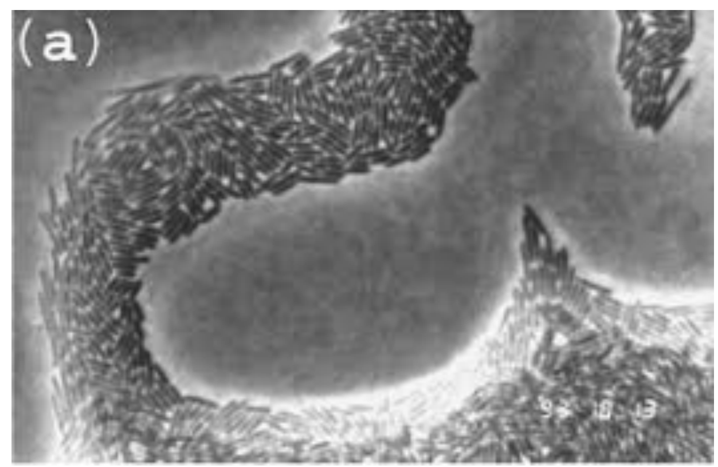
Active matter - overview

18.354 L23

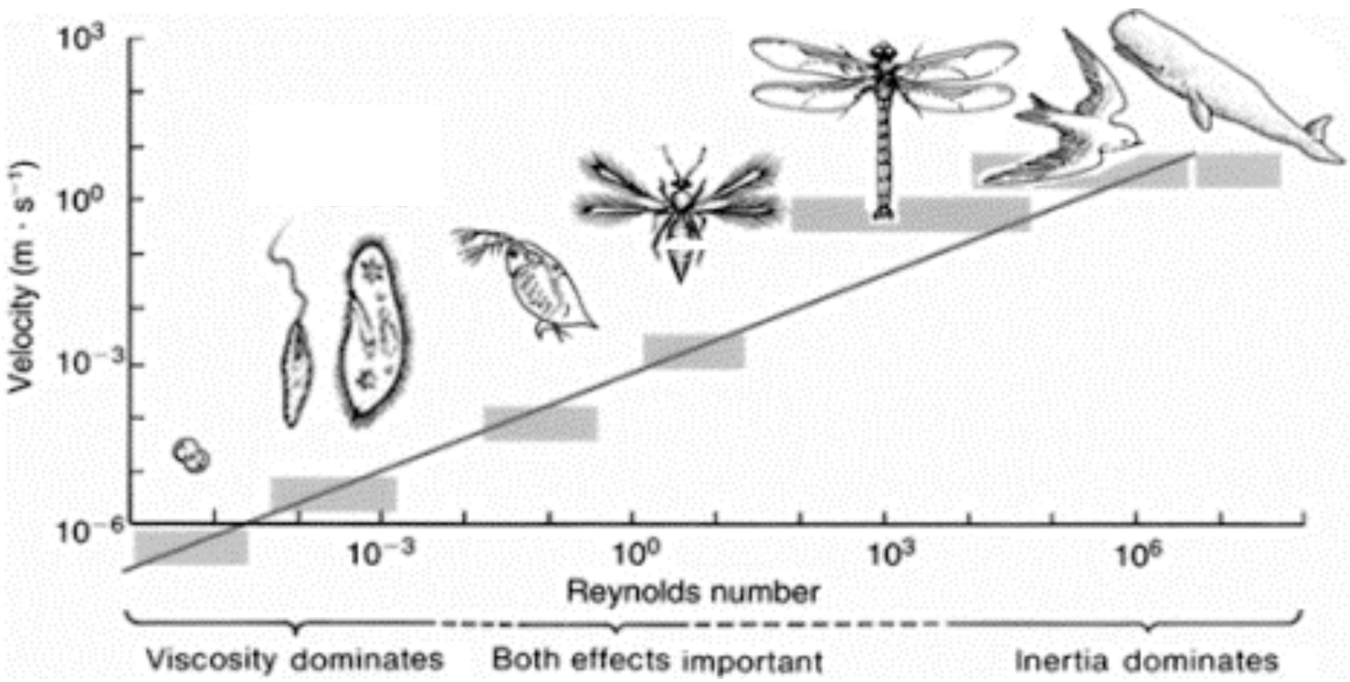
dunkel@math.mit.edu

Active matter



Typical Reynolds numbers

$$Re = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$



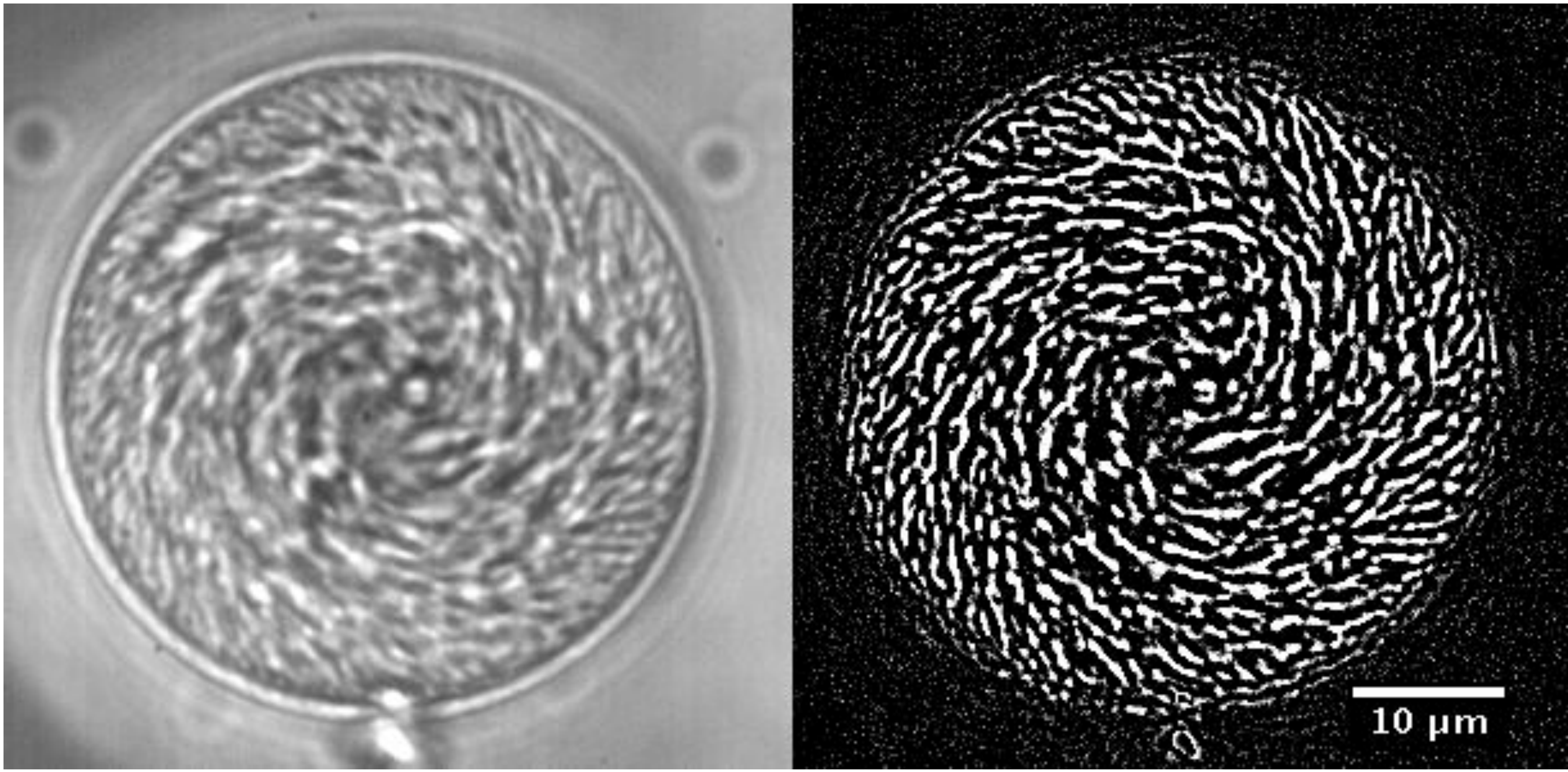
Birds

Fish

Bacteria

Berg lab, Harvard

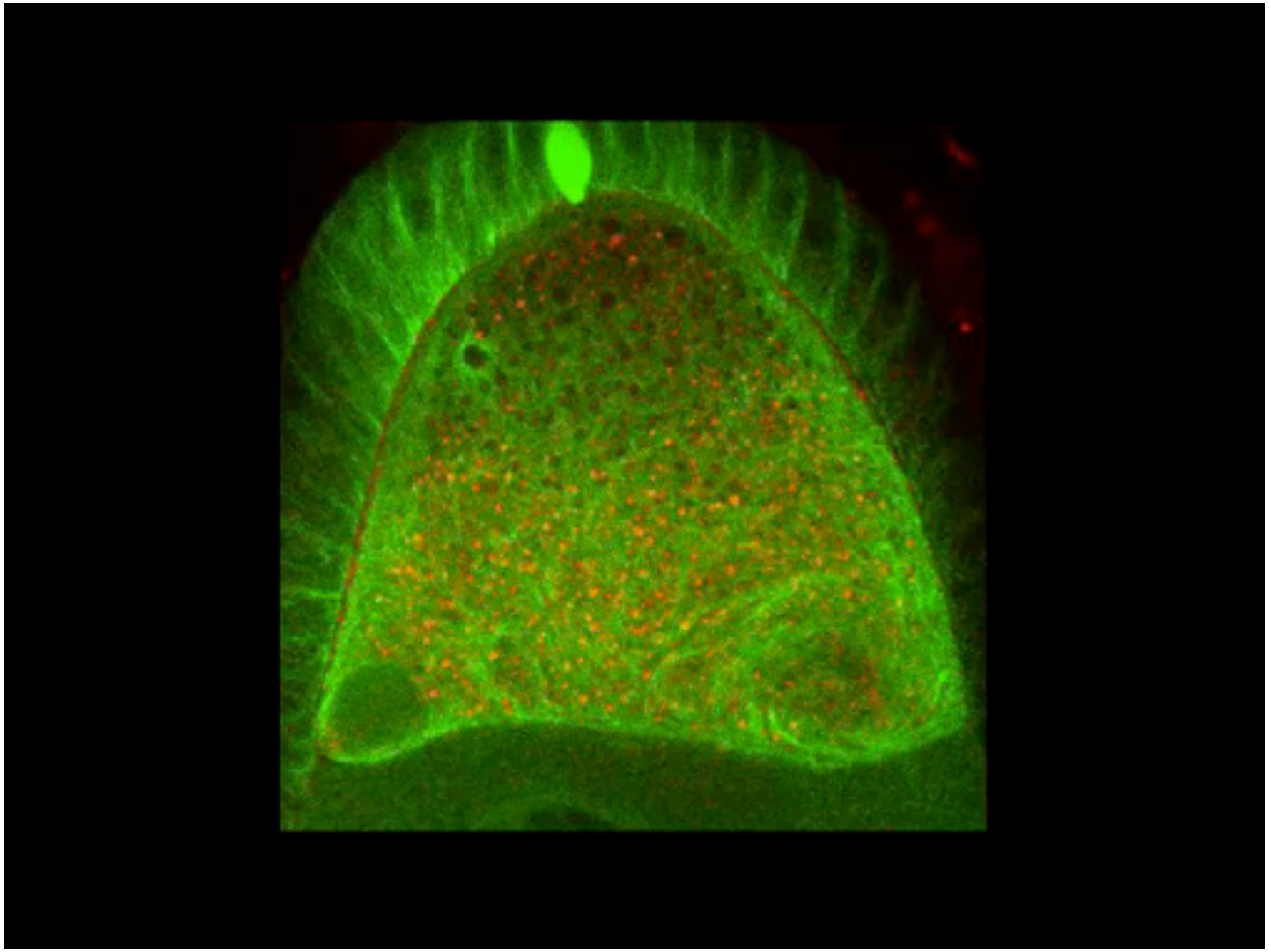
Bacteria



Vortex life time ~ minutes

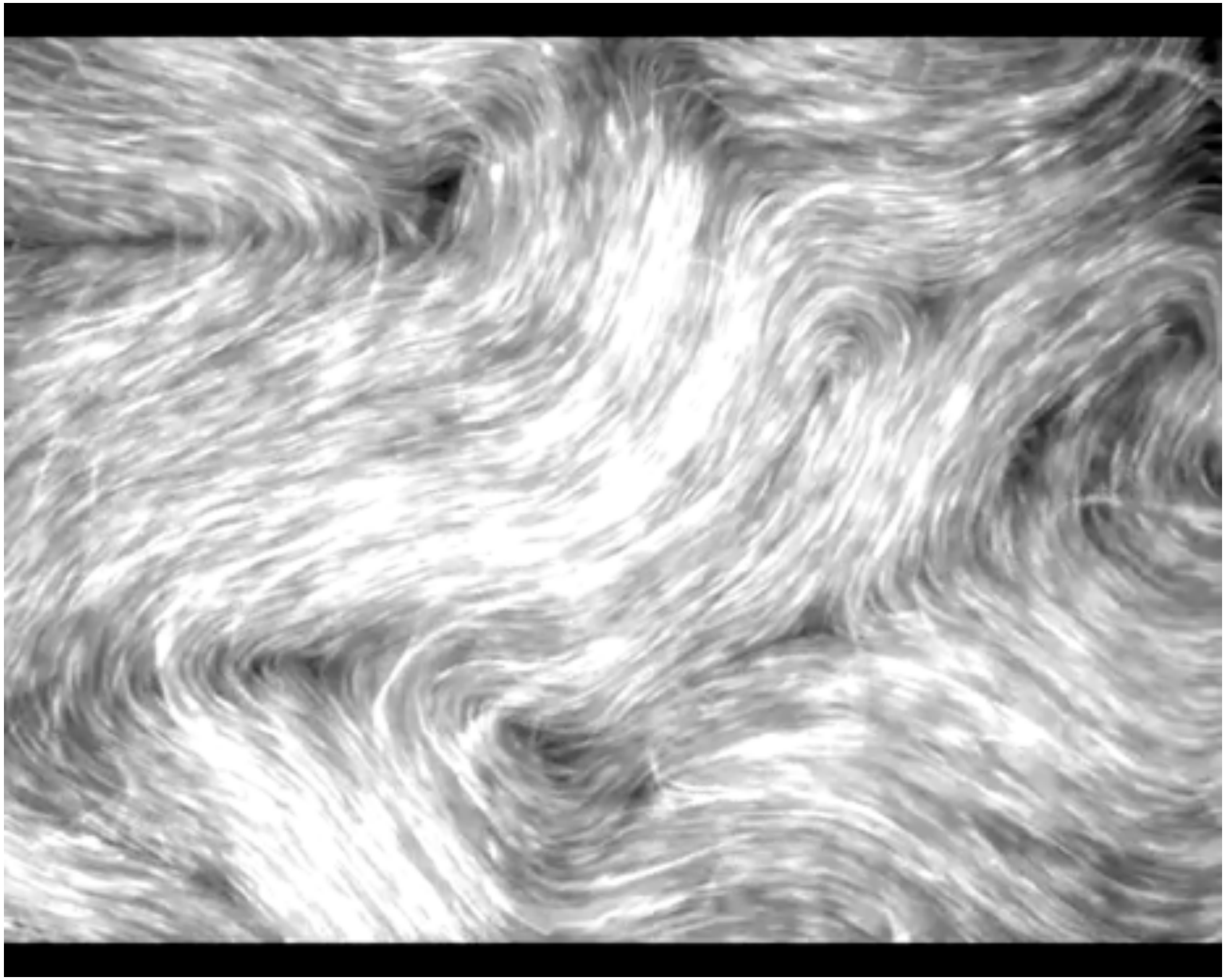
Wioland et al (2013) PRL

Motor-driven filaments



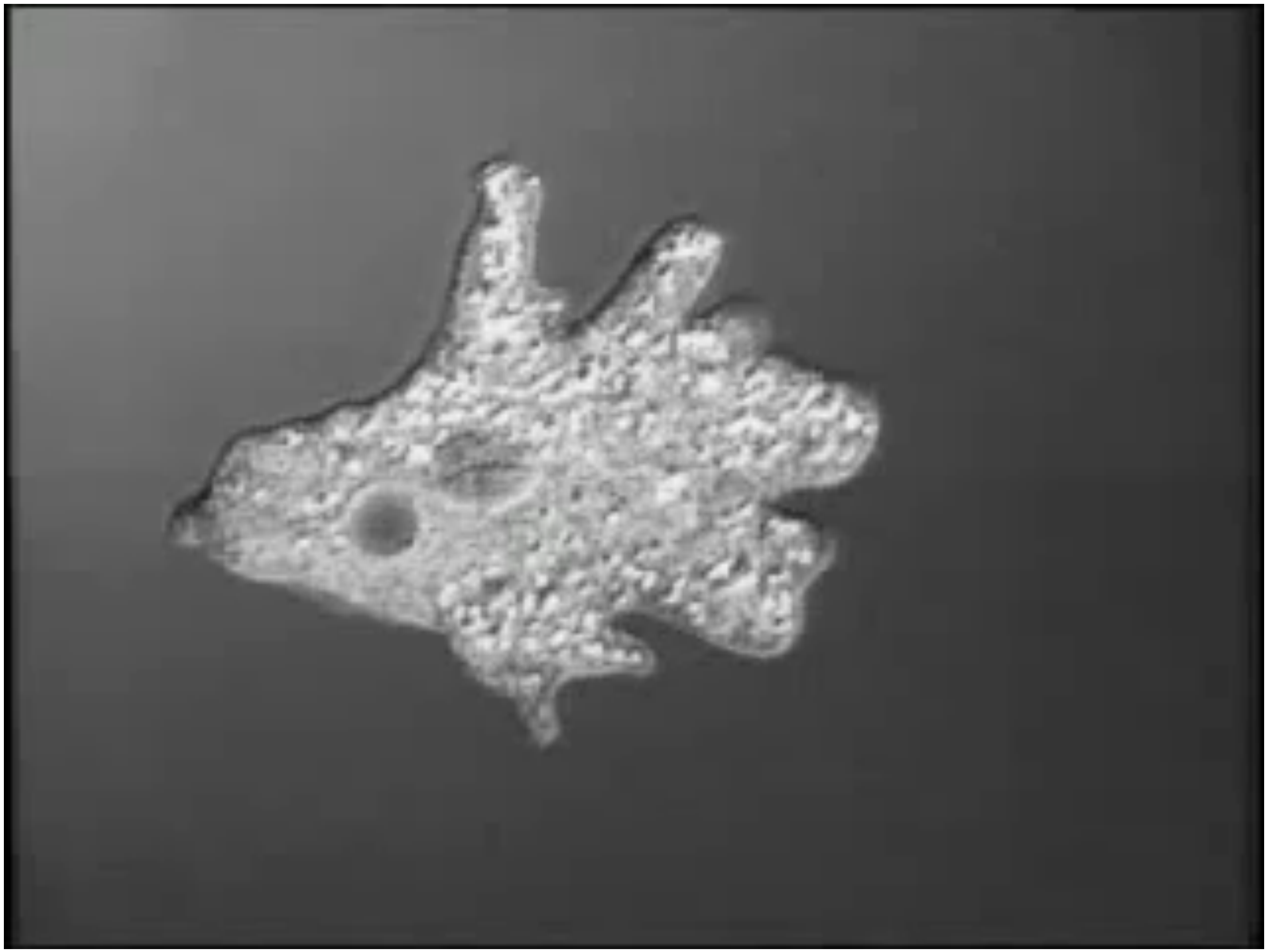
Drosophila embryo, Goldstein lab, Cambridge

Motor-driven filaments

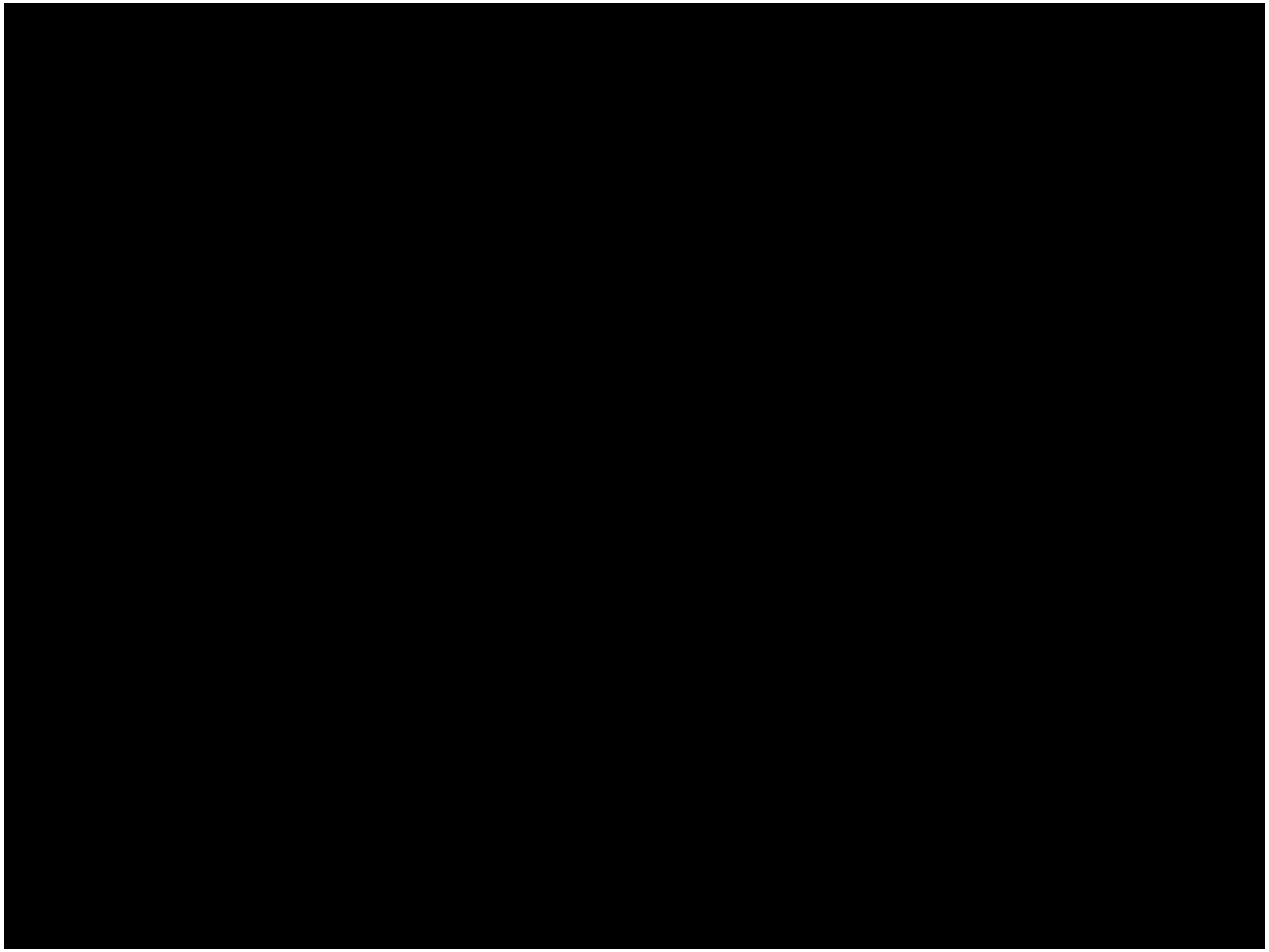


Dogic lab (Brandeis) Nature 2012

Amoeba

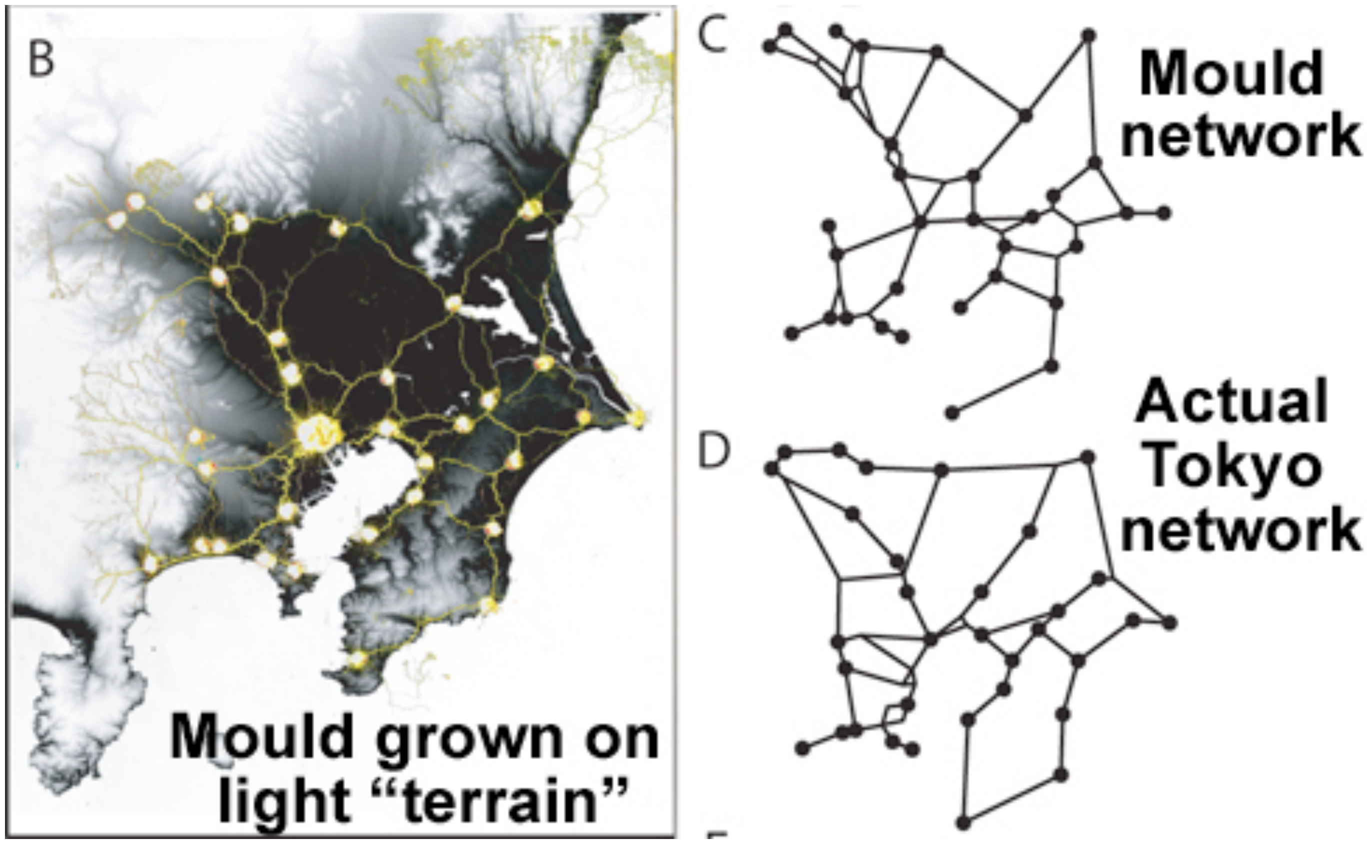


Physarum



Tero et al, Science 2010

Physarum



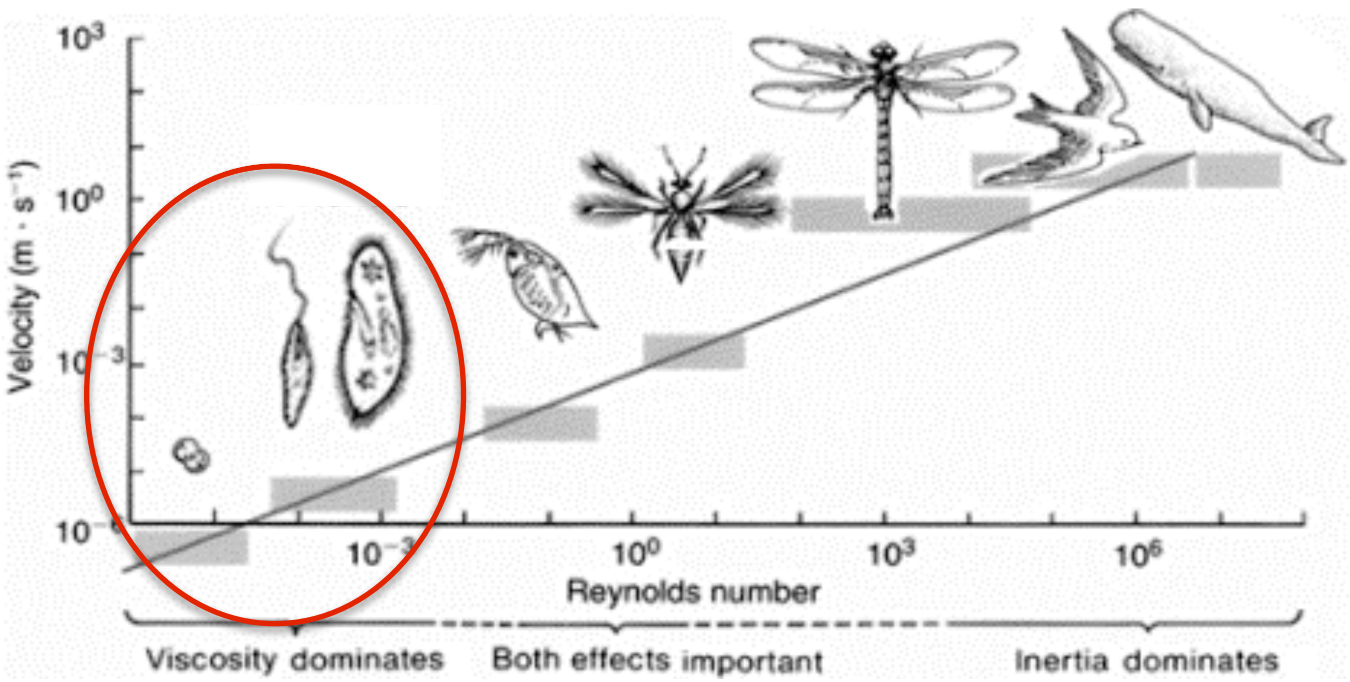
Tero et al, Science 2010

Questions

- universal aspects of collective motion & self-organization ?
- biological functions ?
- information transport ?
- mathematical description? (microscopically, macroscopically, …)
- effects of boundary conditions ?

Typical Reynolds numbers

$$Re = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$



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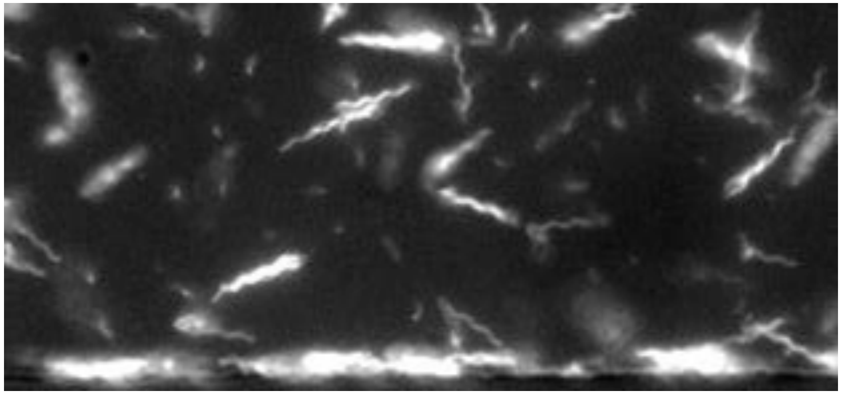
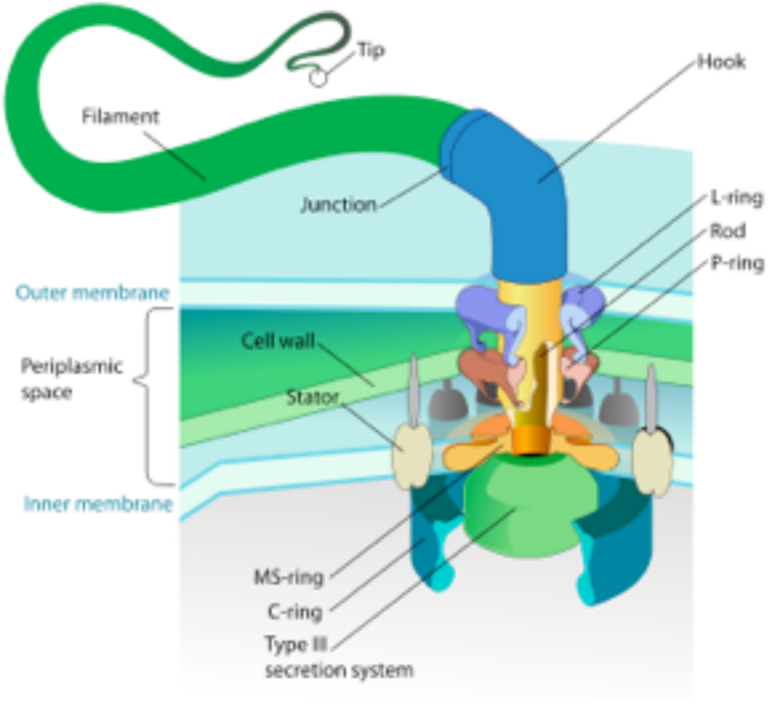
Bacterial motors

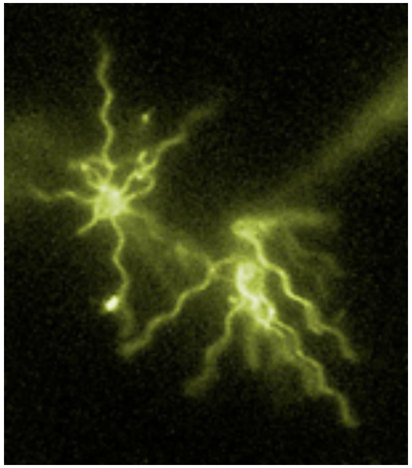
movie: V. Kantsler



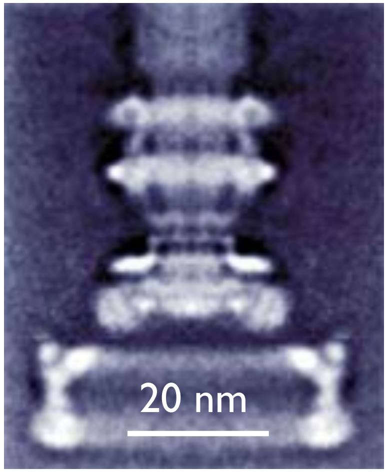
~20 parts



source: wiki



Berg (1999) Physics Today



Chen et al (2011) EMBO Journal

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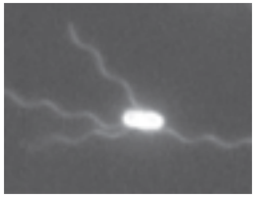
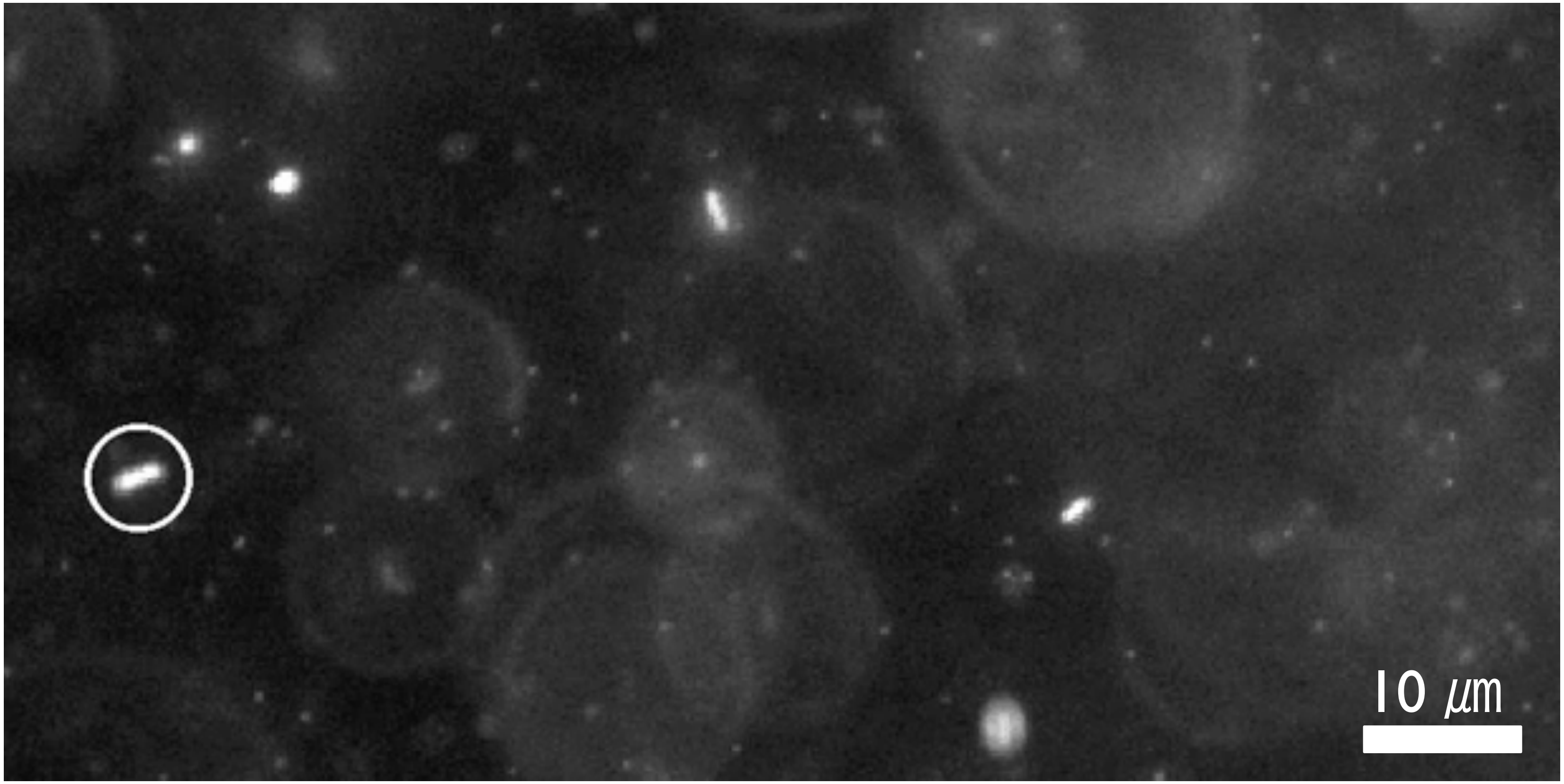
E. coli (non-tumling)

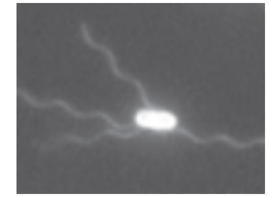


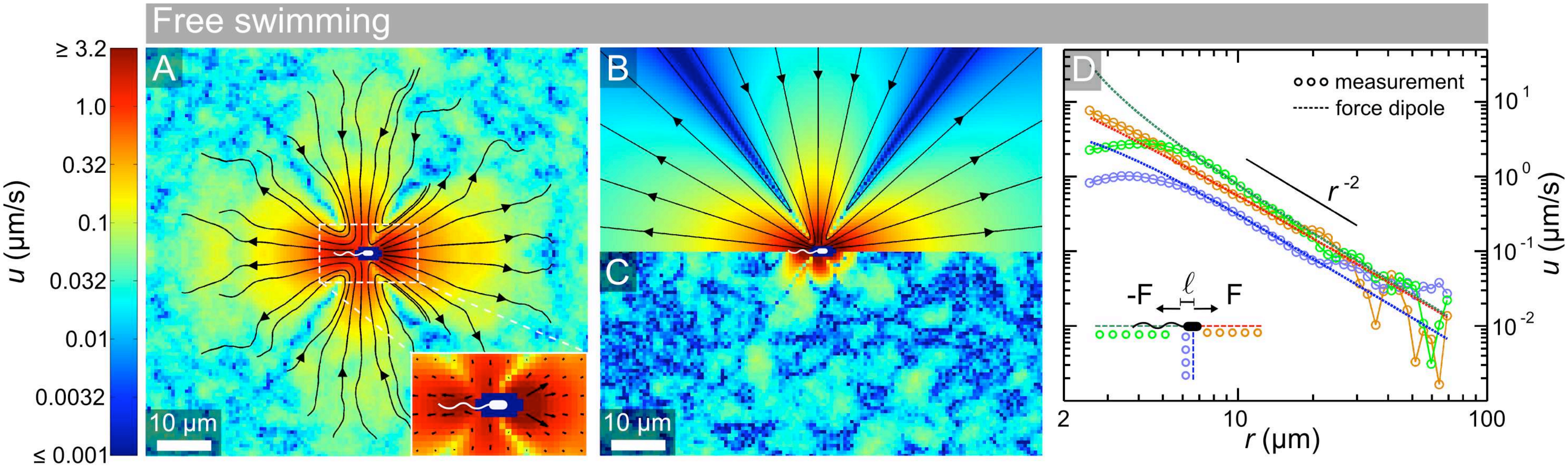
non-tumbling HCB 437



Drescher et al (2011) PNAS

E.coli (non-tumbling HCB 437)





$$\boxed{\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}}$$

$$V_0 = 22 \pm 5 \;\; \mu\text{m/s}$$

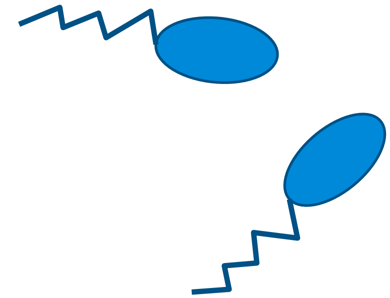
$$\ell = 1.9 \;\; \mu\text{m}$$

$$F = 0.42 \;\; \text{pN}$$

‘pusher’ dipole

Drescher et al (2011) PNAS

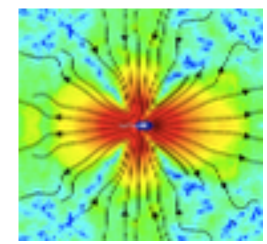
Hydrodynamic scattering



dipole flow $\quad \boldsymbol{v} \sim \dfrac{A}{r^2}$

vorticity $\quad \omega = \nabla \times \boldsymbol{v} \sim \dfrac{A}{r^3}$

encounter time $\quad \tau \sim \ell / V$



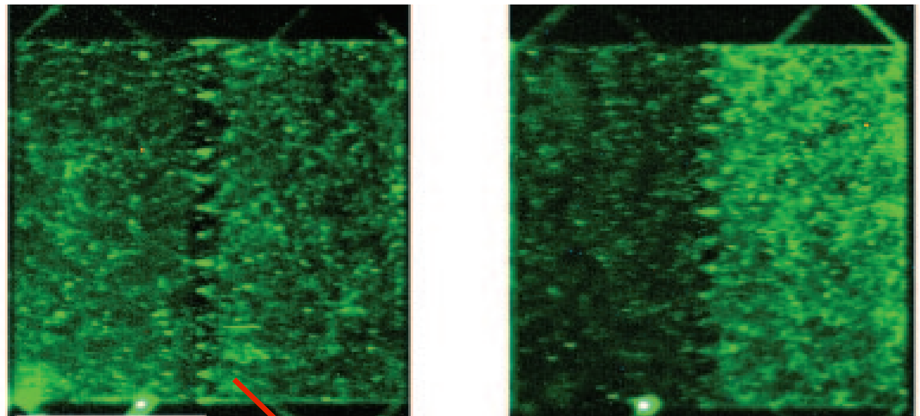
HD rotation $\quad \langle |\Delta\phi|^2 \rangle \sim (\omega\tau)^2 \sim \left(\dfrac{A\tau}{r^3}\right)^2$

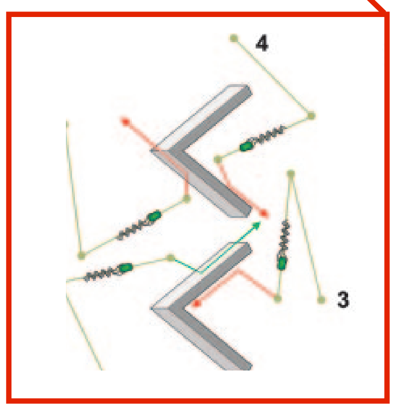
rotational diffusion $\quad \langle |\Delta\phi|^2 \rangle \sim D_r \tau \qquad D_r = 0.057\ \mathrm{rad}^2/\mathrm{s}$

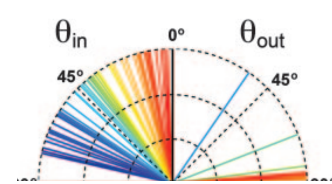
balance $\quad \boxed{r_H \sim \left(\dfrac{A^2\tau}{D_r}\right)^{1/6}} \qquad 3.3\ \mu\mathrm{m}$ for *E. coli*

Drescher et al (2011) PNAS

Rectification of prokaryotic locomotion



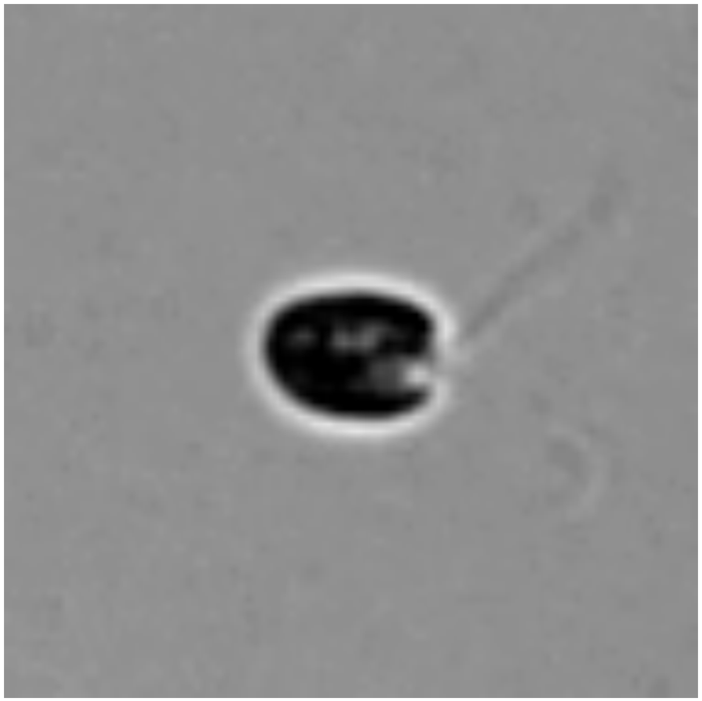




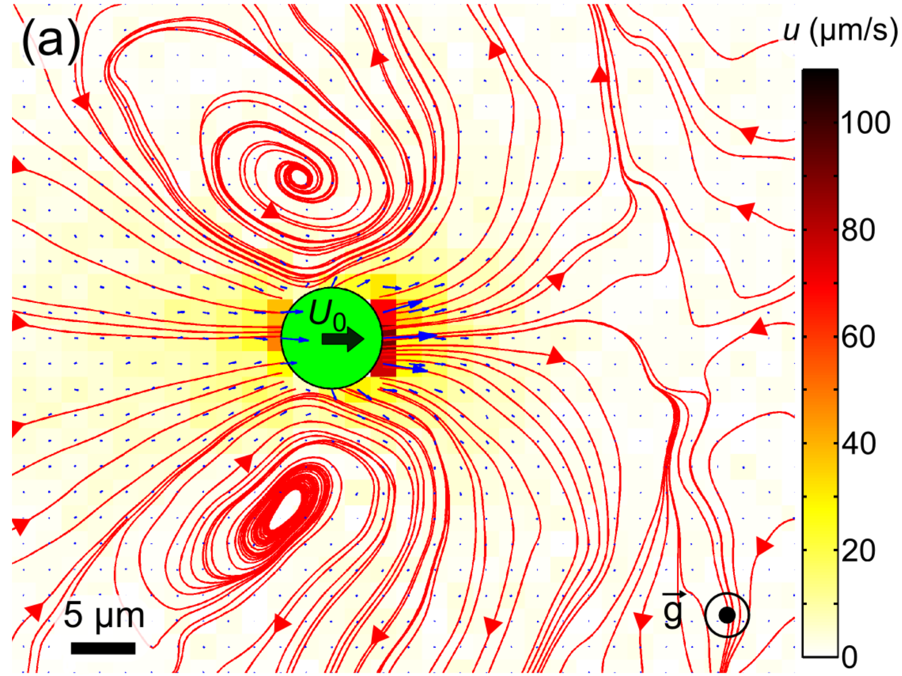
Galadja et al (2009)
J Bacteriology

Austin lab, Princeton, 2009

Chlamydomonas



Movie: Jeff Guasto (TUFTS)

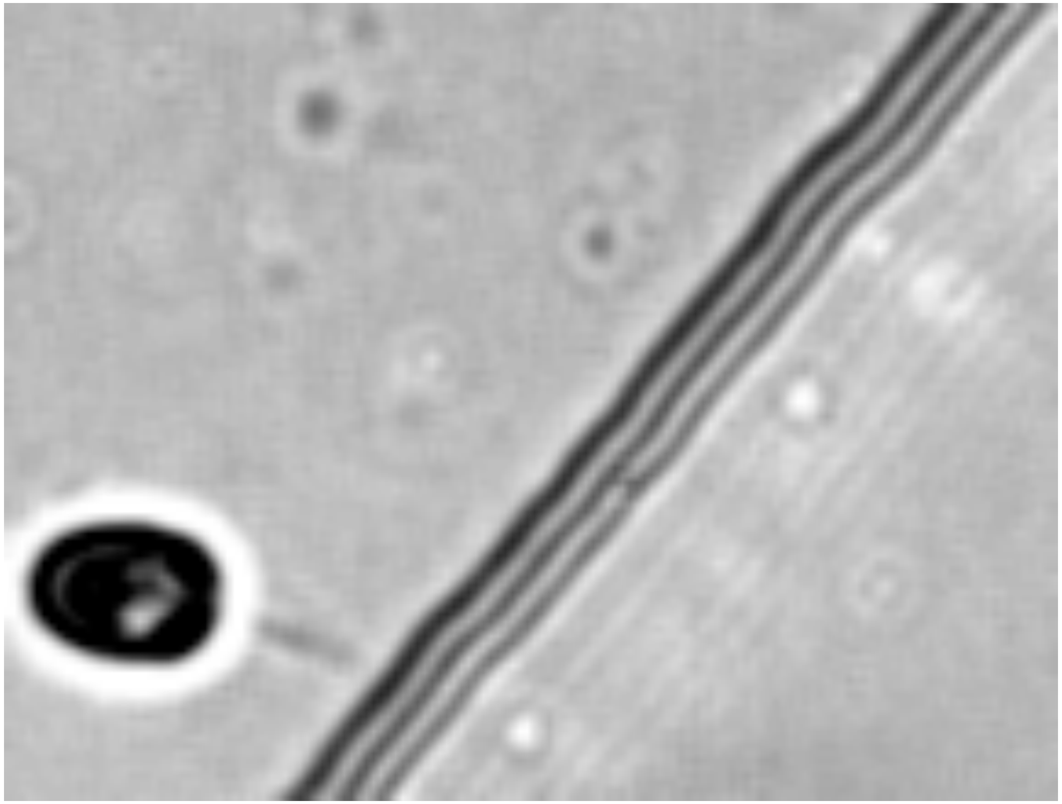


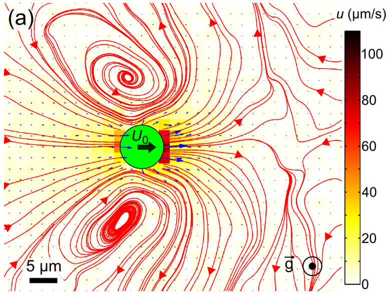
Drescher et al PRL 2010
Guasto et al PRL 2010

'puller'

size ~ 20μm
speed ~ 100μm/s
beat frequency ~30 Hz

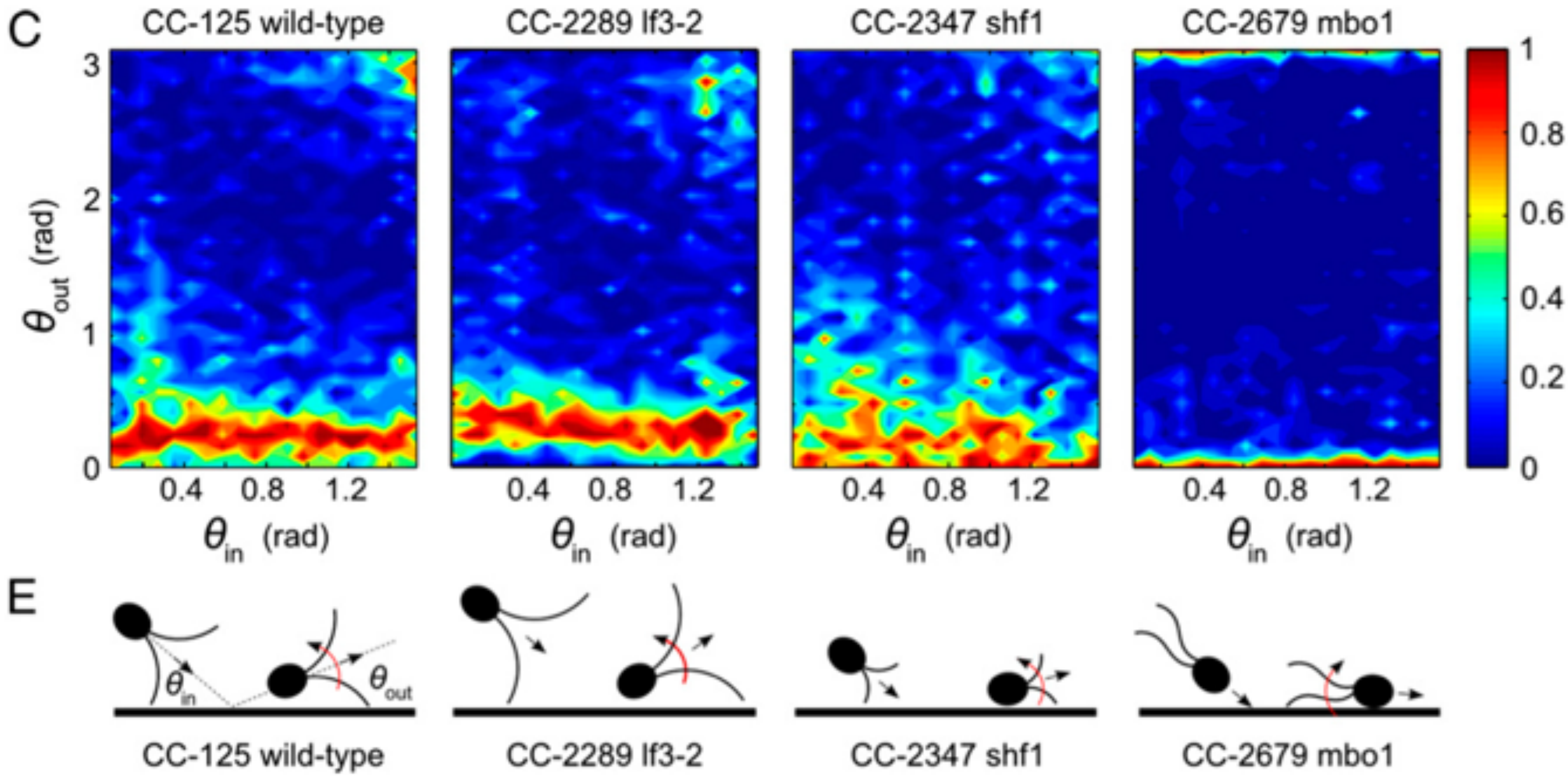
Mechanical control of algal locomotion





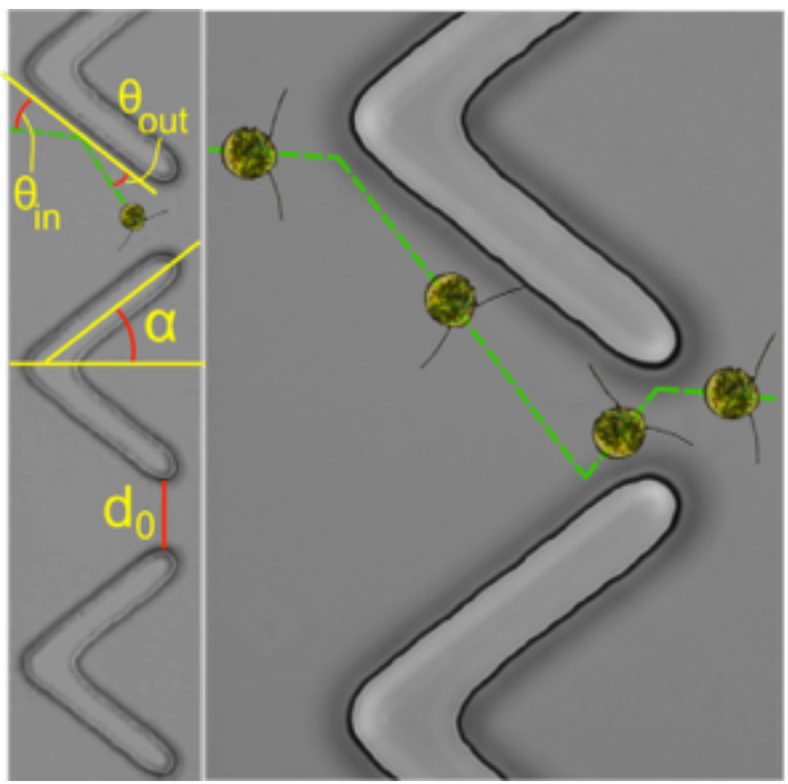
Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

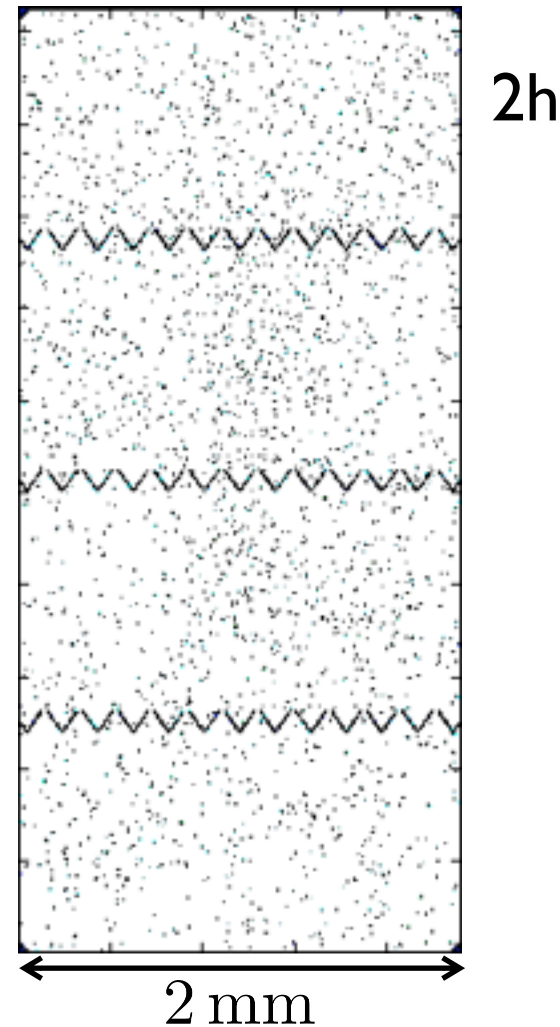
Surface scattering laws



Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

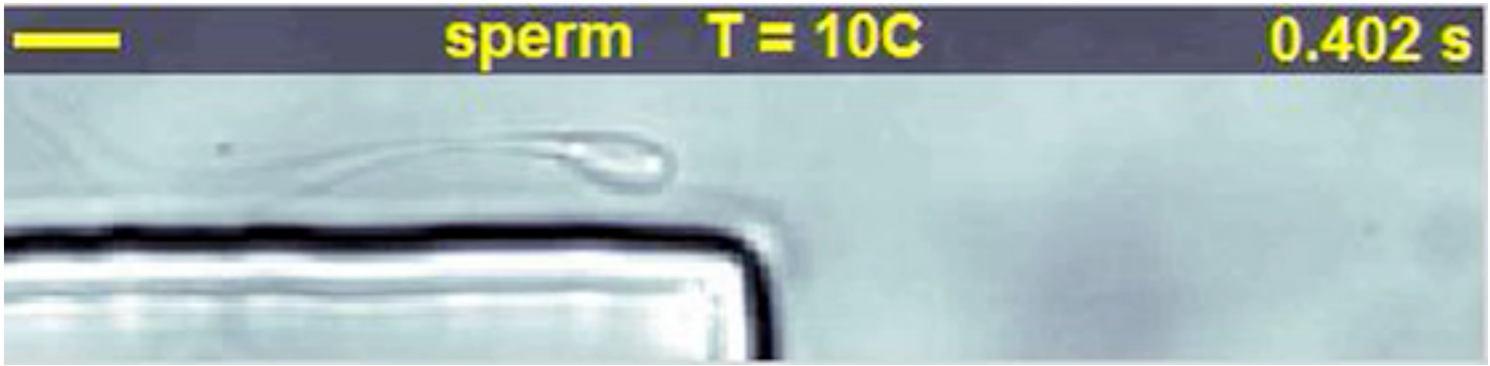
Control of algal locomotion

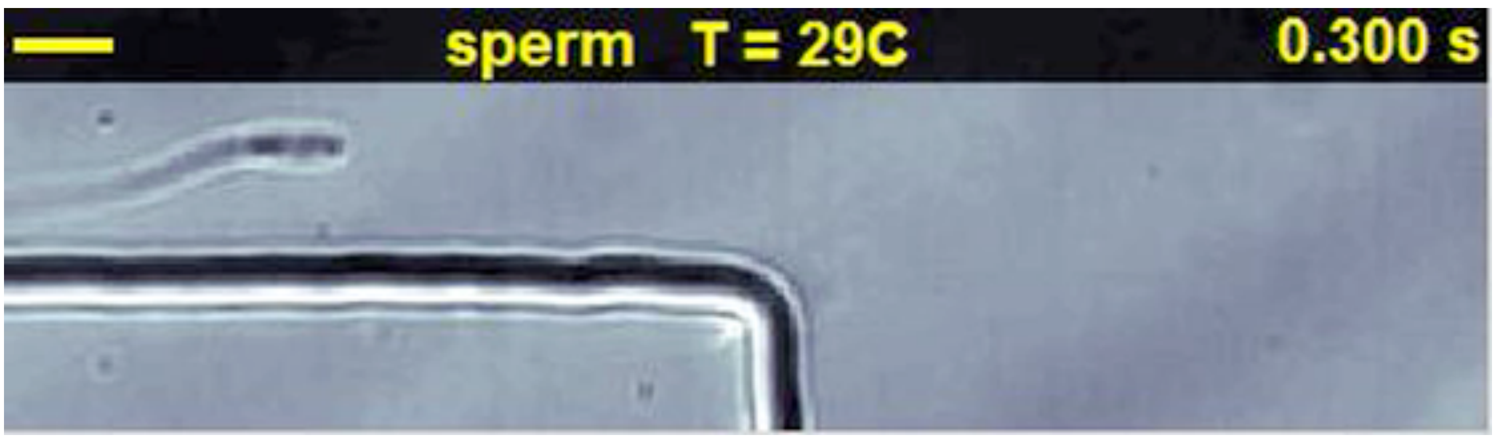


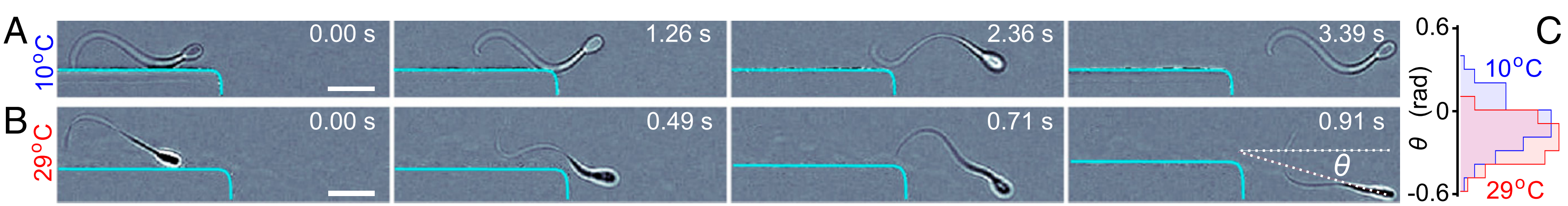


Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

Sperm near surfaces



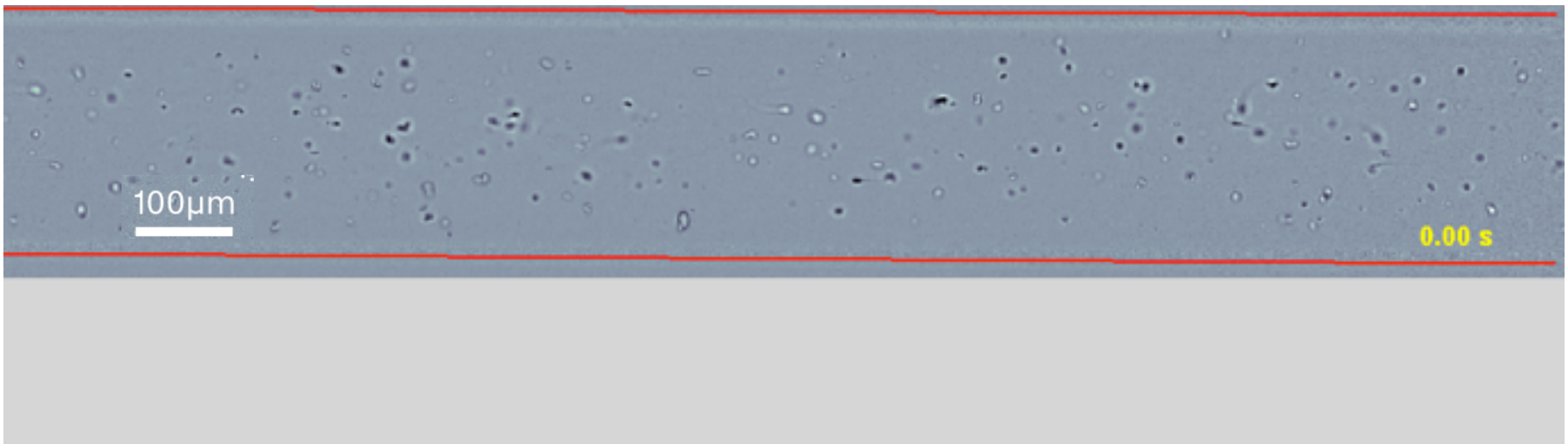




Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

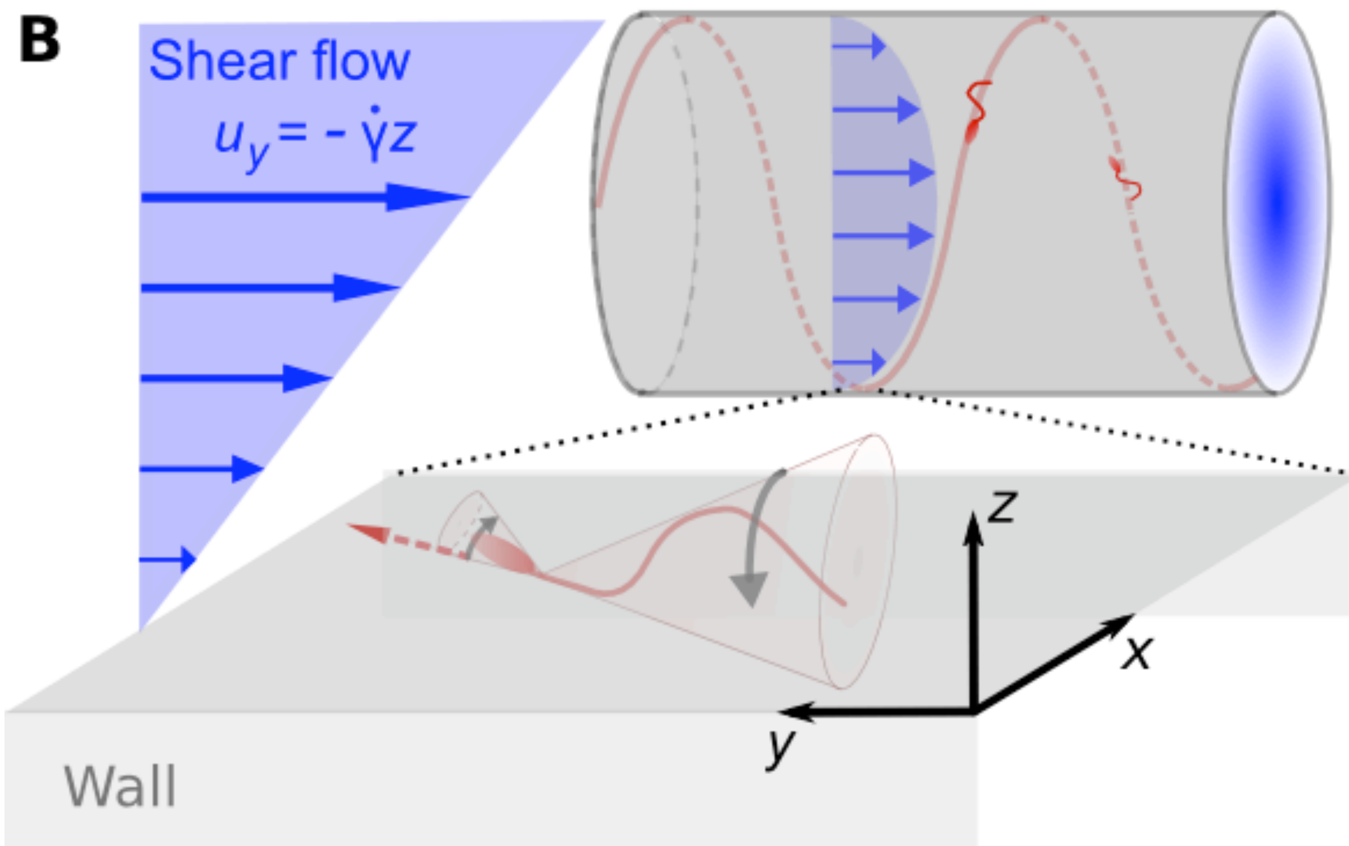
Sperm

Surface + shear flow



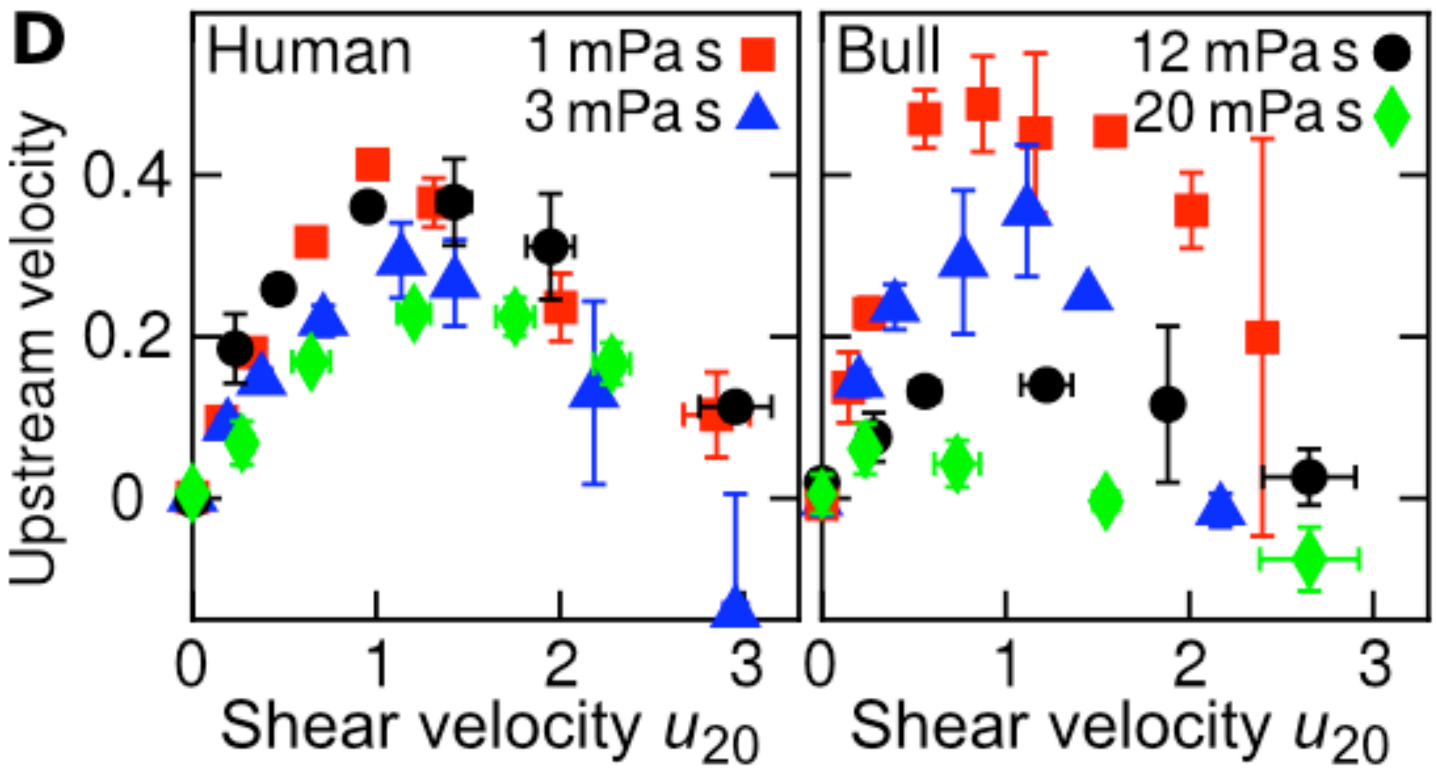
Kantsler et al 2014 (submitted)

Rheotaxis facilitates upstream navigation



Kantsler et al 2014 (submitted)

Viscosity & shear dependence



long distance navigation by rheotaxis ?

2D minimal model

Resistive force theory

$$
\begin{aligned}
0 &= F_i = \int_0^S ds \left\|\frac{d\hat{C}(s)}{ds}\right\| f_i(s), \\
0 &= \tau_i = \int_0^S ds \left\|\frac{d\hat{C}(s)}{ds}\right\| \epsilon_{ijk}[C_j(s) - X_j^*] f_k(s)
\end{aligned}
$$

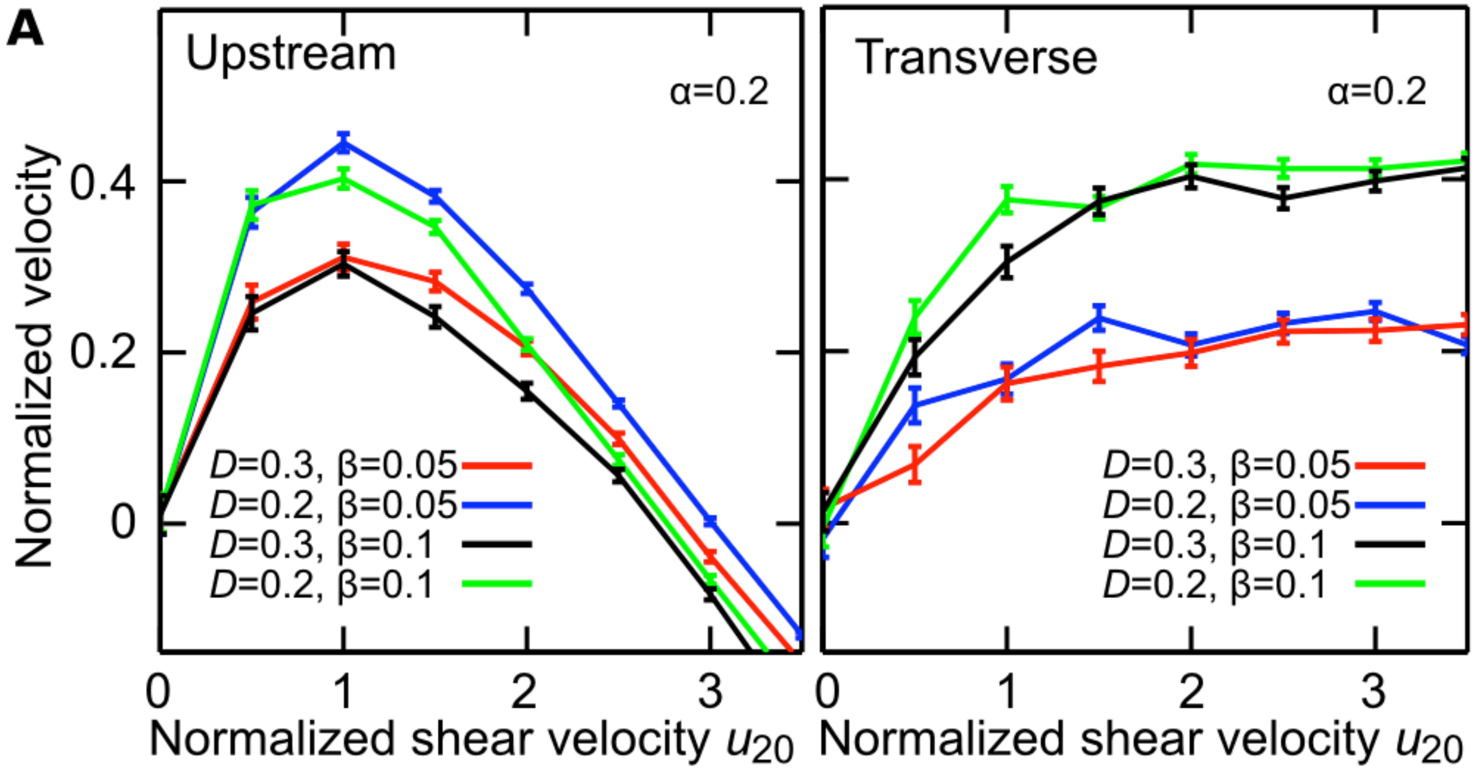
$$
\begin{aligned}
\boldsymbol{f}(s) = {} & \zeta_{||}\left\{\left[\boldsymbol{u}(\boldsymbol{C}(s)) - \dot{\boldsymbol{C}}(s)\right]\cdot \boldsymbol{t}(s)\right\}\boldsymbol{t}(s) + \\
& \zeta_{\perp}\left\{\left[\boldsymbol{u}(\boldsymbol{C}(s)) - \dot{\boldsymbol{C}}(s)\right]\cdot [\boldsymbol{I} - \boldsymbol{t}(s)\boldsymbol{t}(s)]\right\}
\end{aligned}
$$

+ some approximations + noise gives to leading order

$$
\begin{aligned}
\dot{\boldsymbol{R}} &= V\boldsymbol{N} + \sigma\overline{U}\boldsymbol{e}_y, \\
\dot{\boldsymbol{N}} &= \sigma\dot{\gamma}\alpha\begin{pmatrix} N_x N_y \\ N_y^2 - 1 \end{pmatrix} + \sigma\dot{\gamma}\chi\beta\begin{pmatrix} N_x^2 - 1 \\ N_x N_y \end{pmatrix} + (2D)^{1/2}(\boldsymbol{I} - \boldsymbol{N}\boldsymbol{N})\cdot\boldsymbol{\xi}(t).
\end{aligned}
$$

Kantsler et al 2014 (submitted)

2D minimal model



Kantsler et al 2014 (submitted)

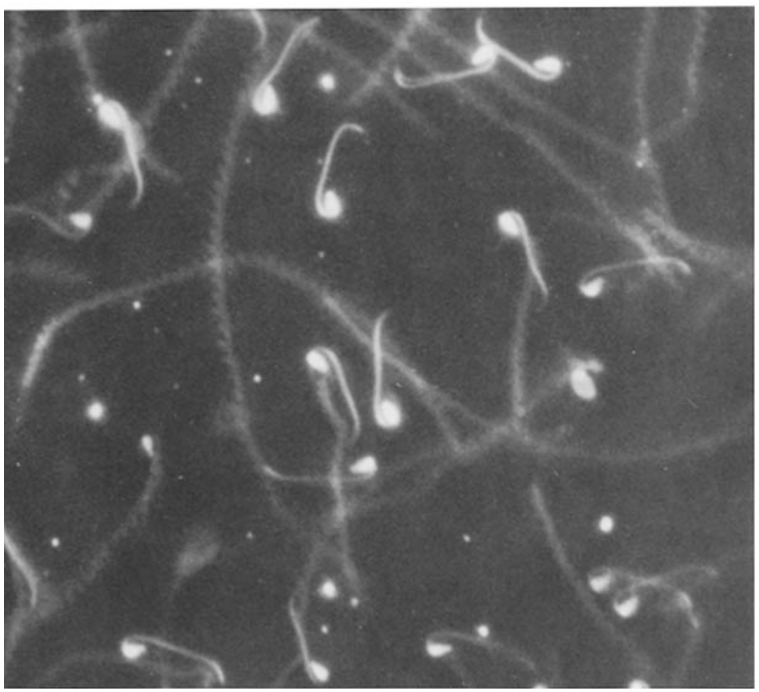
Collective motion

Broken reflection-symmetry at surfaces

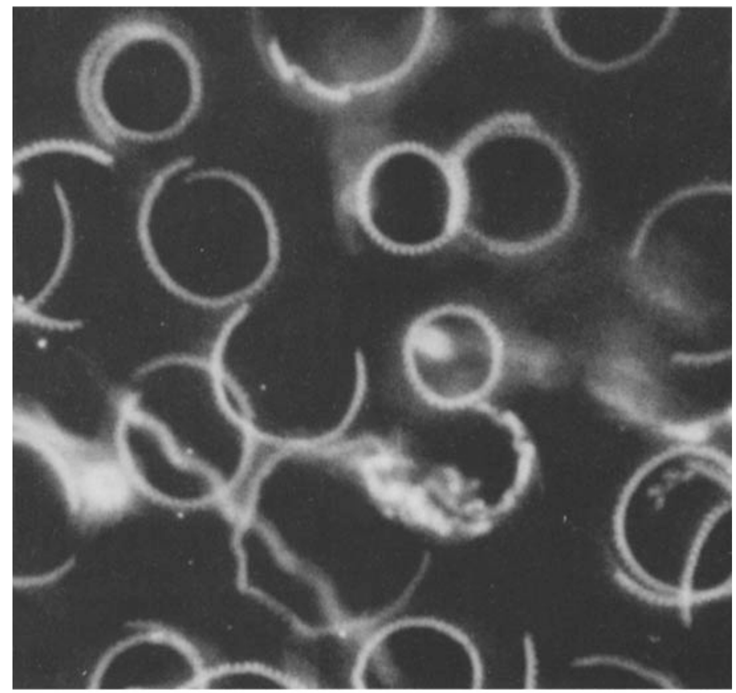
Gibbons (1980) JCB

Sea urchin sperm

in bulk (dilute)



near surface (dilute)



similar for bacteria (E. coli): Di Luzio et al (2005) Nature

2d Swift-Hohenberg model

arxiv: 1208.4464

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

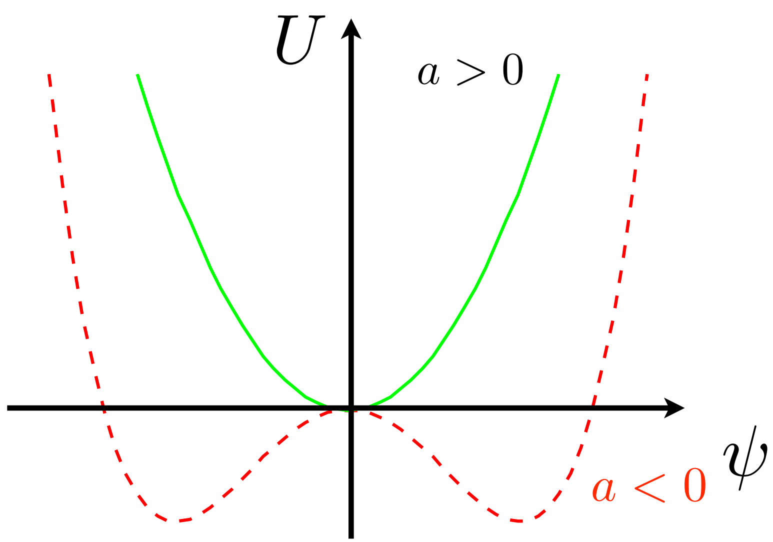
$$U(\psi) = \frac{a}{2}\psi^2 + \cancel{\frac{b}{3}\psi^3} + \frac{c}{4}\psi^4$$

$$\psi(t, \boldsymbol{x}) = \nabla \times \boldsymbol{v}$$

reflection-symmetry

$$b = 0$$

$$\psi \mapsto -\psi$$



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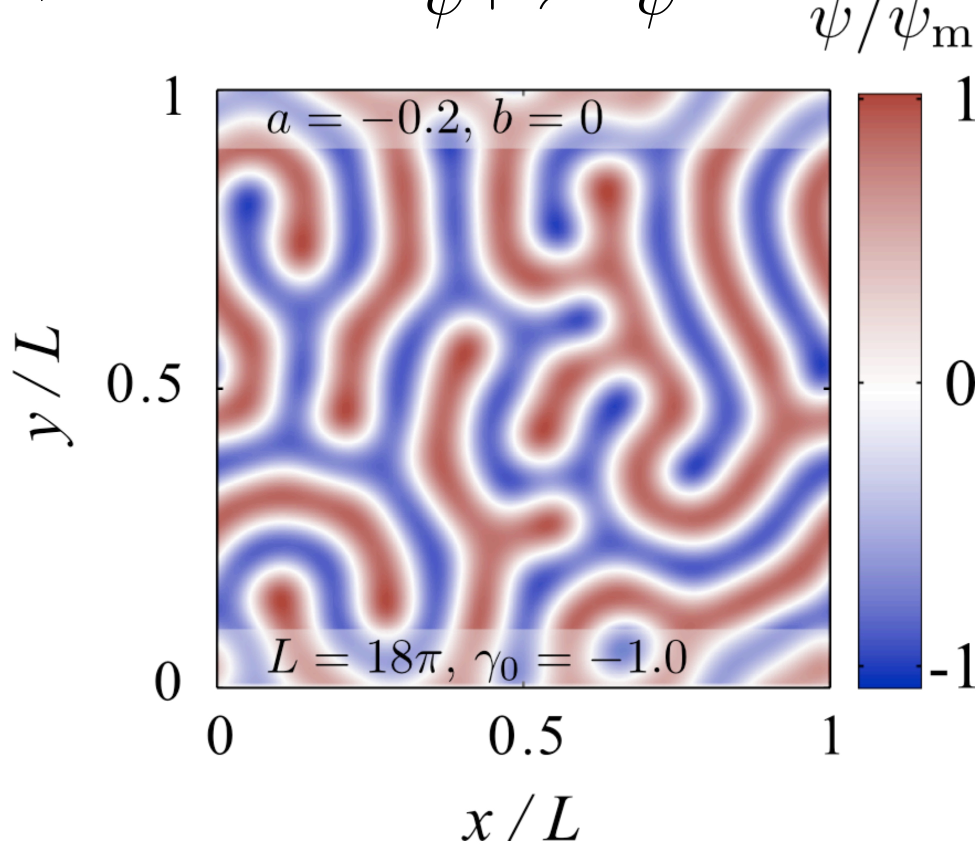

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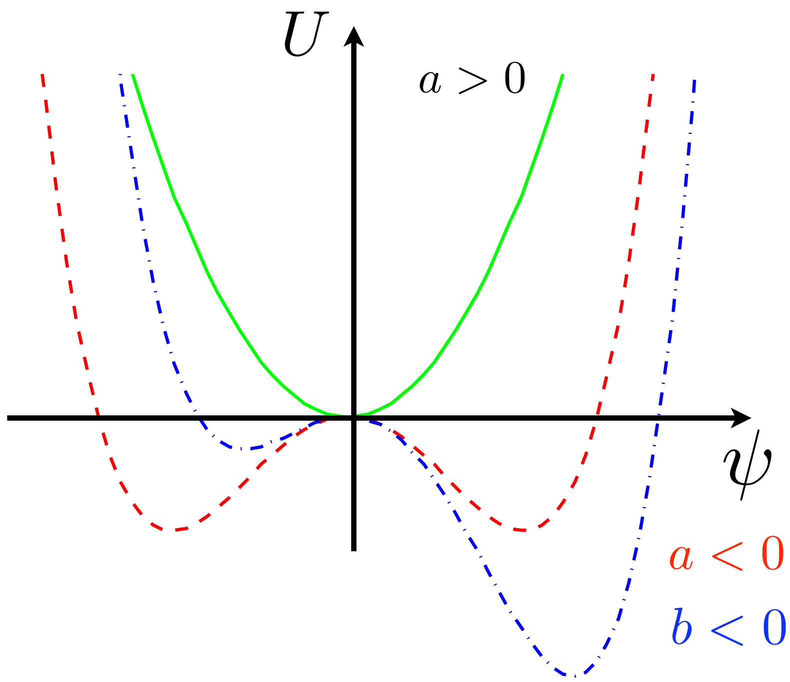
broken

reflection-symmetry

$$b \neq 0$$

$$\psi \not\mapsto -\psi$$



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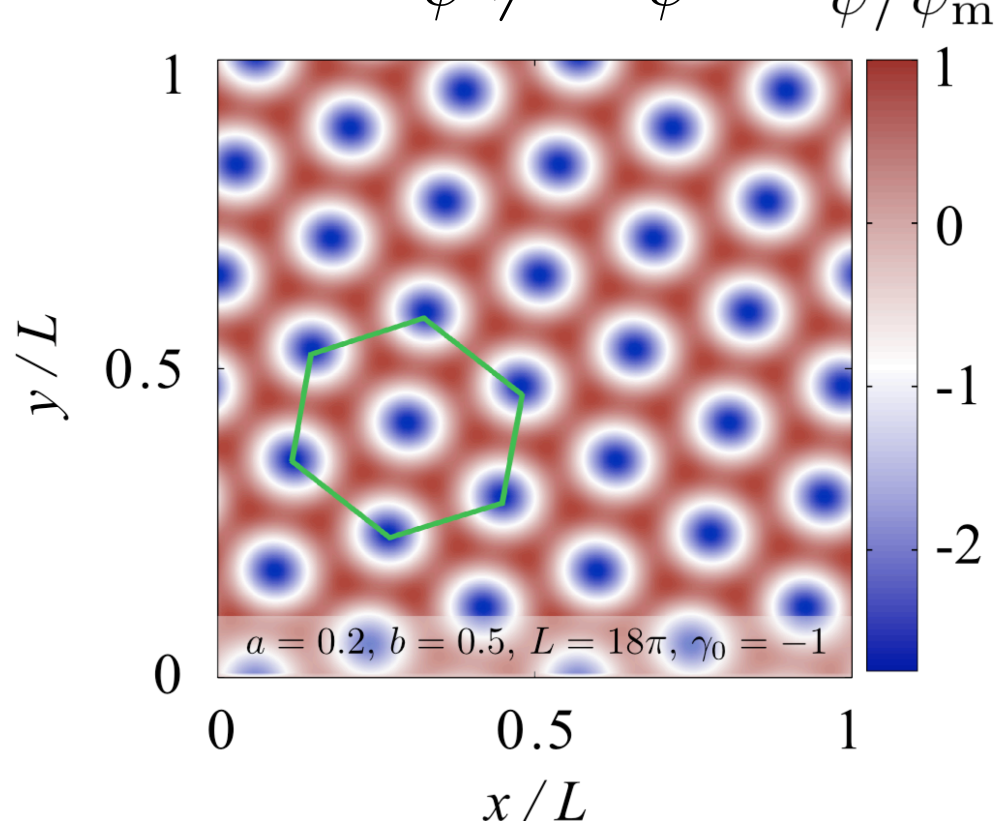
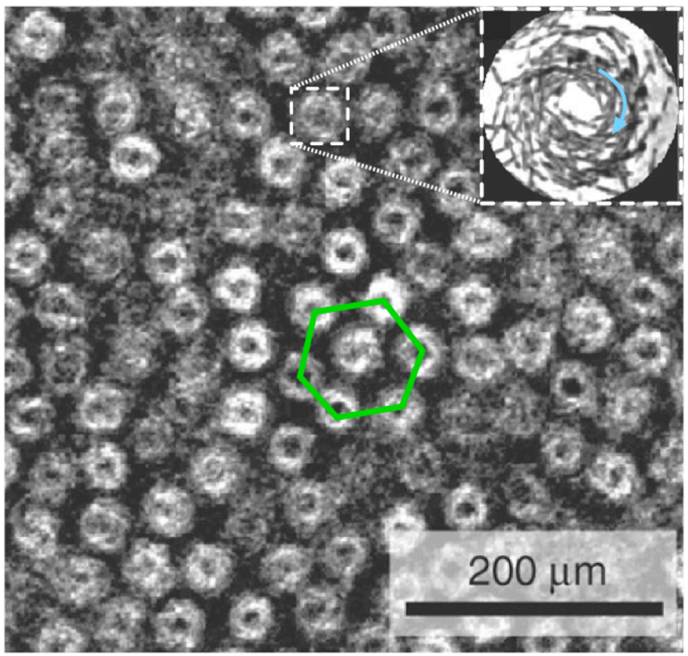


2d Swift-Hohenberg model

arxiv: 1208.4464

Sea urchin sperm cells
near surface
(high concentration)

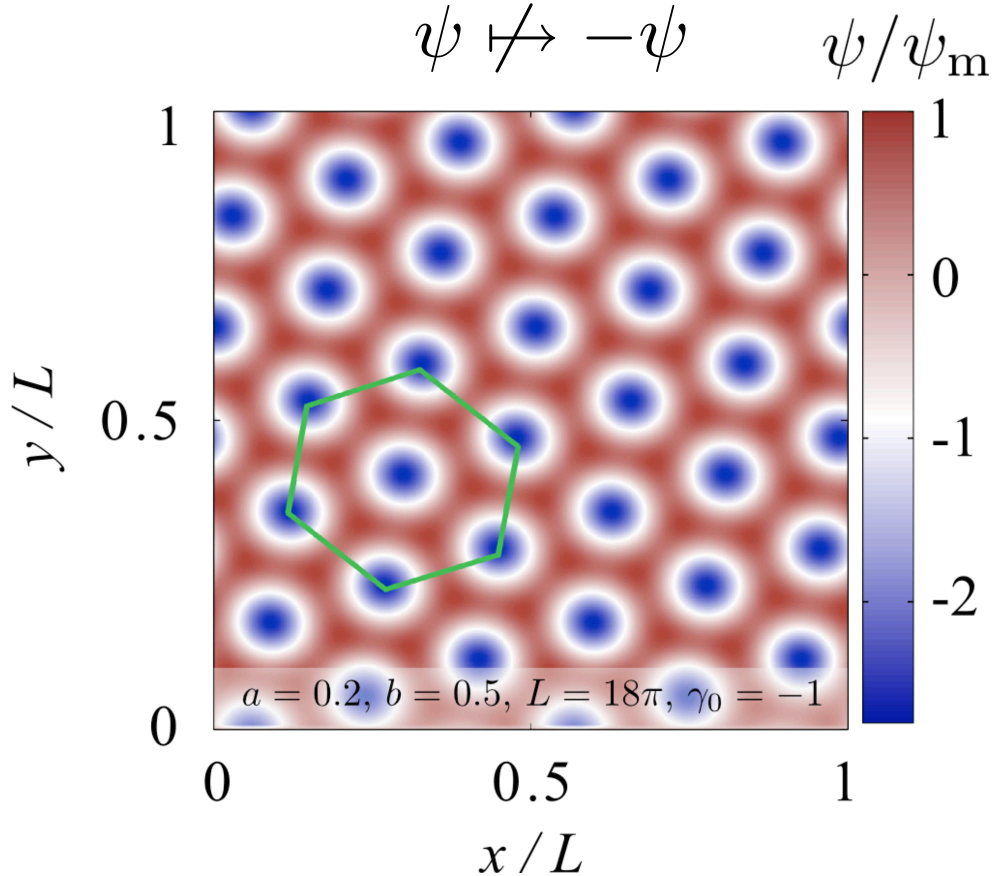


Riedel et al (2007) Science

broken
reflection-symmetry

$$b \neq 0$$

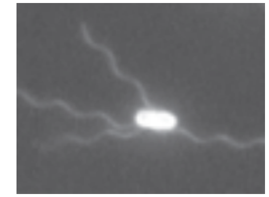
$$\psi \not\mapsto -\psi$$

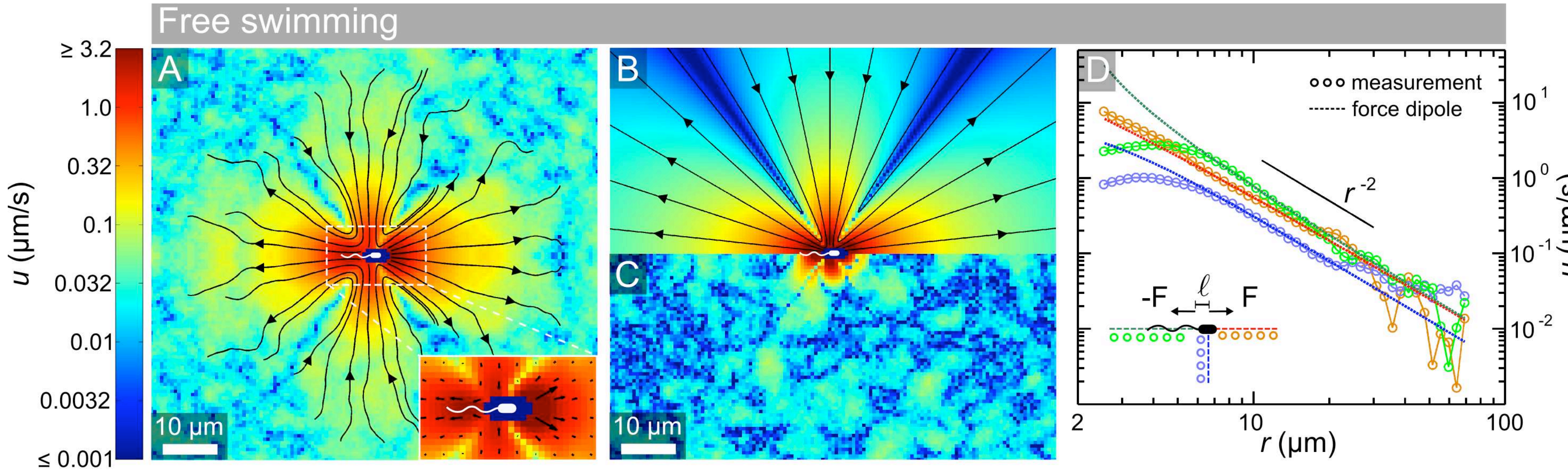


Active polar fluids

(things with a head and tail)

E.coli (non-tumbling HCB 437)





$$\boxed{\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}}$$

$$V_0 = 22 \pm 5 \ \ \mu\mathrm{m/s}$$

$$\ell = 1.9 \ \mu\mathrm{m}$$

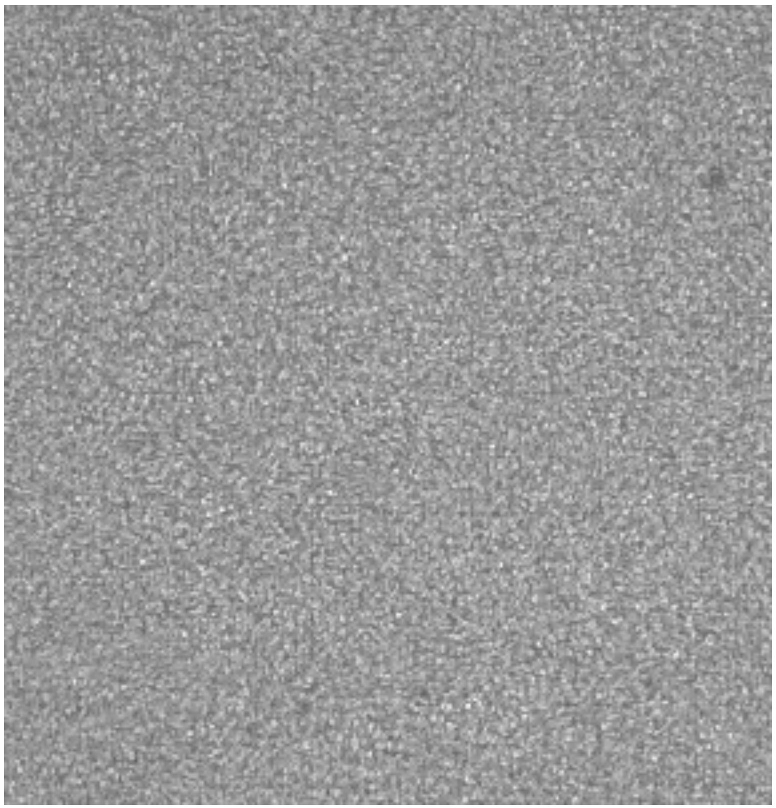
$$F = 0.42 \ \mathrm{pN}$$

‘pusher’ dipole

Drescher et al (2011) PNAS

Active polar fluids

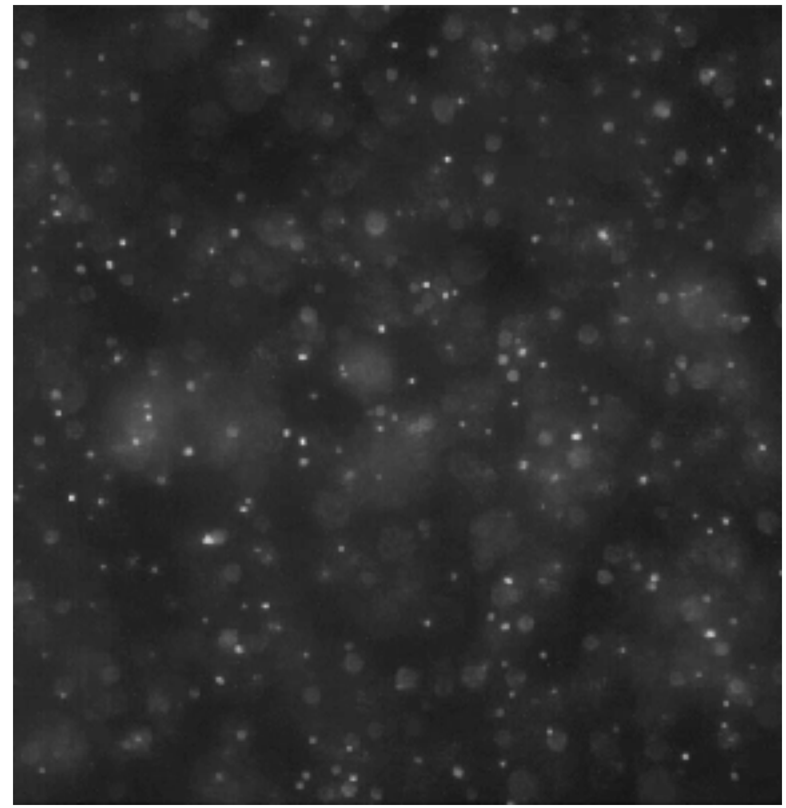
B. subtilis



bright field

Wensink et al PNAS 2012

tracers

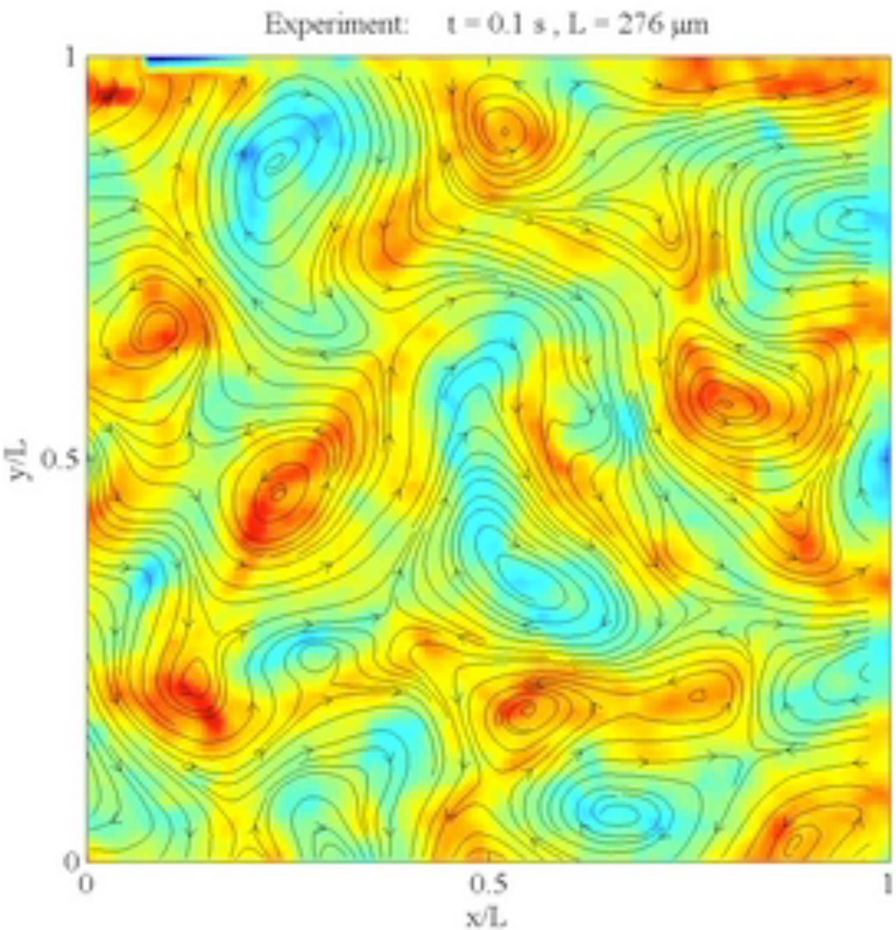


fluorescence

Dunkel et al PRL 2013

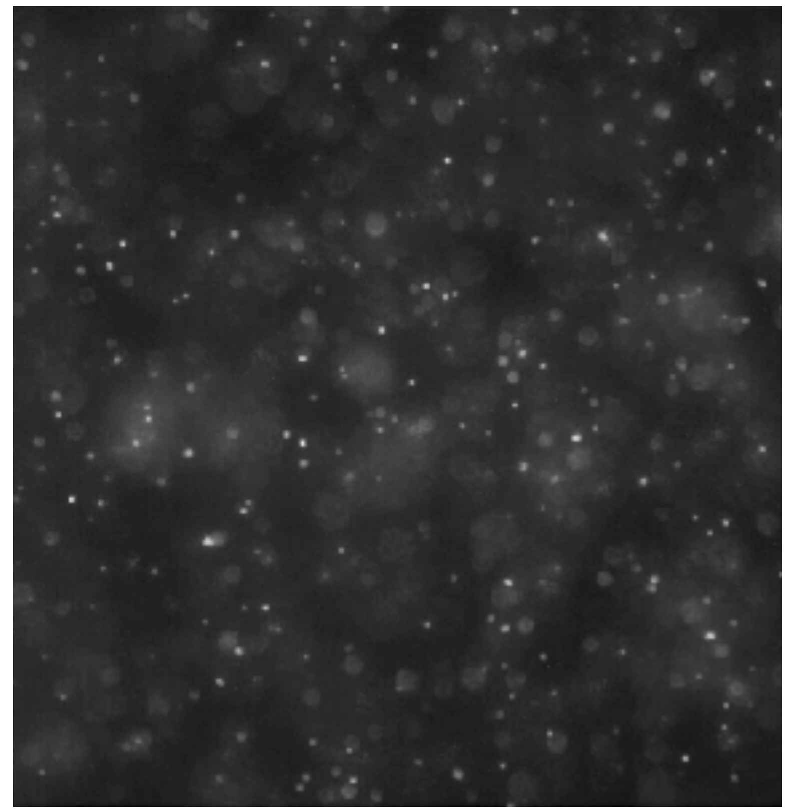
Bacterial 'turbulence'

PIV



Vortex diameter ~ 70µm
Vortex life time ~ 1 sec

tracers



fluorescence

Dunkel et al PRL 2013

Minimal continuum theory for bacterial velocity field

incompressibility $\nabla \cdot \boldsymbol{v} = 0$

$\alpha < 0$
polar alignment

nematic stresses

$$(\partial_t + \lambda_0 \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla(p + \lambda_1 \boldsymbol{v}^2) - (\beta \boldsymbol{v}^2 + \alpha)\boldsymbol{v} + \\ + \Gamma_0 \nabla^2 \boldsymbol{v} - \Gamma_2 (\nabla^2)^2 \boldsymbol{v}$$

vortices

PNAS 2012 New J Phys 2013 PRL 2013

Isotropic fixed-point $\quad (p, \boldsymbol{v}) = (p_0, \mathbf{0})$

$$0 = \boldsymbol{k} \cdot \boldsymbol{\varepsilon}$$

$$\sigma \boldsymbol{\varepsilon} = -i\boldsymbol{k}\eta - \alpha\boldsymbol{\varepsilon} - \Gamma_0 k^2 \boldsymbol{\varepsilon} - \Gamma_2 k^4 \boldsymbol{\varepsilon}$$

$$(\eta, \boldsymbol{\varepsilon}) = (\hat{\eta}, \hat{\boldsymbol{\varepsilon}}) e^{i\boldsymbol{k}\cdot\boldsymbol{x} + \sigma t}$$

$$\sigma(\boldsymbol{k}) = -(\alpha + \Gamma_0 k^2 + \Gamma_2 k^4)$$

Polar fixed-point

$$(p, \boldsymbol{v}) = (p_0, \boldsymbol{v}_0)$$

$$\begin{aligned} 0 &= \boldsymbol{k} \cdot \hat{\varepsilon}, \\ \sigma\, \hat{\varepsilon} &= -i(\hat{\eta} - 2v_0\lambda_1\hat{\varepsilon}_{||})\boldsymbol{k} + \boldsymbol{A}\hat{\varepsilon} \end{aligned}$$

$$|\boldsymbol{v}_0| = \sqrt{|\alpha|/\beta}$$

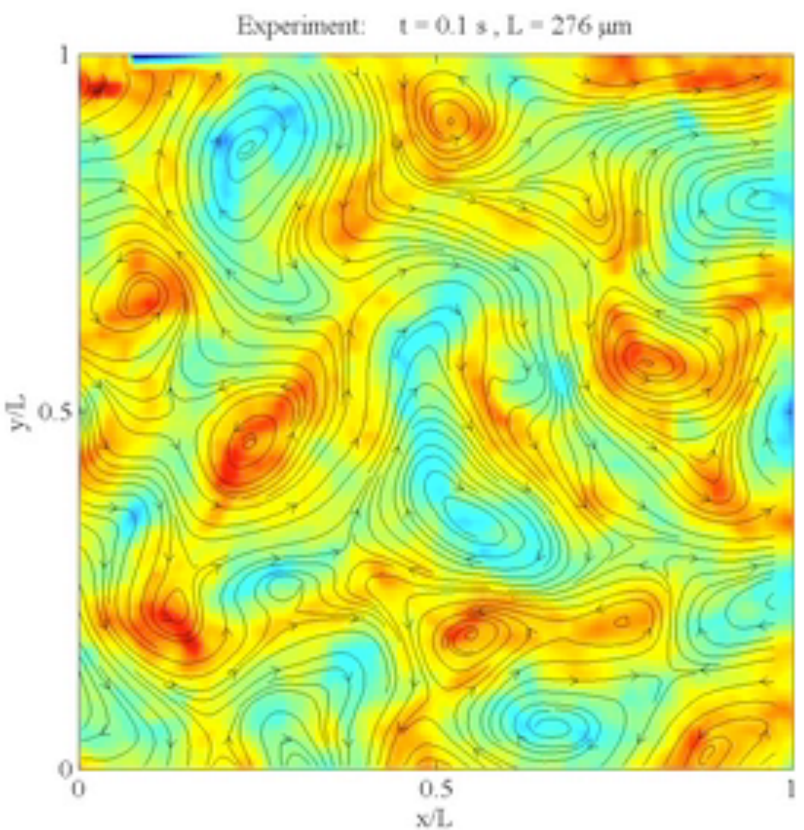
$$\lambda_0 = 1 - S$$

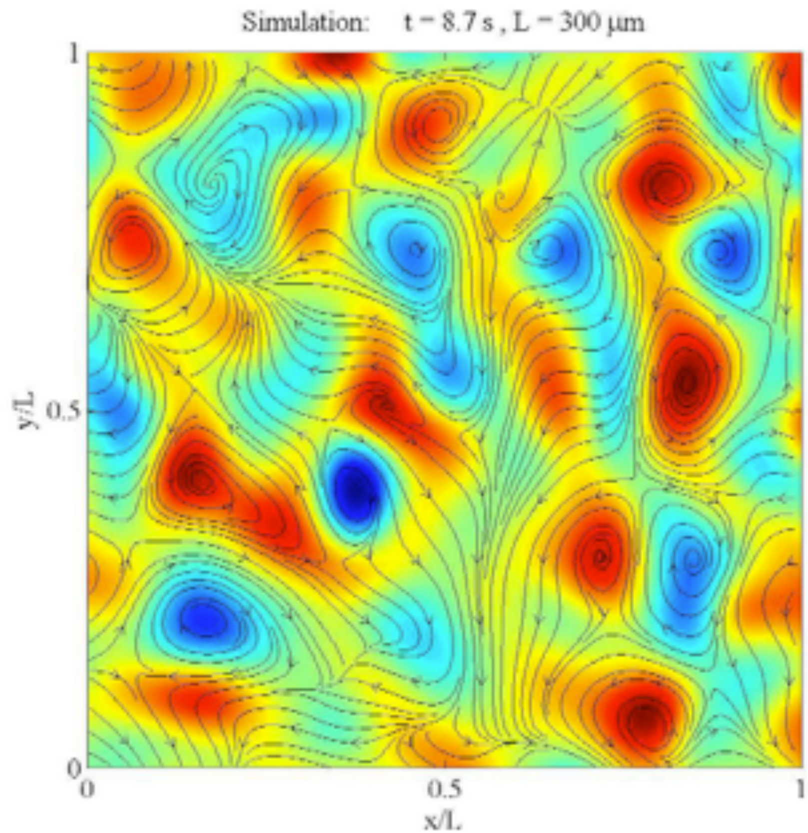
$$\boldsymbol{A} = \begin{pmatrix} 2\alpha & 0 \\ 0 & 0 \end{pmatrix} - (\Gamma_0 k^2 + \Gamma_2 k^4 + i\lambda_0 k_x v_0)\boldsymbol{I}$$

Eigenvalues of $\boldsymbol{A}_\perp = \Pi(k)\boldsymbol{A}$ determine stability

$$\sigma(\boldsymbol{k}) \in \left\{0, -\left(\Gamma_0 k^2 + \Gamma_2 k^4 - 2\alpha \frac{k_x^2}{k^2}\right) - i\lambda_0 v_0 k_x \right\}.$$

experiment *vs.* theory





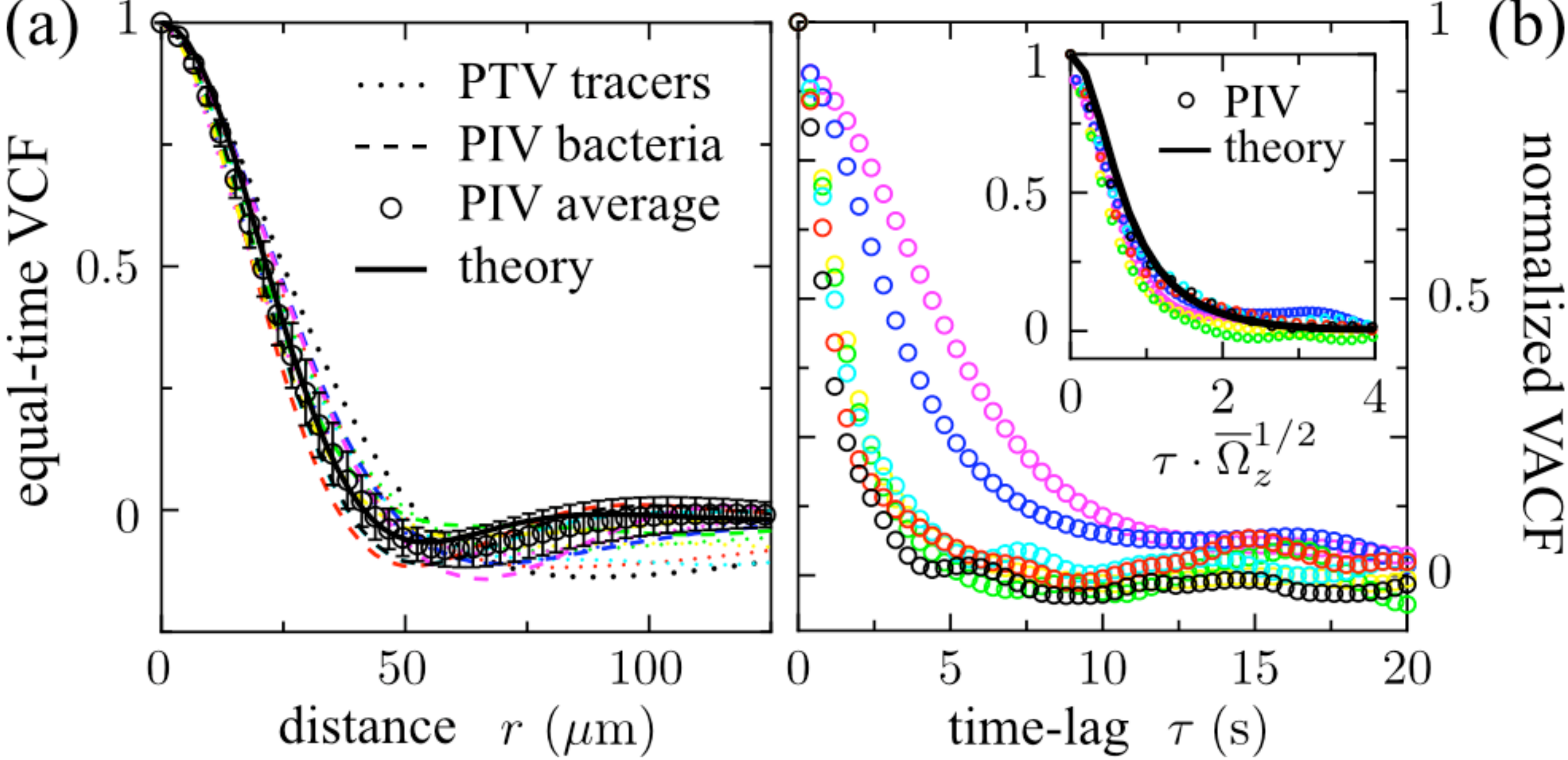
quasi-2D slice

2D slice
from 3D simulation

Dunkel et al PRL 2013

Velocity correlations

$$\Omega_z(t) = \langle \omega_z^2/2 \rangle$$



Vortex diameter ~ 70μm Vortex life time ~ seconds

Dunkel et al PRL 2013

Continuum theory for bacterial velocity field

incompressibility $\nabla \cdot \boldsymbol{v} = 0$

non-conservative

$$(\partial_t + \lambda_0 \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla(p + \lambda_1 \boldsymbol{v}^2) - (\beta \boldsymbol{v}^2 + \alpha)\boldsymbol{v} + \\ + \Gamma_0 \nabla^2 \boldsymbol{v} - \Gamma_2 (\nabla^2)^2 \boldsymbol{v}$$

Conserved dynamics ?

Flow equations

$$\begin{aligned} 0 &= \nabla \cdot \boldsymbol{v} \\ \partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} &= -\nabla p + \nabla \cdot \boldsymbol{\sigma} \end{aligned}$$

with stress tensor

$$\boldsymbol{\sigma} = [\Gamma_0 - \Gamma_2(\nabla^2) + \Gamma_4(\nabla^2)^2](\nabla^\top \boldsymbol{v} + \nabla \boldsymbol{v}^\top)$$

Interpretation: effective flow-field for
passive solvent + active component
that creates non-local stresses

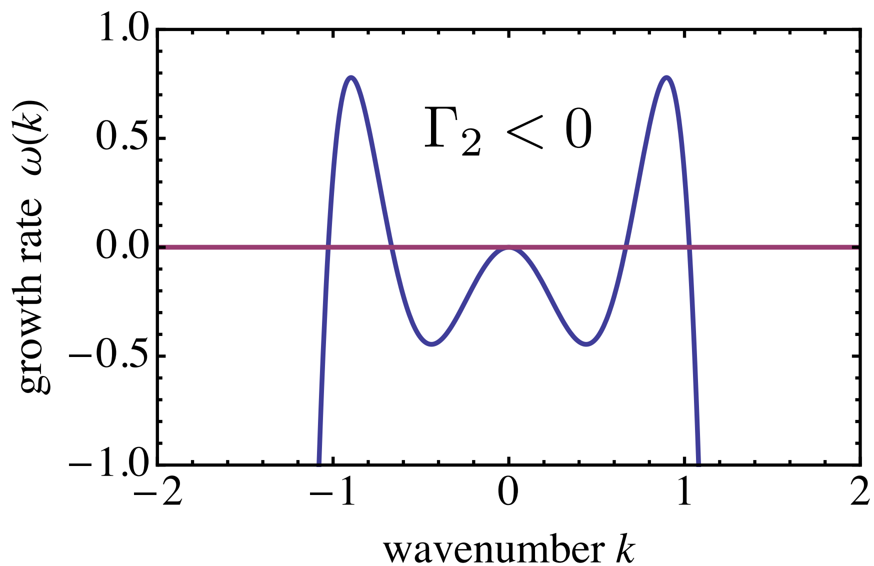
Conserved dynamics ?

Flow equations

$$
\begin{aligned}
0 &= \nabla \cdot \boldsymbol{v} \\
\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} &= -\nabla p + \nabla \cdot \boldsymbol{\sigma}
\end{aligned}
$$

with stress tensor

$$
\boldsymbol{\sigma} = [\Gamma_0 - \Gamma_2(\nabla^2) + \Gamma_4(\nabla^2)^2](\nabla^\top \boldsymbol{v} + \nabla \boldsymbol{v}^\top)
$$



linear stability

Conserved dynamics ?

Flow equations

$$
\begin{aligned}
0 &= \nabla \cdot \boldsymbol{v} \\
\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} &= -\nabla p + \nabla \cdot \boldsymbol{\sigma}
\end{aligned}
$$

with stress tensor

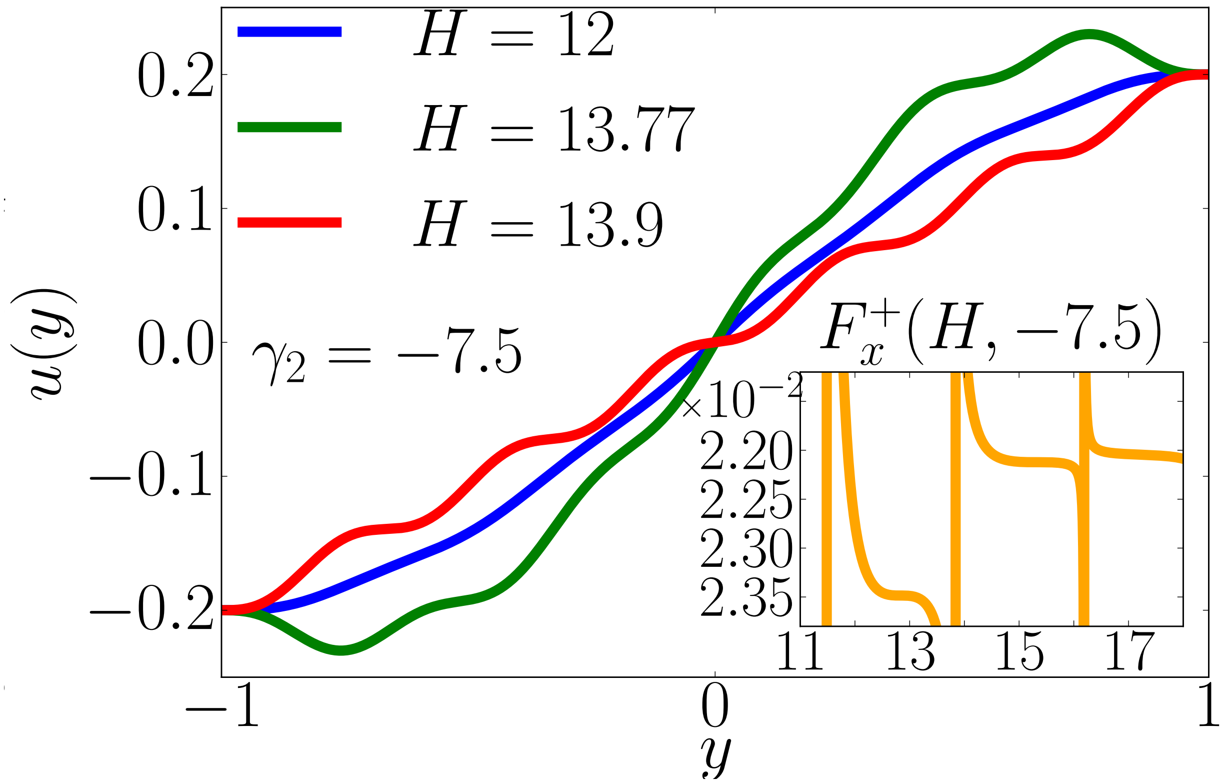
$$
\boldsymbol{\sigma} = [\Gamma_0 - \Gamma_2(\nabla^2) + \Gamma_4(\nabla^2)^2](\nabla^\top \boldsymbol{v} + \nabla \boldsymbol{v}^\top)
$$

6th order PDE

S-type: First and second-order derivatives vanish.

W-type: Second and fourth-order derivatives vanish.

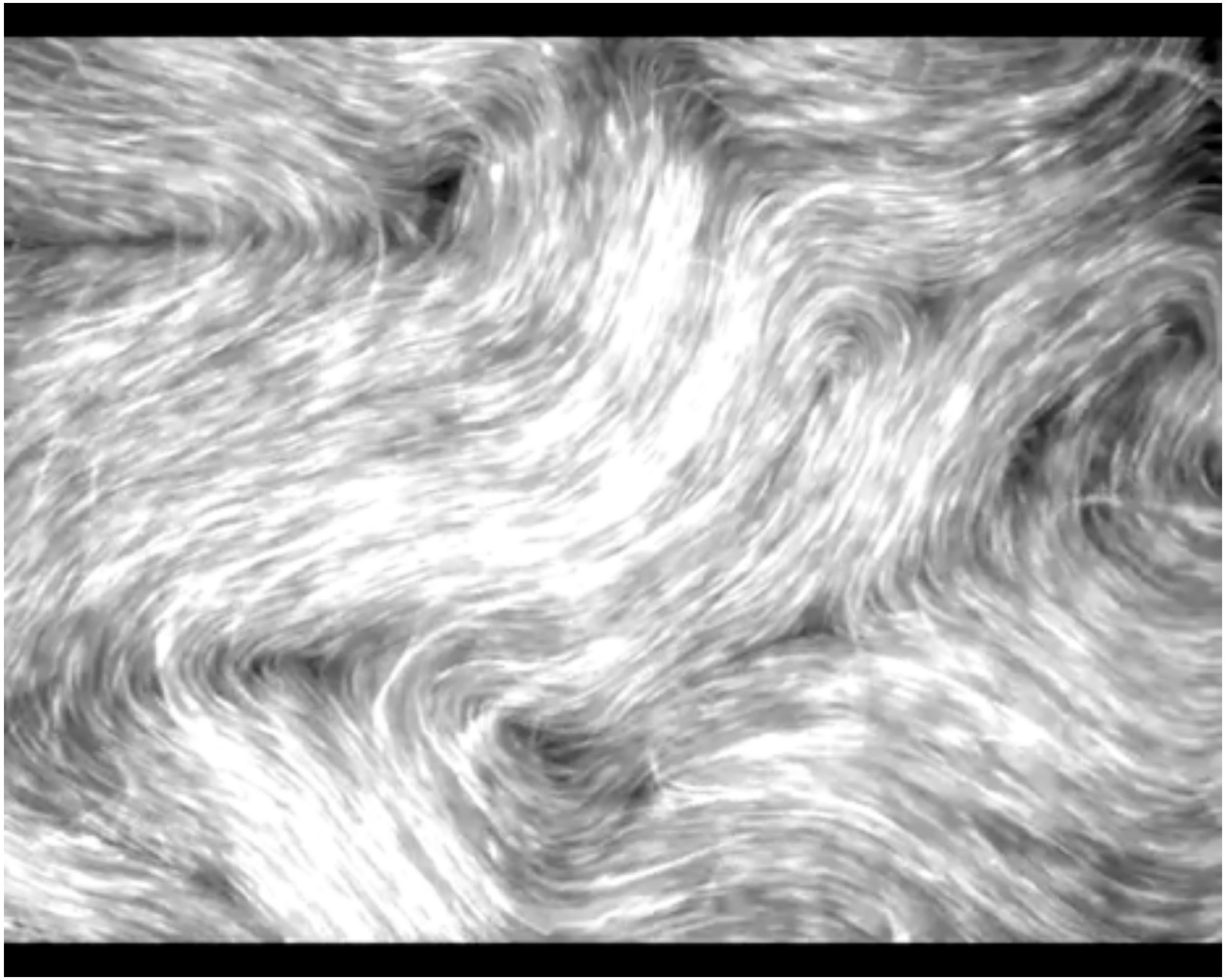
Mean field prediction for shear flow between two plates



$$u'(\pm H/2) = u''(\pm H/2) = 0$$

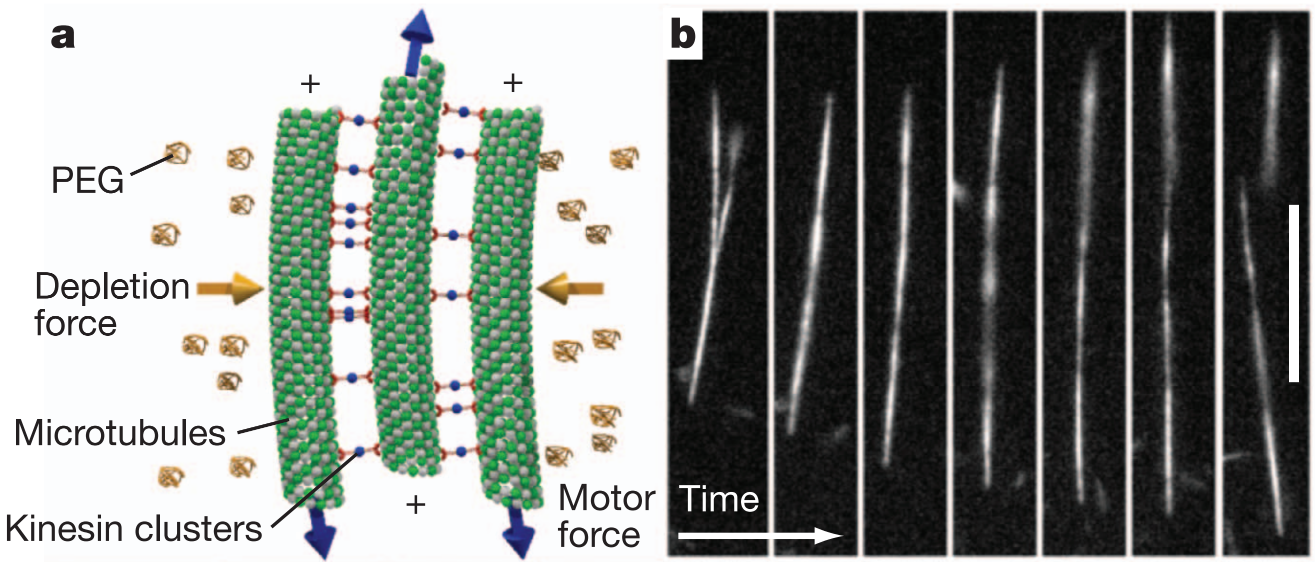
+ periodic BCs in other directions

Active nematics



Dogic lab (Brandeis) Nature 2012

Active nematics



Dogic lab (Brandeis) Nature 2012

no head or tail ⇒ Q-tensor order-parameter

$$Q_{ij} = Q_{ji} \ , \qquad \mathrm{Tr}\, Q = 0 \qquad Q = \begin{pmatrix} \lambda & \mu \\ \mu & -\lambda \end{pmatrix} .$$

$$\Delta = \sqrt{\lambda^2 + \mu^2}, \qquad \Lambda^{\pm} = \pm \Delta$$

Active nematics

$$u_{ij} = (\partial_i v_j + \partial_j v_i)/2$$

$$\omega_{ij} = (\partial_i v_j - \partial_j v_i)/2$$

$$D_{ij} = D_0 \delta_{ij} + D_1 Q_{ij}$$

$$\boxed{\begin{aligned} \frac{Dc}{Dt} &= \partial_i [D_{ij} \partial_j c + \alpha_1 c^2 \partial_j Q_{ij}], \\ \rho \frac{Dv_i}{Dt} &= \eta \nabla^2 v_i - \partial_i p + \partial_j \sigma_{ij}, \\ \frac{DQ_{ij}}{Dt} &= \lambda S u_{ij} + Q_{ik} \omega_{kj} - \omega_{ik} Q_{kj} + \gamma^{-1} H_{ij}, \end{aligned}}$$

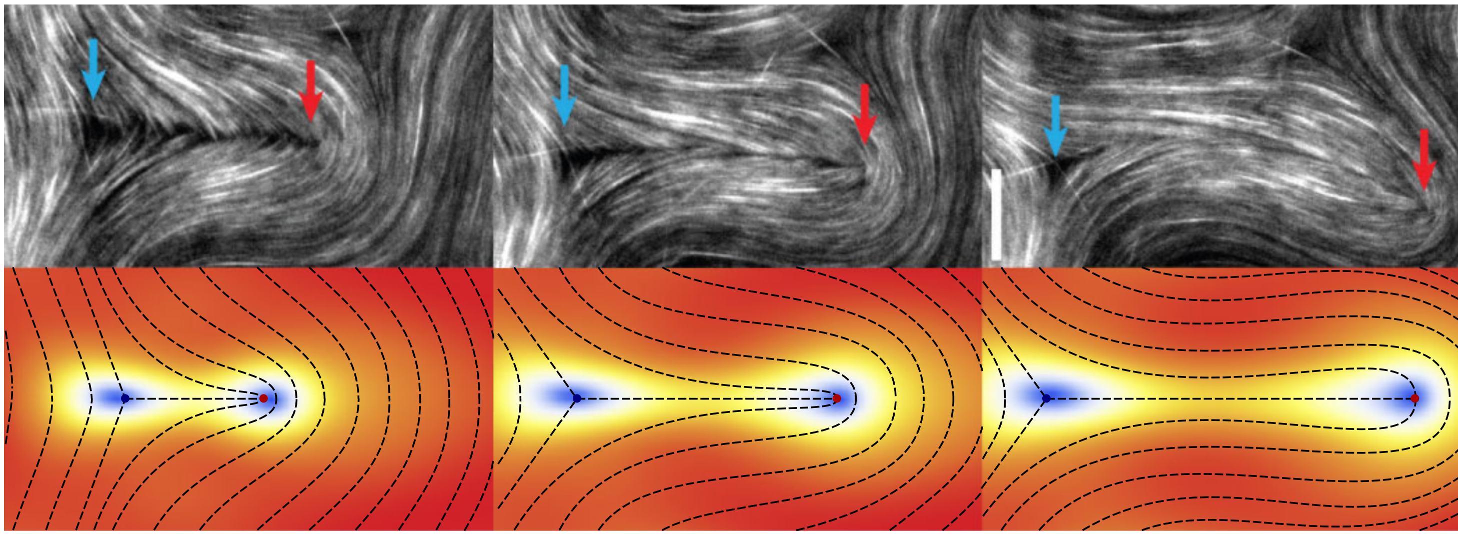
$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0,$$

$$H_{ij} = -\delta F / \delta Q_{ij},$$

$$F/K = \int dA \left[\frac{1}{4}(c - c^{\star}) \mathrm{tr} \boldsymbol{Q}^2 + \frac{1}{4} c (\mathrm{tr} \boldsymbol{Q}^2)^2 + \frac{1}{2} |\nabla \boldsymbol{Q}|^2 \right],$$

Giomi et al PRL 2012

Active nematics



$\nabla \cdot \boldsymbol{v} = 0,$

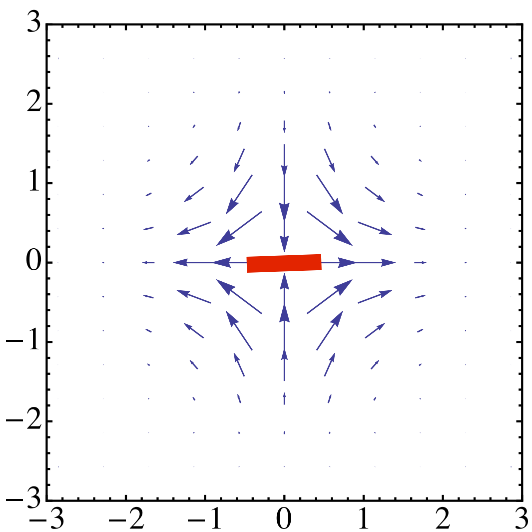
not consistent with experimental setup

Giomi et al PRL 2012

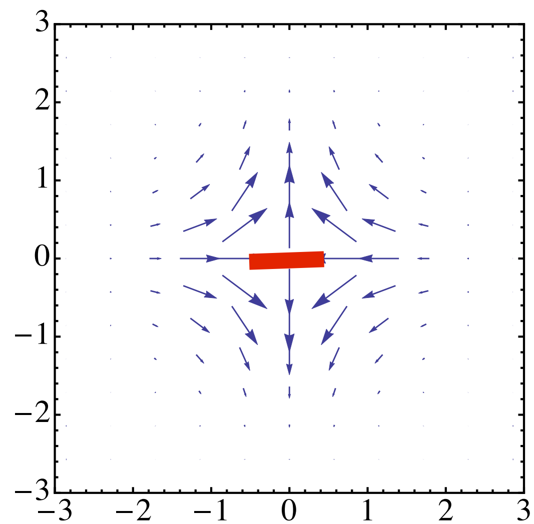
Alternative approach

$$\partial_t Q_{ij} + v_k \partial_k Q_{ij} = -\frac{\delta \mathcal{F}}{\delta Q_{ij}}$$

$$v_k \;=\; D\,\partial_n Q_{nk}$$



D<0 extensile



D>0 contractile

Alternative approach

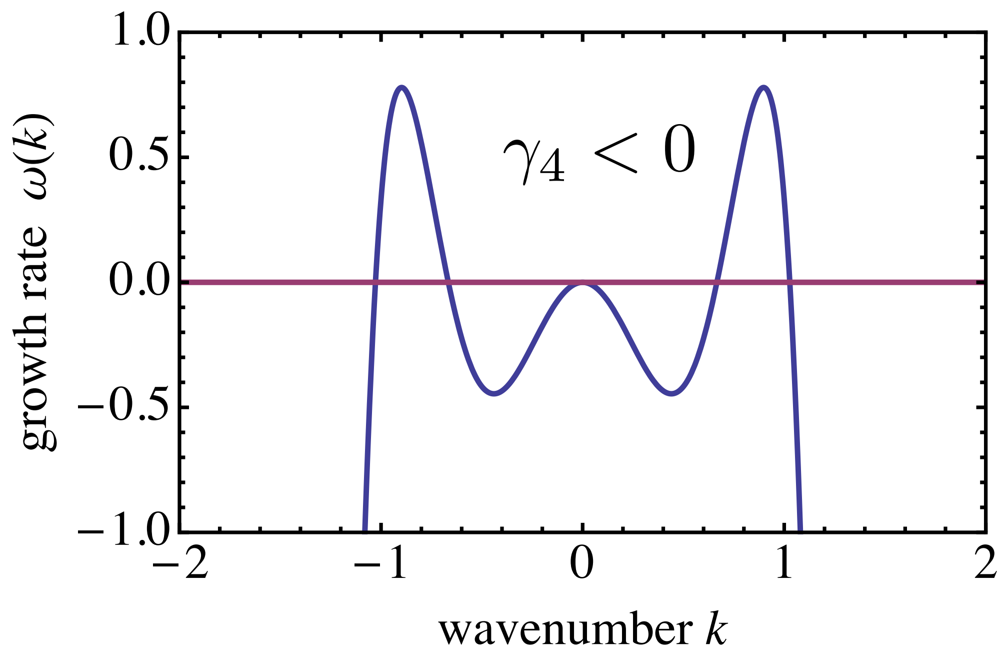
$$\partial_t Q_{ij} + v_k \partial_k Q_{ij} = -\frac{\delta \mathcal{F}}{\delta Q_{ij}}$$

$$v_k \;=\; D\,\partial_n Q_{nk}$$

$$\boxed{\begin{aligned}\partial_t Q + D[(\nabla \cdot Q) \cdot \nabla]Q \;=\; & -aQ - bQ^3 + \\ & \gamma_2 \nabla^2 Q - \gamma_4 (\nabla^2)^2 Q + \gamma_6 (\nabla^2)^3 Q\end{aligned}}$$

Alternative approach

$$\boxed{\begin{aligned}\partial_t Q + D[(\nabla\cdot Q)\cdot\nabla]Q \;=\; & -aQ - bQ^3 + \\ & \gamma_2\nabla^2 Q - \gamma_4(\nabla^2)^2 Q + \gamma_6(\nabla^2)^3 Q\end{aligned}}$$

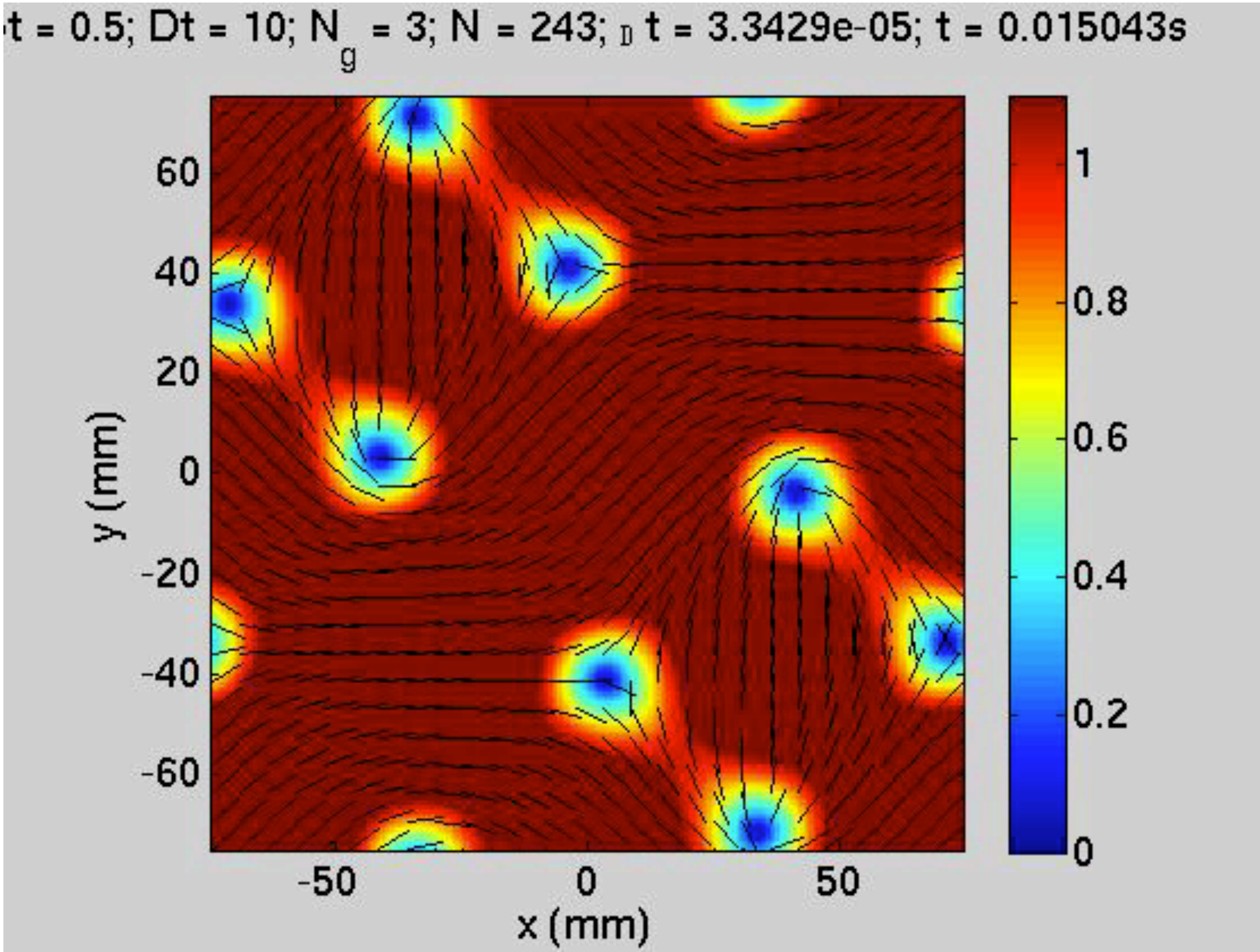


linear stability

Chaos possible?

Prelim. simulation results

$D<0$ extensile



Prelim. simulation results

D>0 contractile