Topological Defects

18.354 L24

Order Parameters, Broken Symmetry, and Topology

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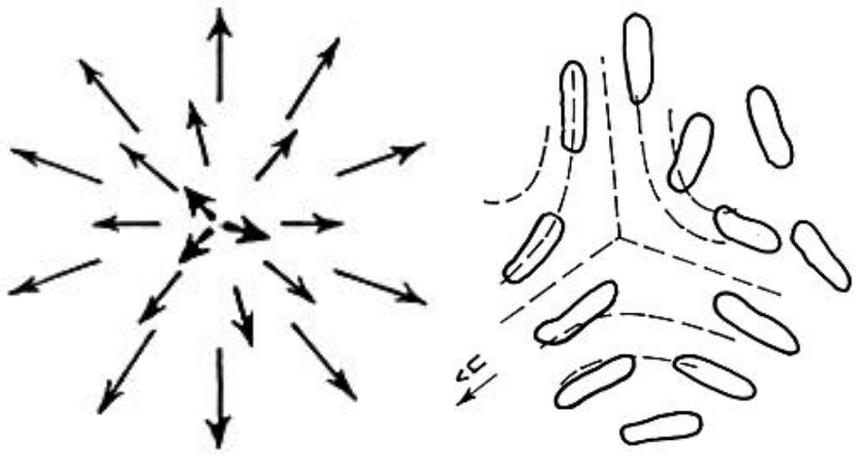
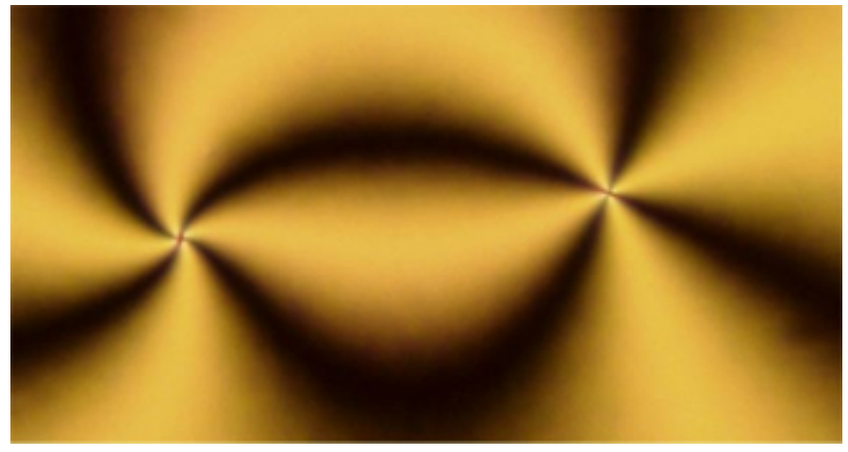
Topological defects are discontinuities in order-parameter fields



- optical effects
- work hardening, etc



"umbilic defects" in a nematic liquid crystal

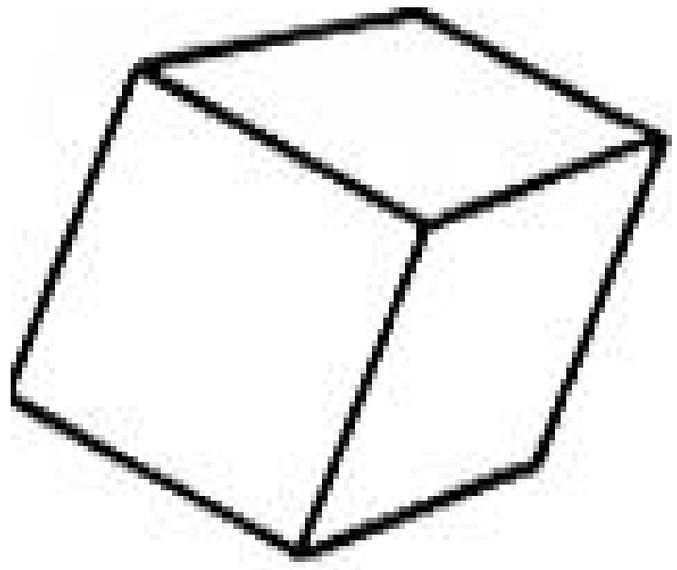
order = symmetry = invariance

(under certain group actions)

symmetry groups can be discrete,
continous, Lie-groups, ...

More or less symmetric ?

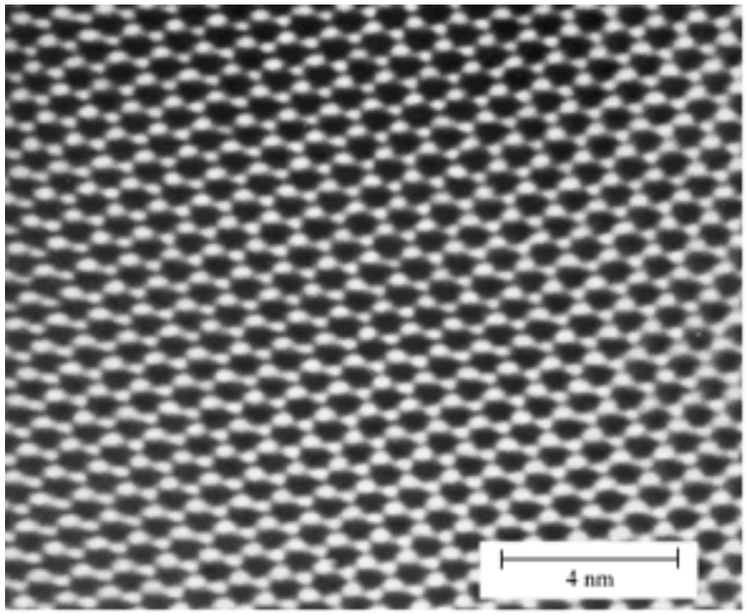
A

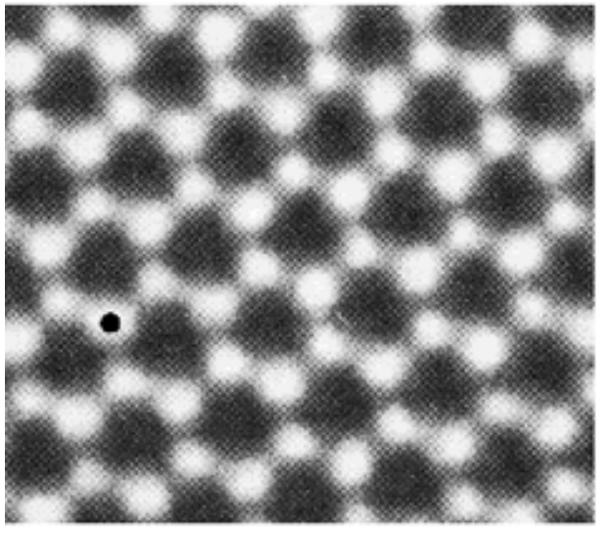


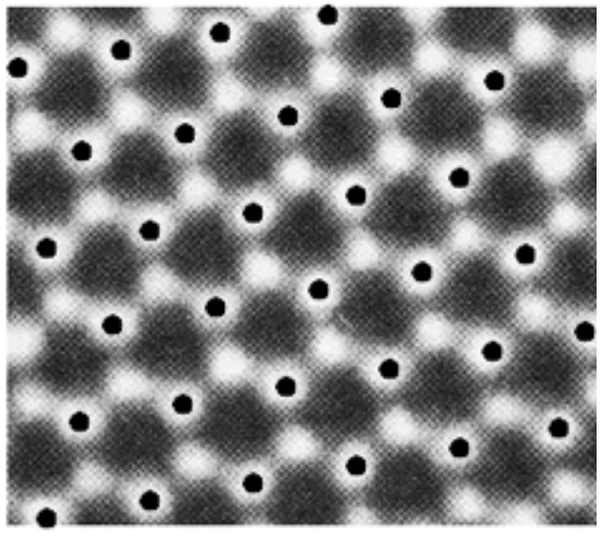
B

More or less symmetric ?

$Mg_2Al_4Si_5O_{18}$

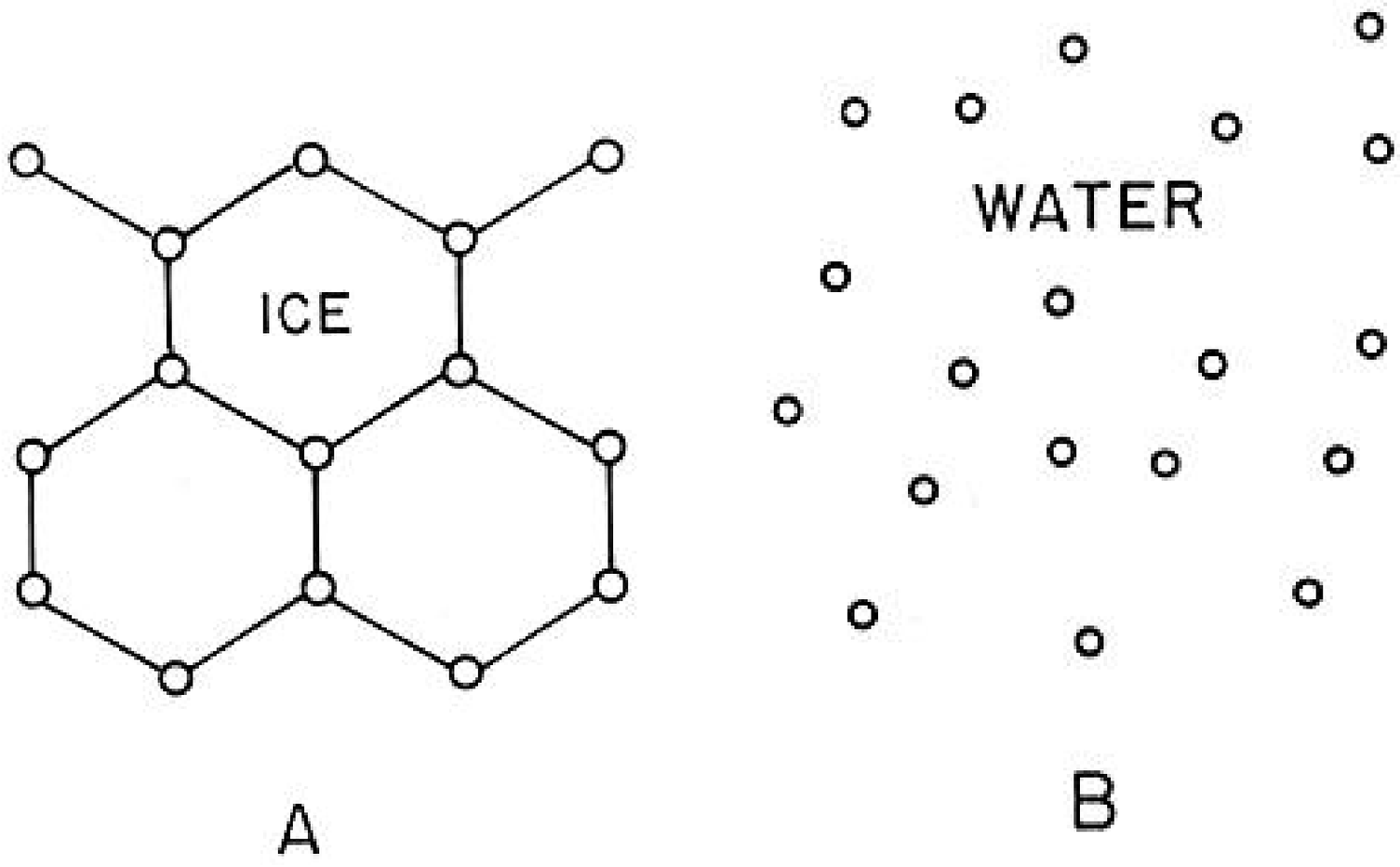






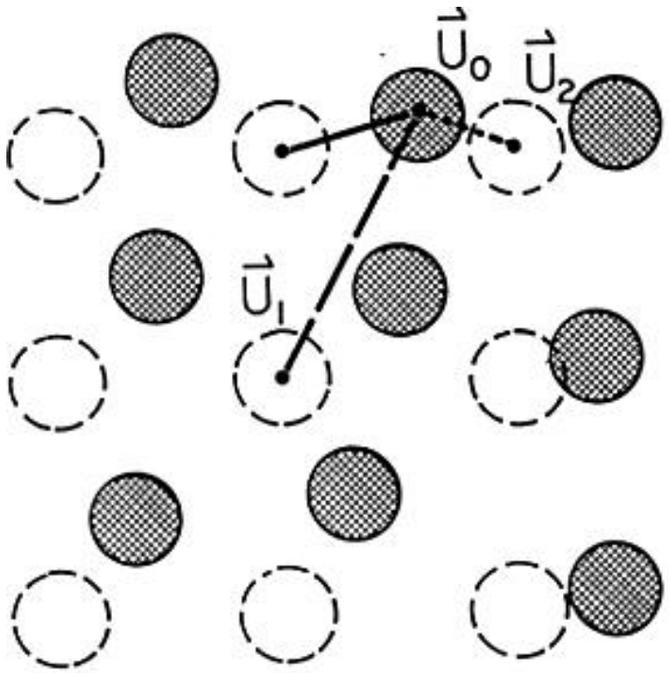
http://www.doitpoms.ac.uk/tlplib/crystallography3/printall.php

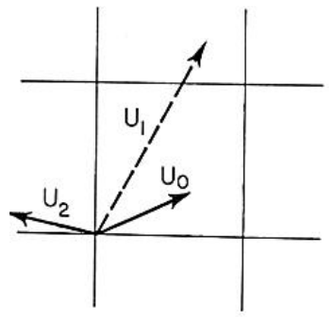
More or less symmetric ?

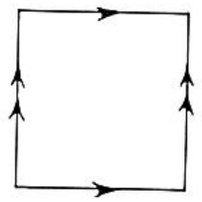


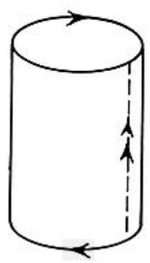
broken continuous
translation/rotation
symmetry (invariance)

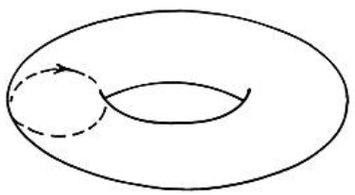
Order parameters: 2D crystal







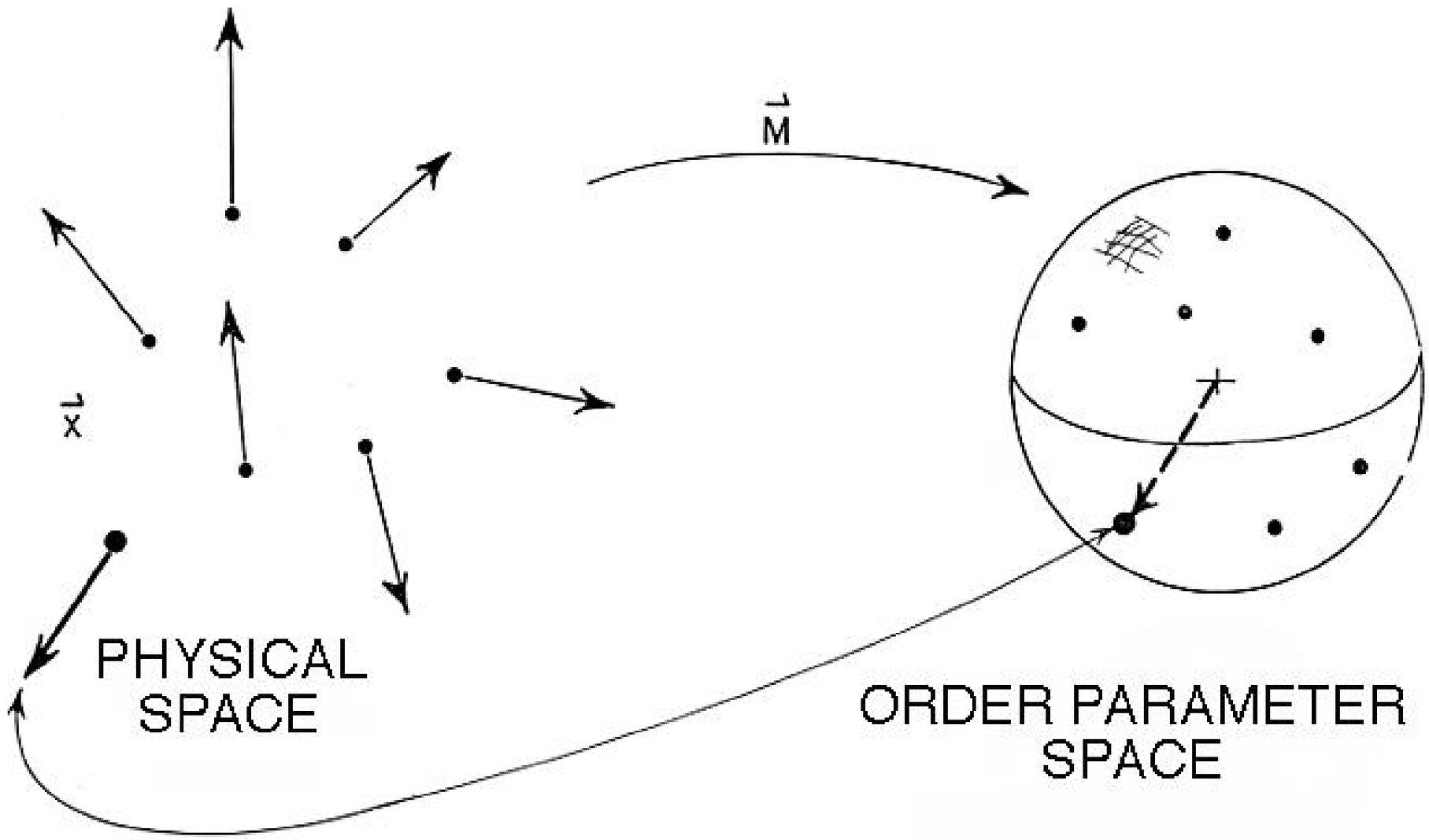




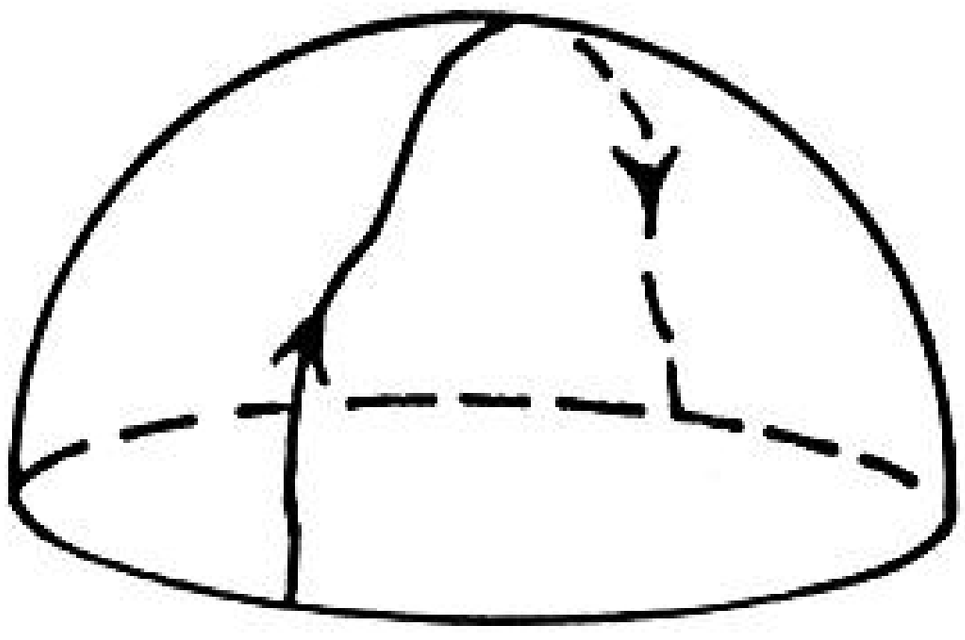
$$\vec{u} \equiv \vec{u} + a\hat{x} = \vec{u} + ma\hat{x} + na\hat{y}.$$

$$\mathcal{E} = \int dx\,(\kappa/2)(du/dx)^2.$$

Order parameters: magnets

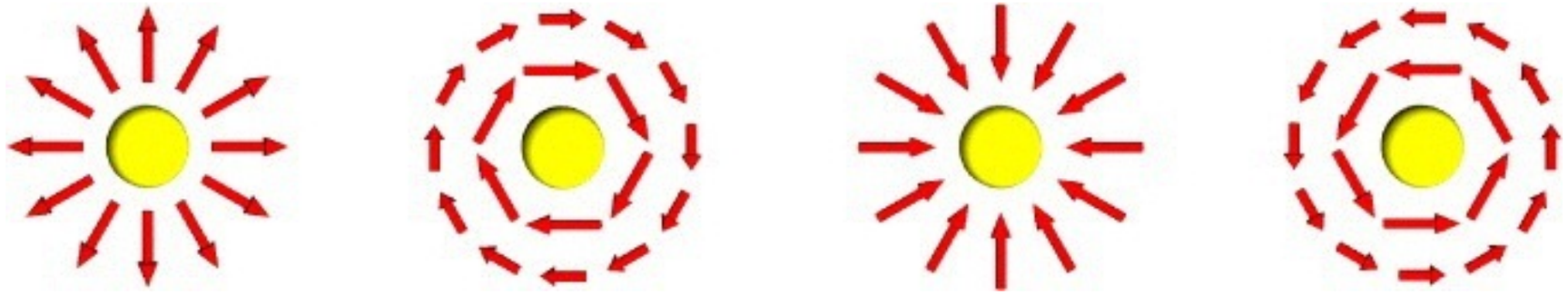


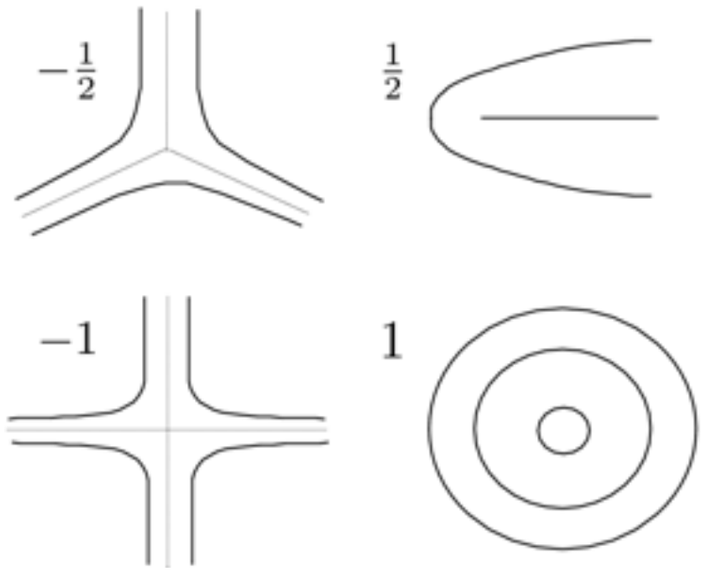
Order parameters: nematic liquid crystals

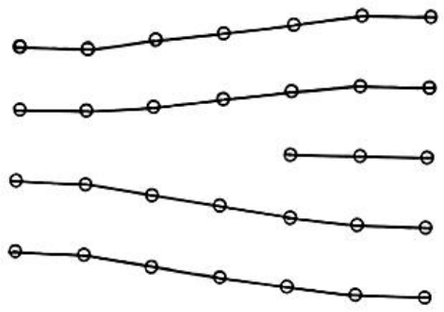


"projective plane" = half-sphere with opposite points on equator identified

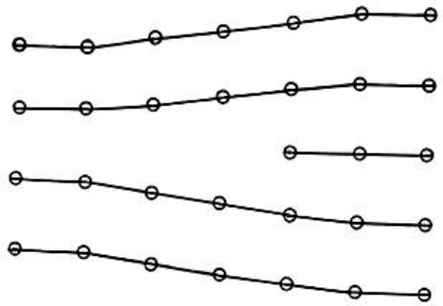
Topological defects



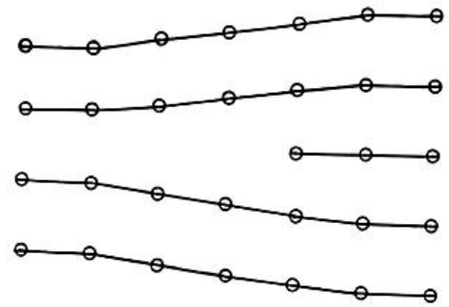




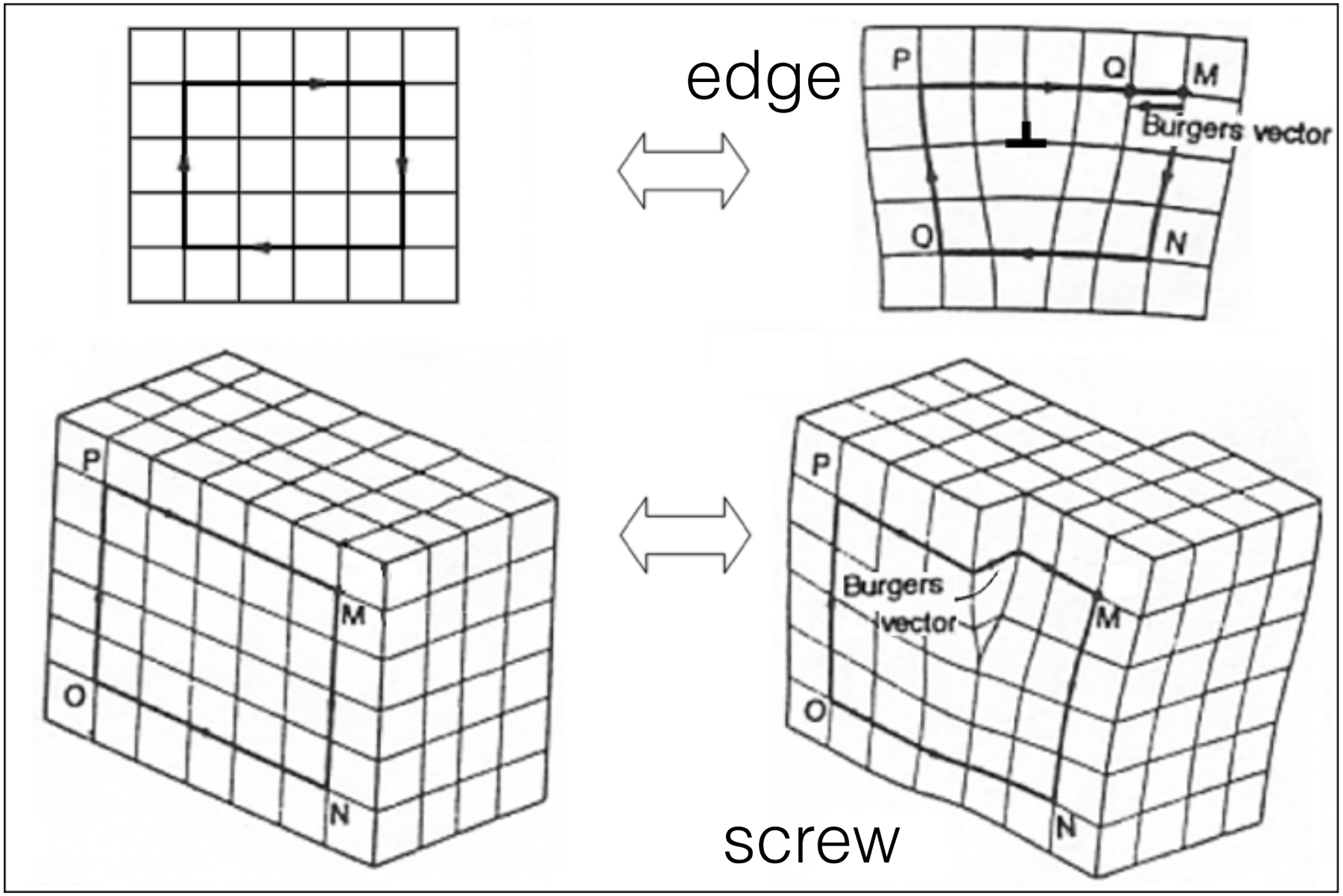
Work hardening

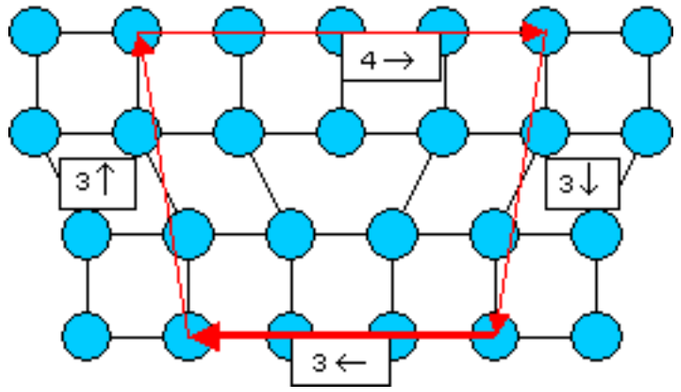


Disclineations



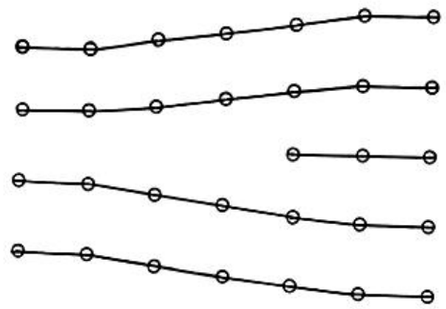
edge

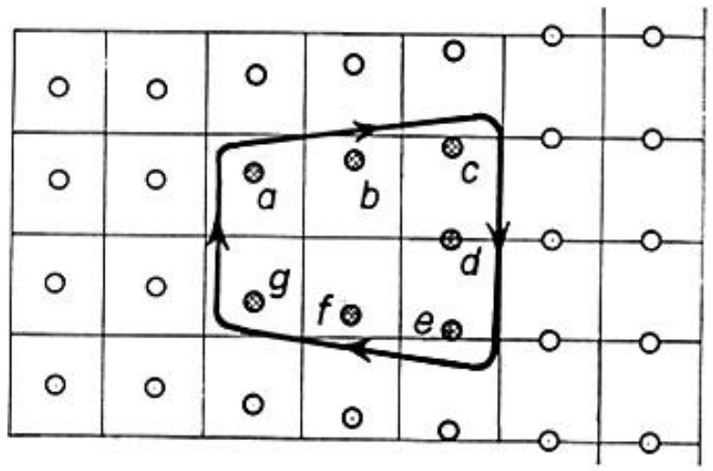


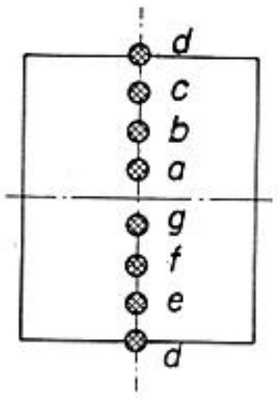


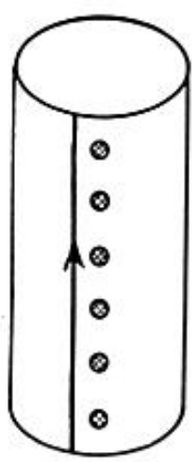
$$\|\mathbf{b}\| = (a/2)\sqrt{h^2 + k^2 + l^2}$$

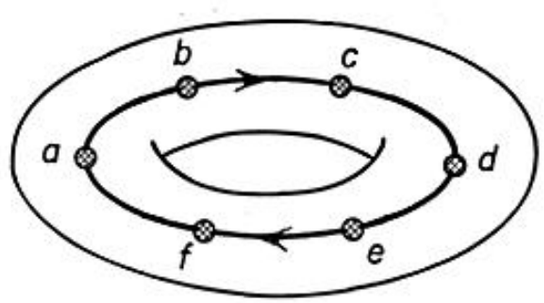
Disclineations





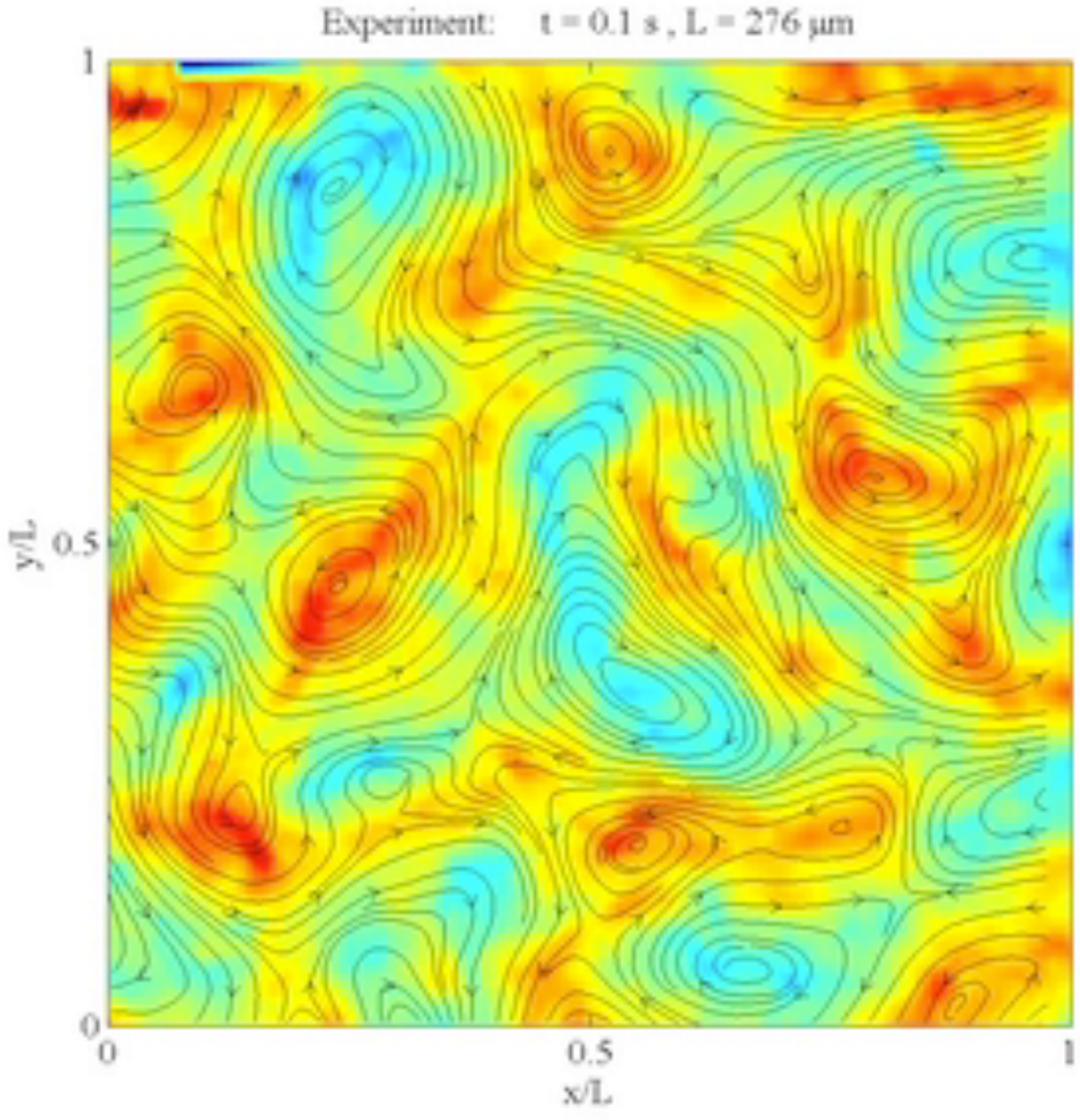


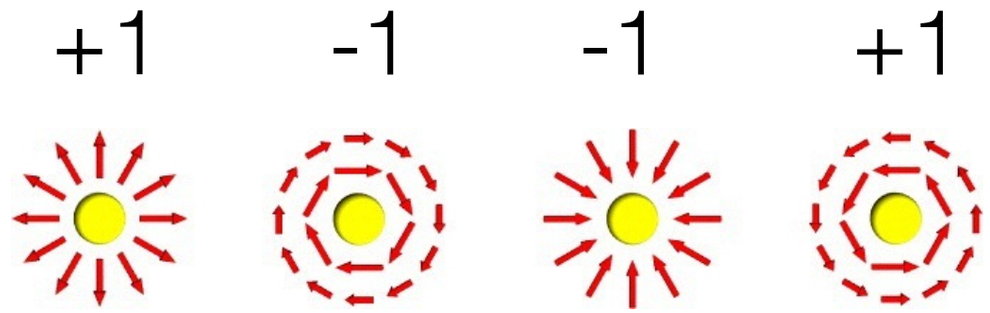


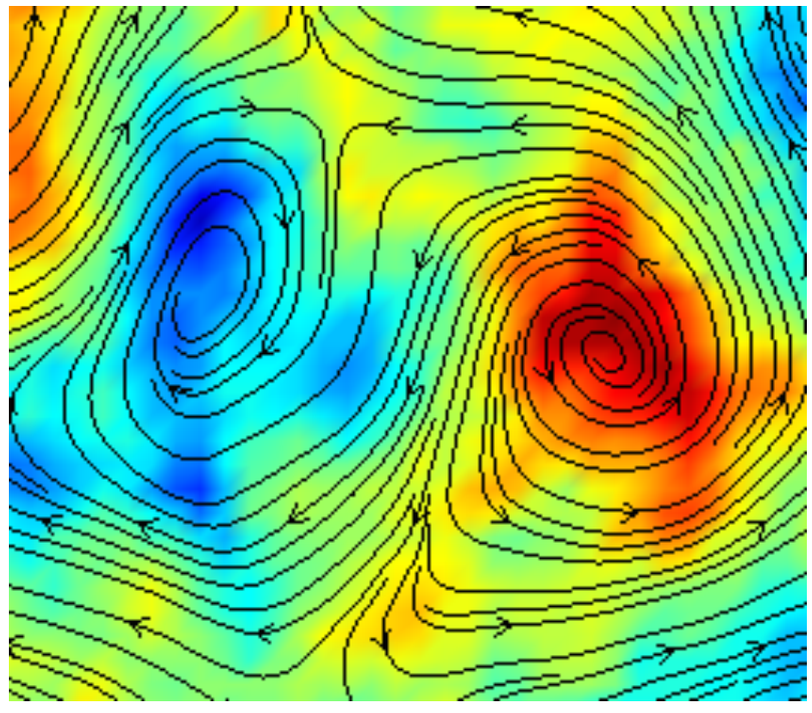


Bacterial vortices

PIV

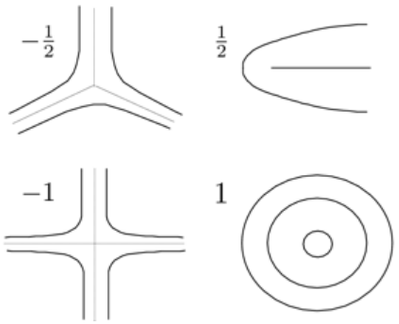


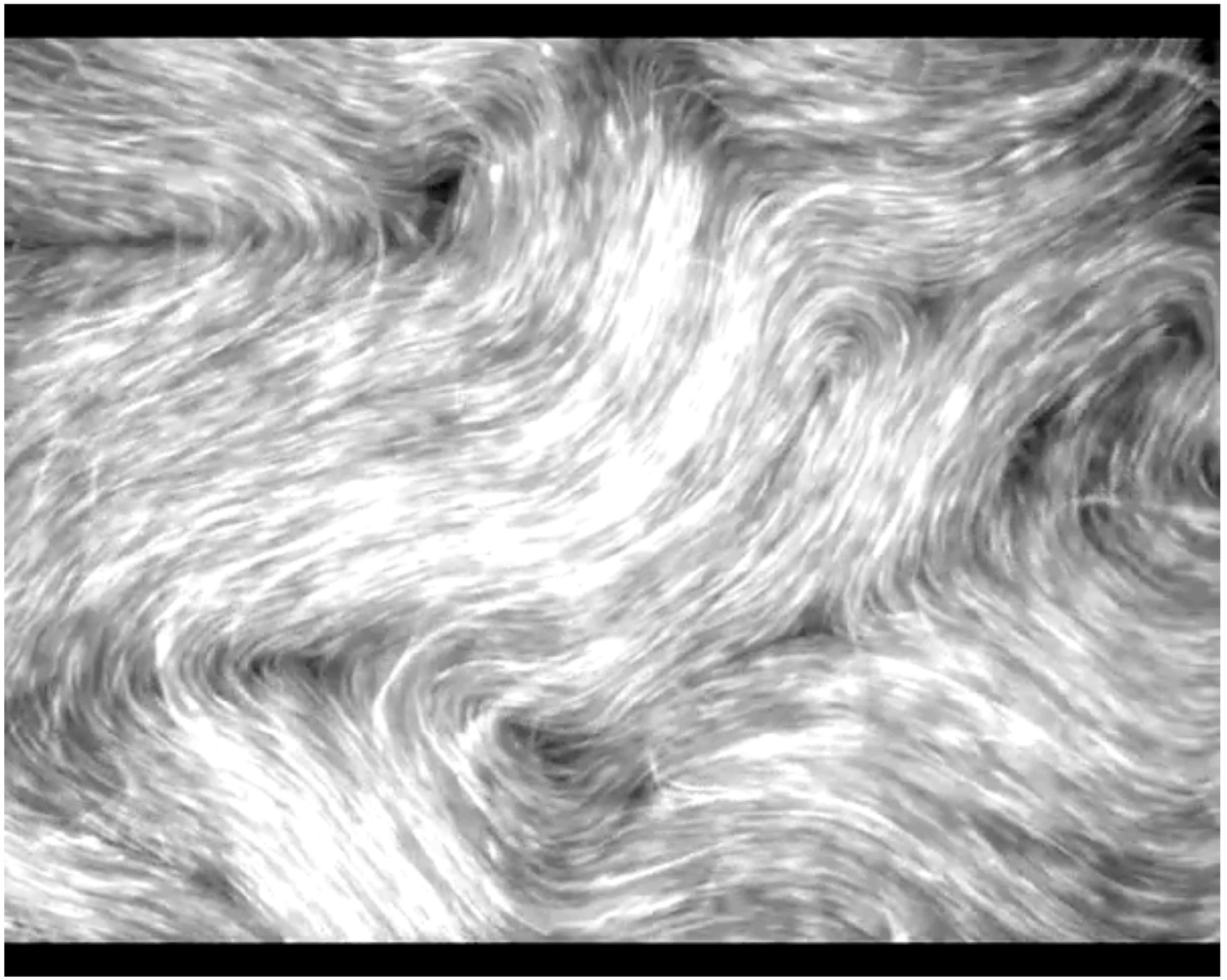




Dunkel et al PRL 2013

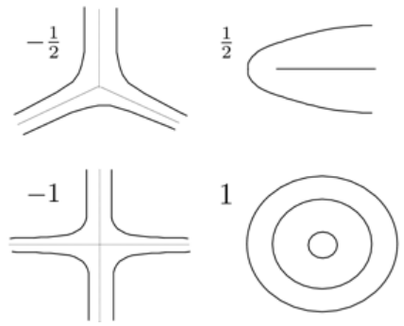
Active nematics

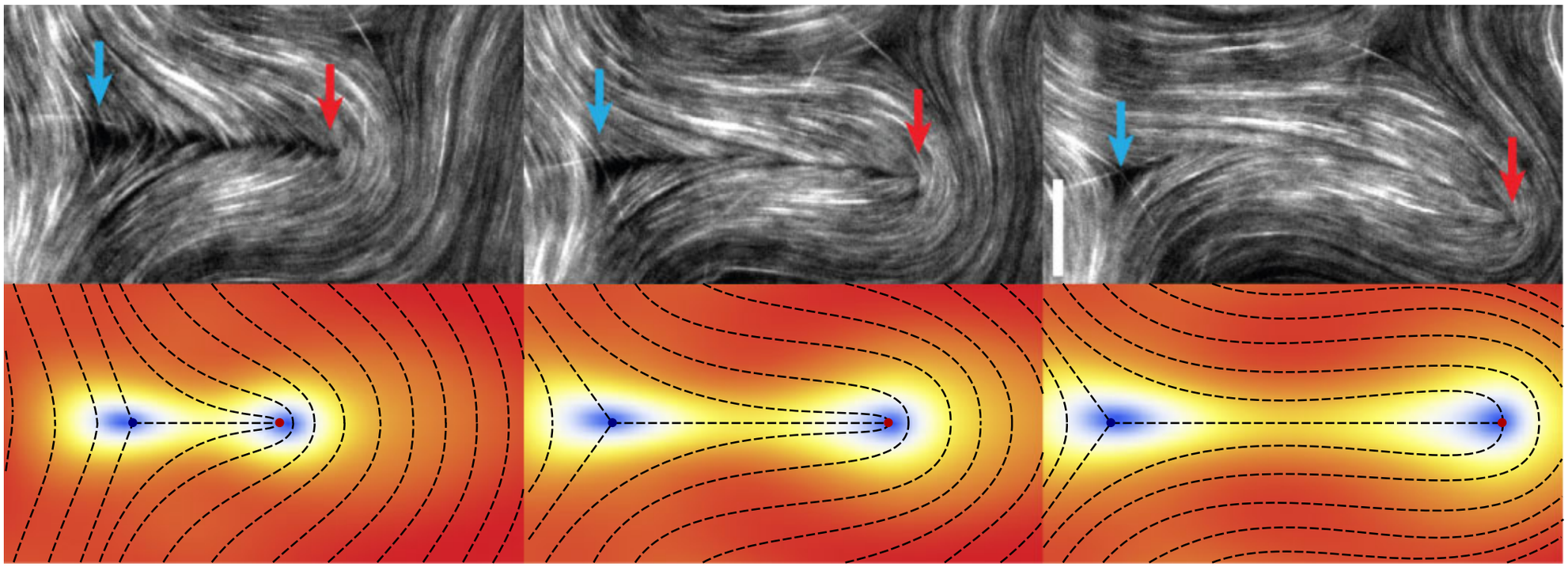




Dogic lab (Brandeis) Nature 2012

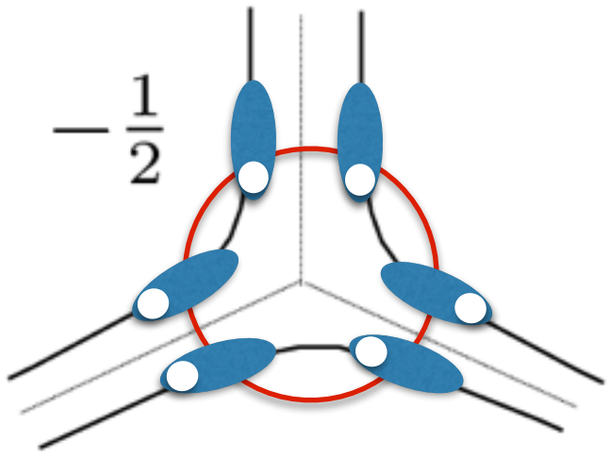
Active nematics

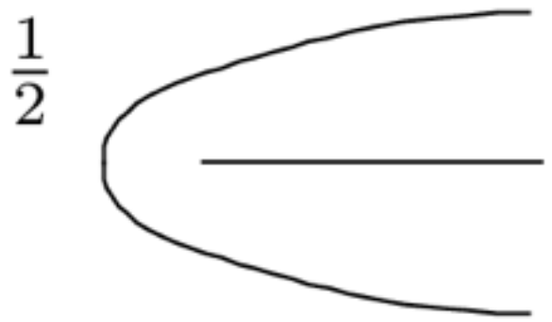




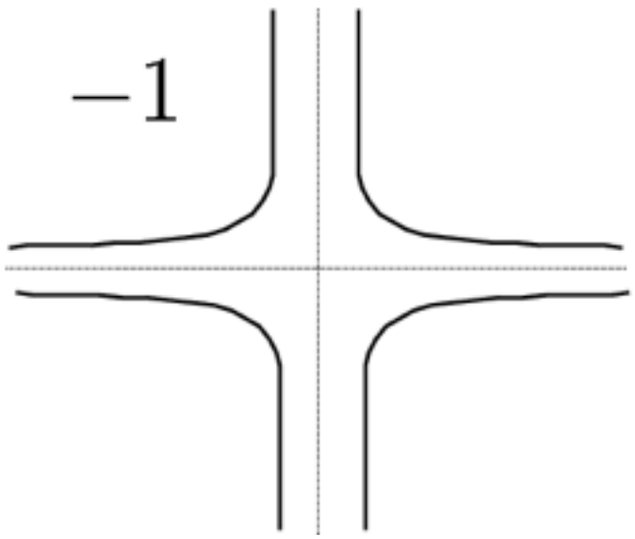
Giomi et al PRL 2012

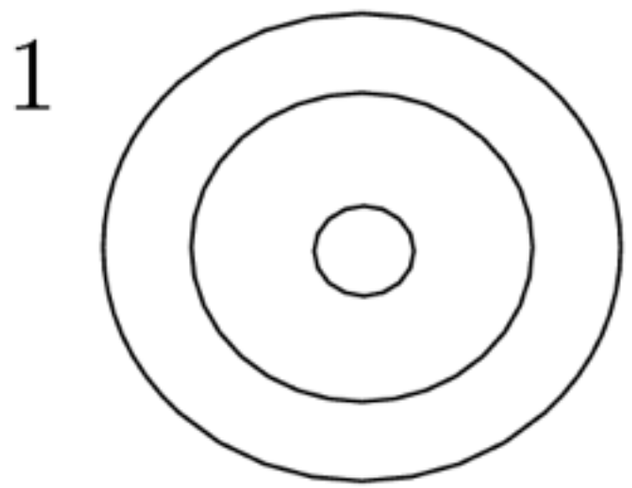
Defects in nematics



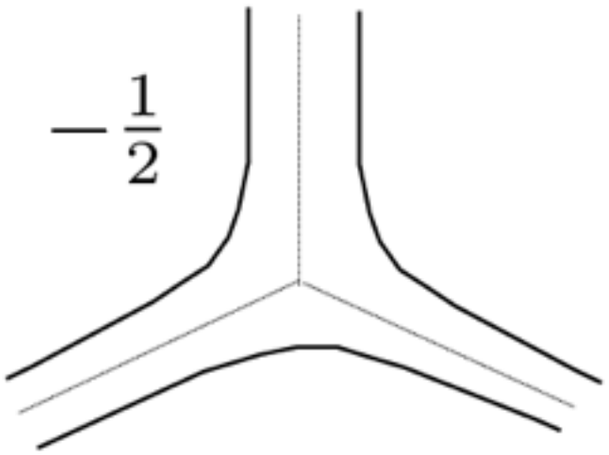


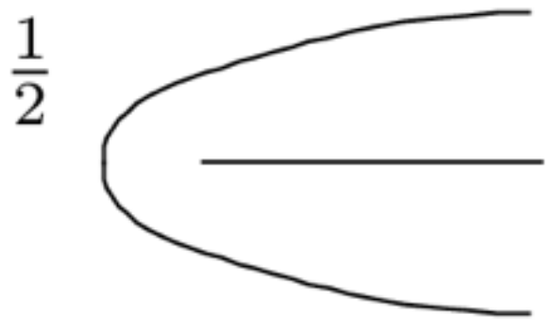
winding number





Defects in nematics





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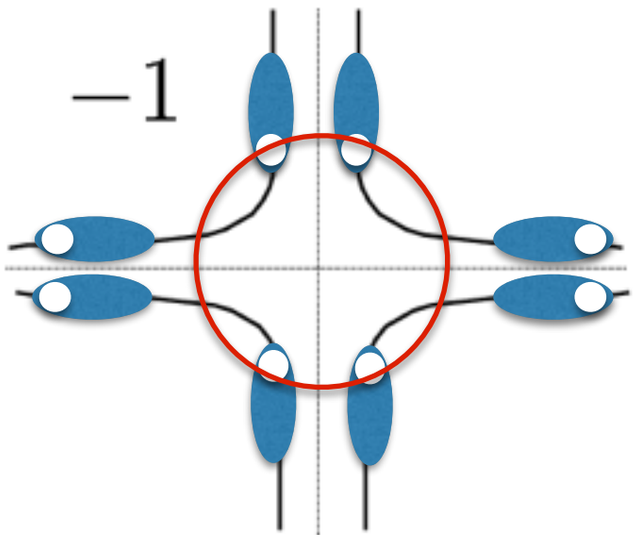
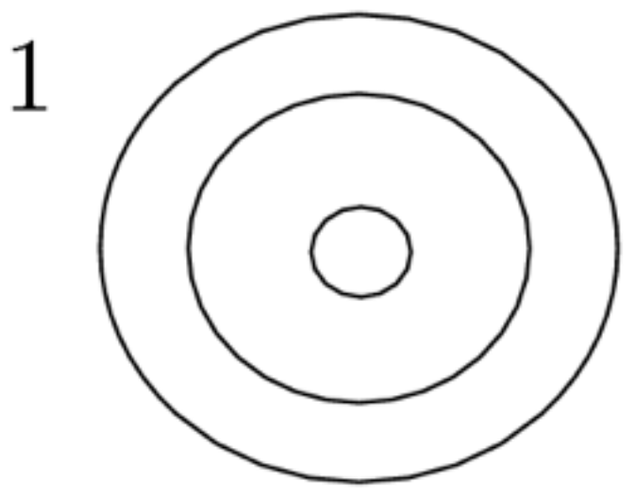
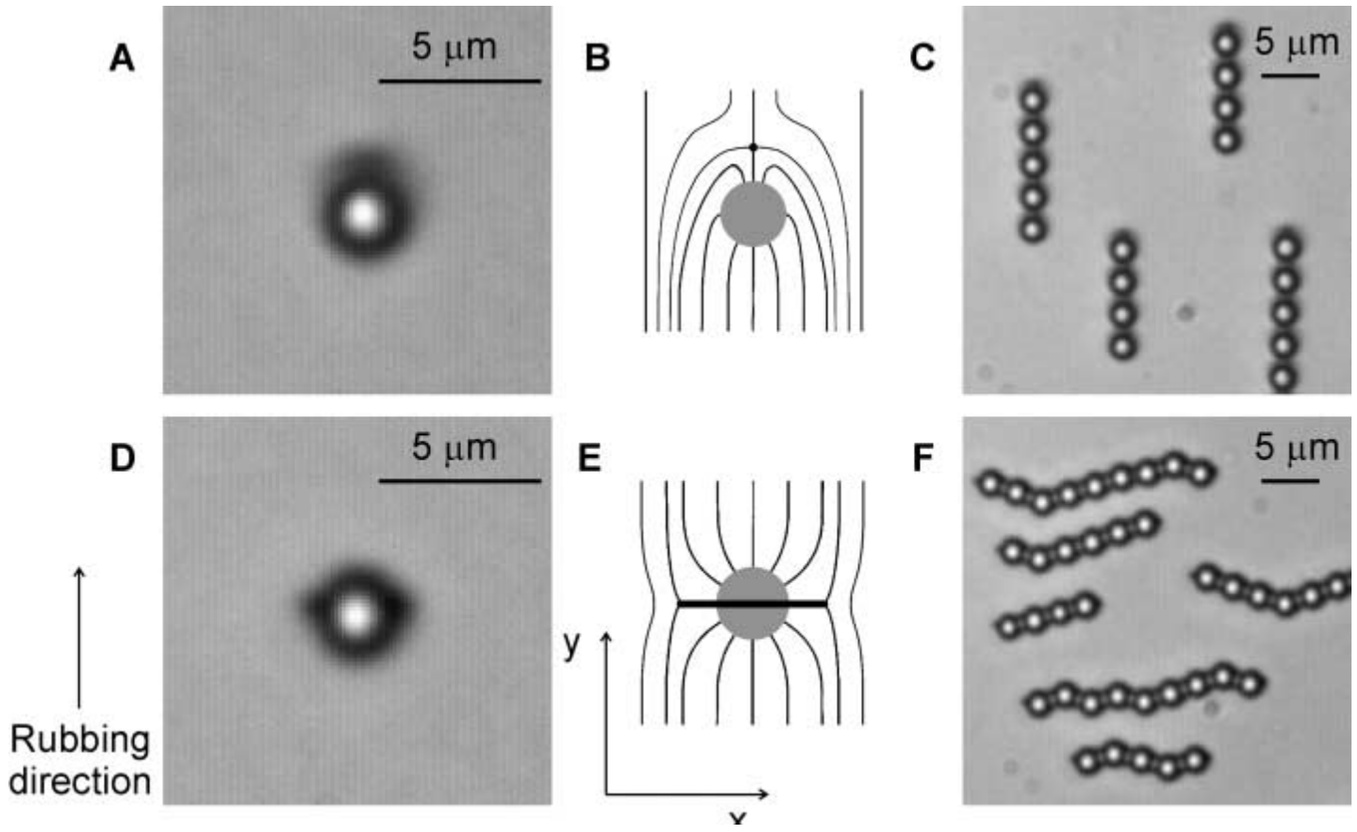




Two-Dimensional Nematic Colloidal Crystals Self-Assembled by Topological Defects

Igor Musevic *et al.*

Science **313**, 954 (2006);
DOI: 10.1126/science.1129660



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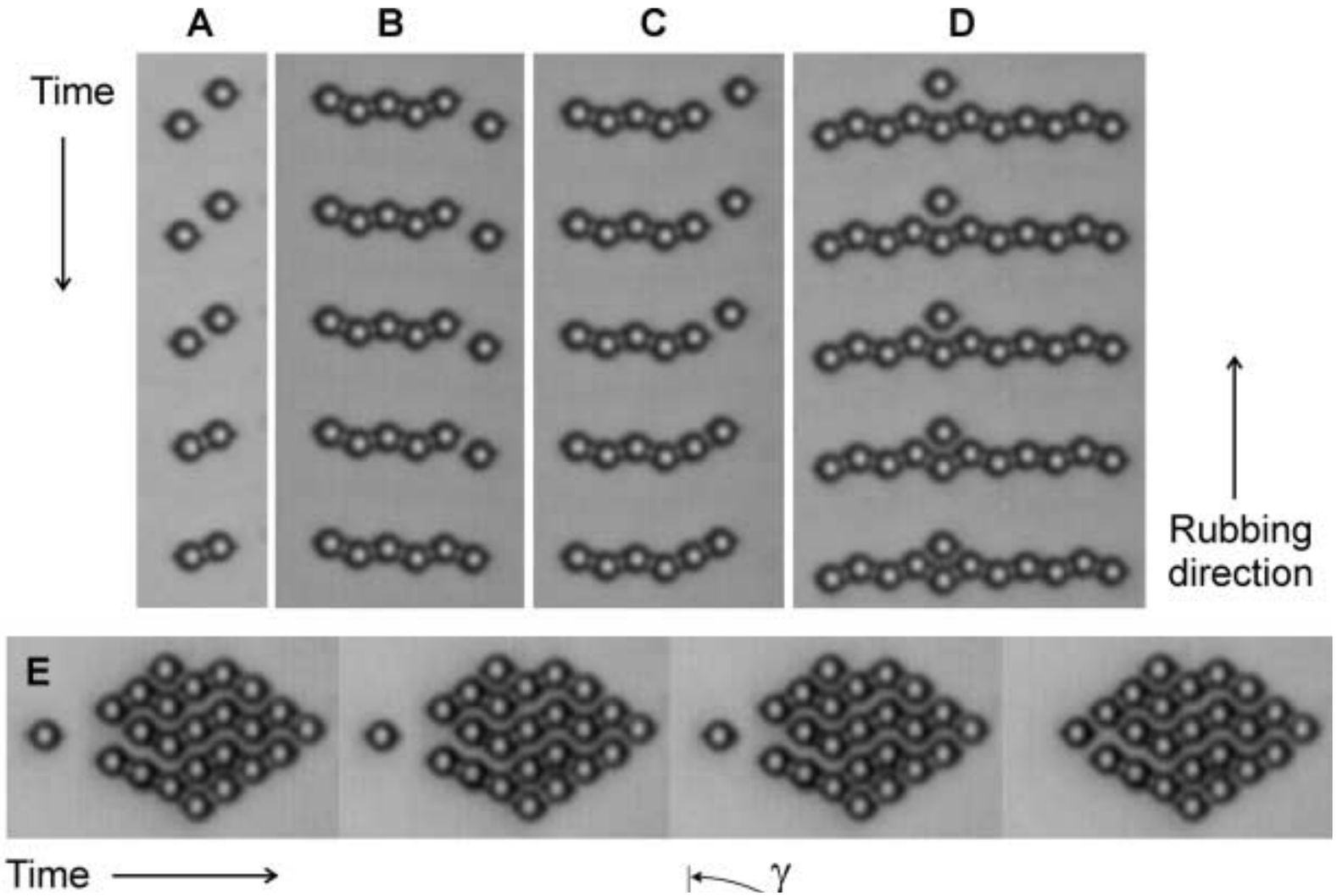

Reconfigurable Knots and Links in Chiral Nematic Colloids

Uros Tkalec *et al.*

Science **333**, 62 (2011);
DOI: 10.1126/science.1205705