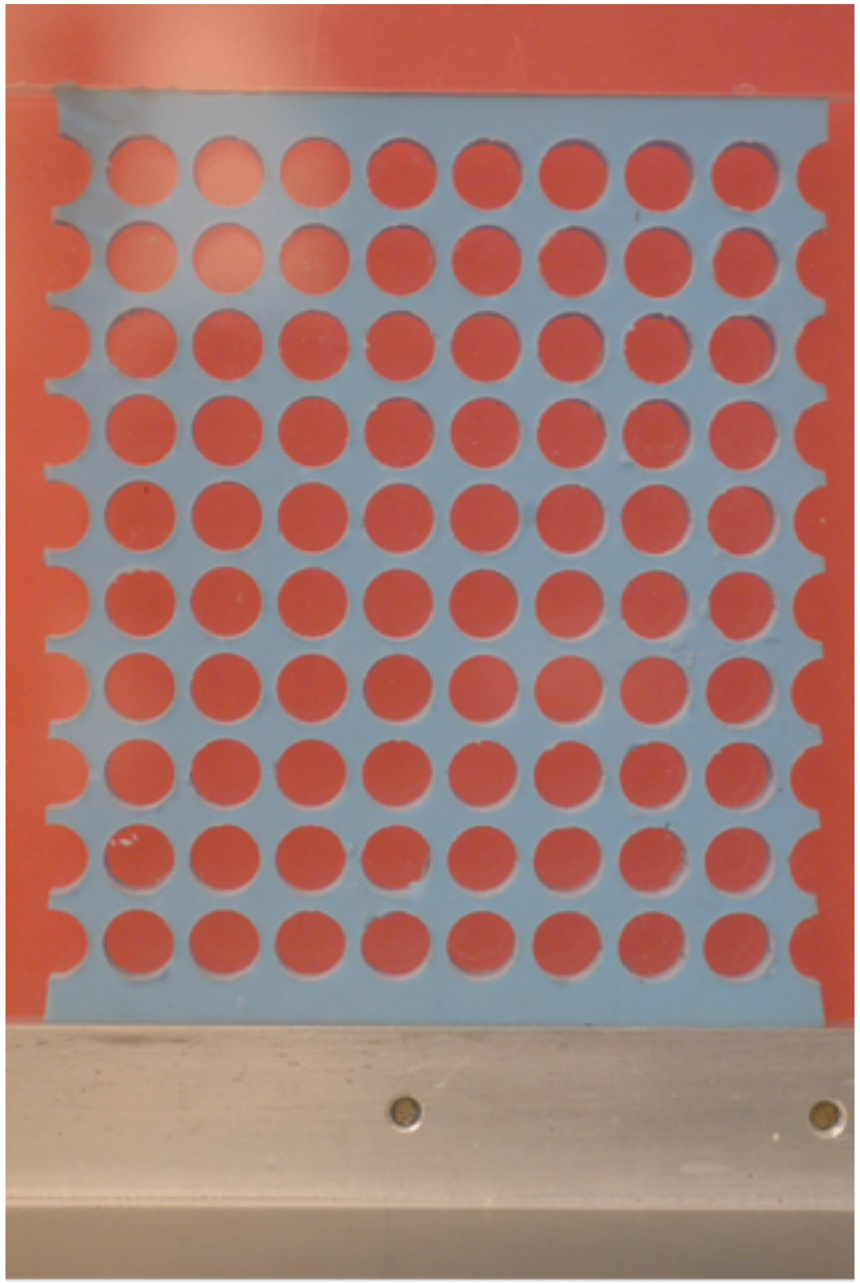
$$0 = \frac{\partial f}{\partial Y}-\frac{d}{dx}\frac{\partial f}{\partial Y'}$$

Surface tension



Elasticity





Hydrodynamics

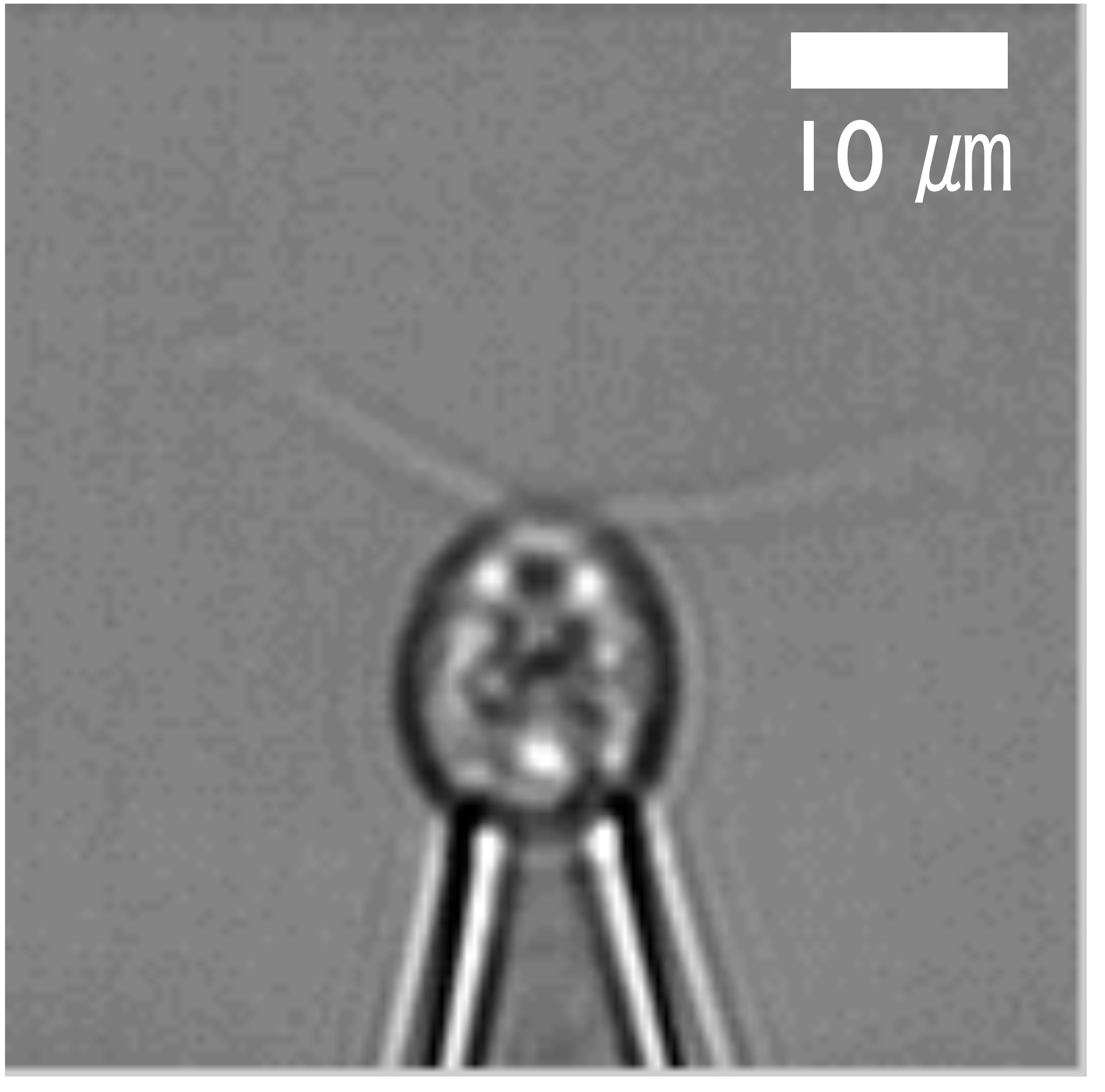
$$\int_V \frac{\partial \rho}{\partial t} dV = -\int_S \rho \mathbf{u} \cdot \mathbf{n} dS = -\int_V \nabla \cdot (\rho \mathbf{u}) dV.$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0.$$

$$\int_{V(t)} \rho \frac{D\mathbf{u}}{Dt} dV = \int_{V(t)} (-\nabla p + \rho \mathbf{g}) dV$$

$$\frac{D\mathbf{u}}{Dt} = \frac{-\nabla p}{\rho} + \mathbf{g}.$$

Low Re

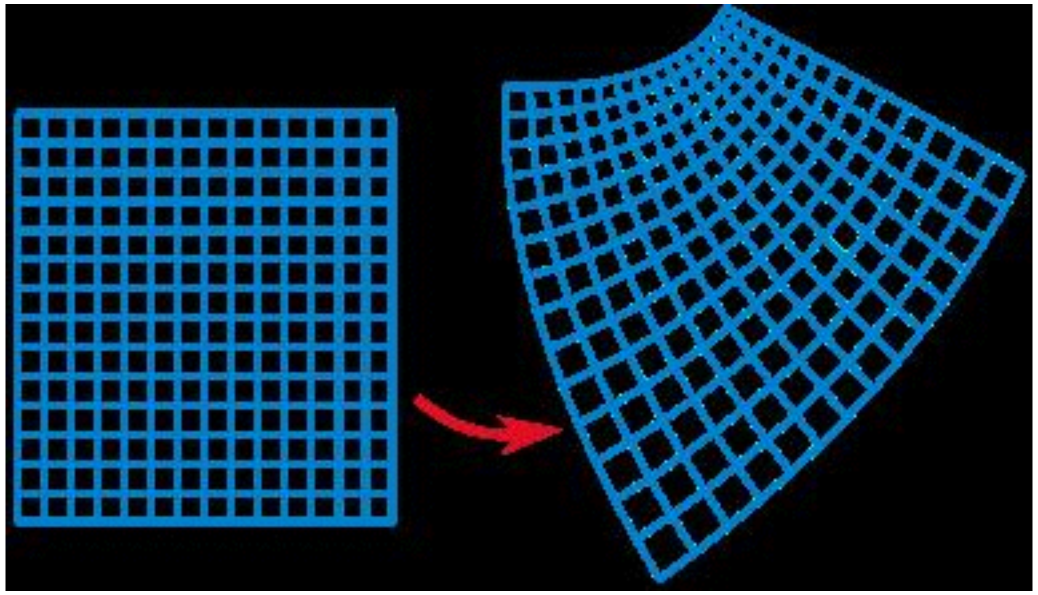


Singular perturbations

$$\epsilon \frac{d^2u}{dx^2} + \frac{du}{dx} = 1.$$

Conformal mappings

$$\frac{dw}{dz} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} = u - iv.$$



$$W(Z) = u_0\left(Ze^{-i\alpha} + \frac{R^2}{Z}e^{i\alpha}\right) - \frac{i\Gamma}{2\pi}\ln Z.$$

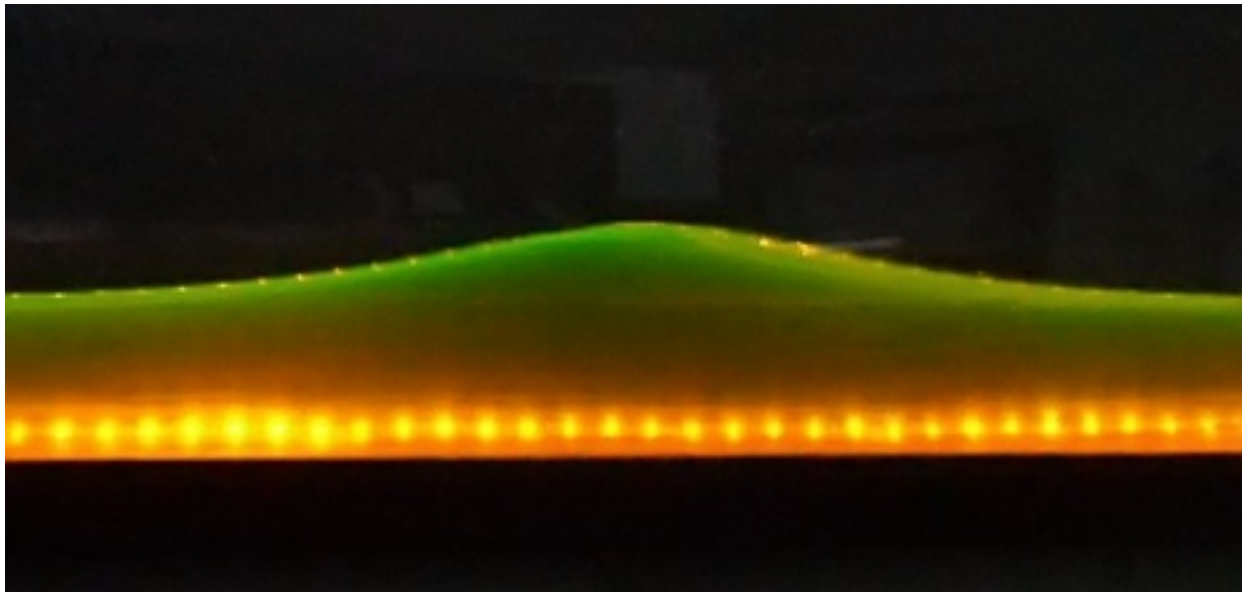
Rotating flows

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \Omega \times (\Omega \times \boldsymbol{r}) = -\frac{1}{\rho} \nabla p_{\Omega} + \nu \nabla^2 \boldsymbol{u} - 2\Omega \times \boldsymbol{u},$$

$$\nabla \cdot \boldsymbol{u} = 0.$$

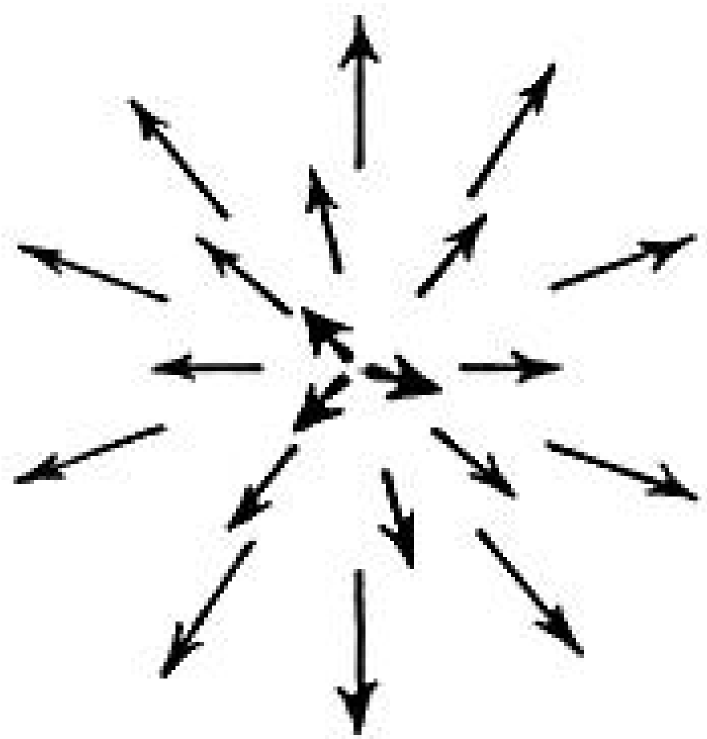
Taylor columns, etc

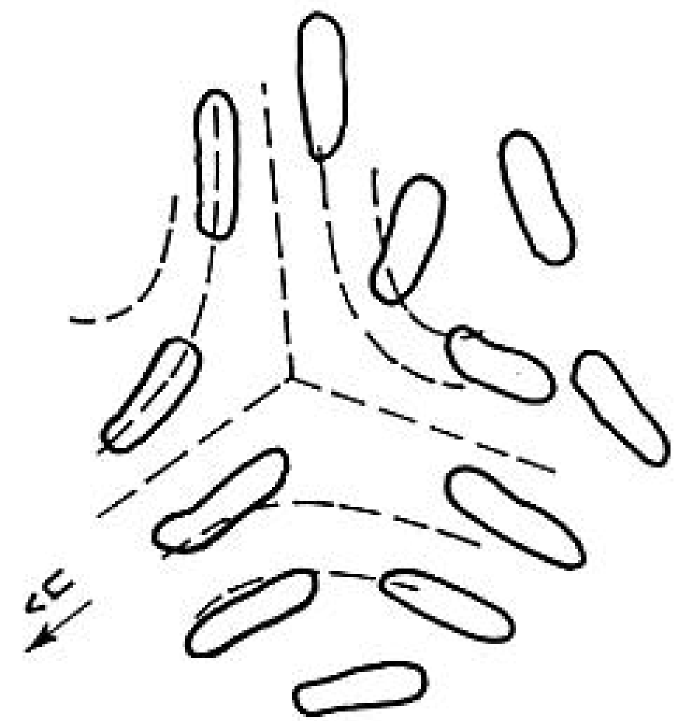
Solitons



KdV equation

Topological defects





Active matter