Beyond diffusion: Patterns

18.354 - L5

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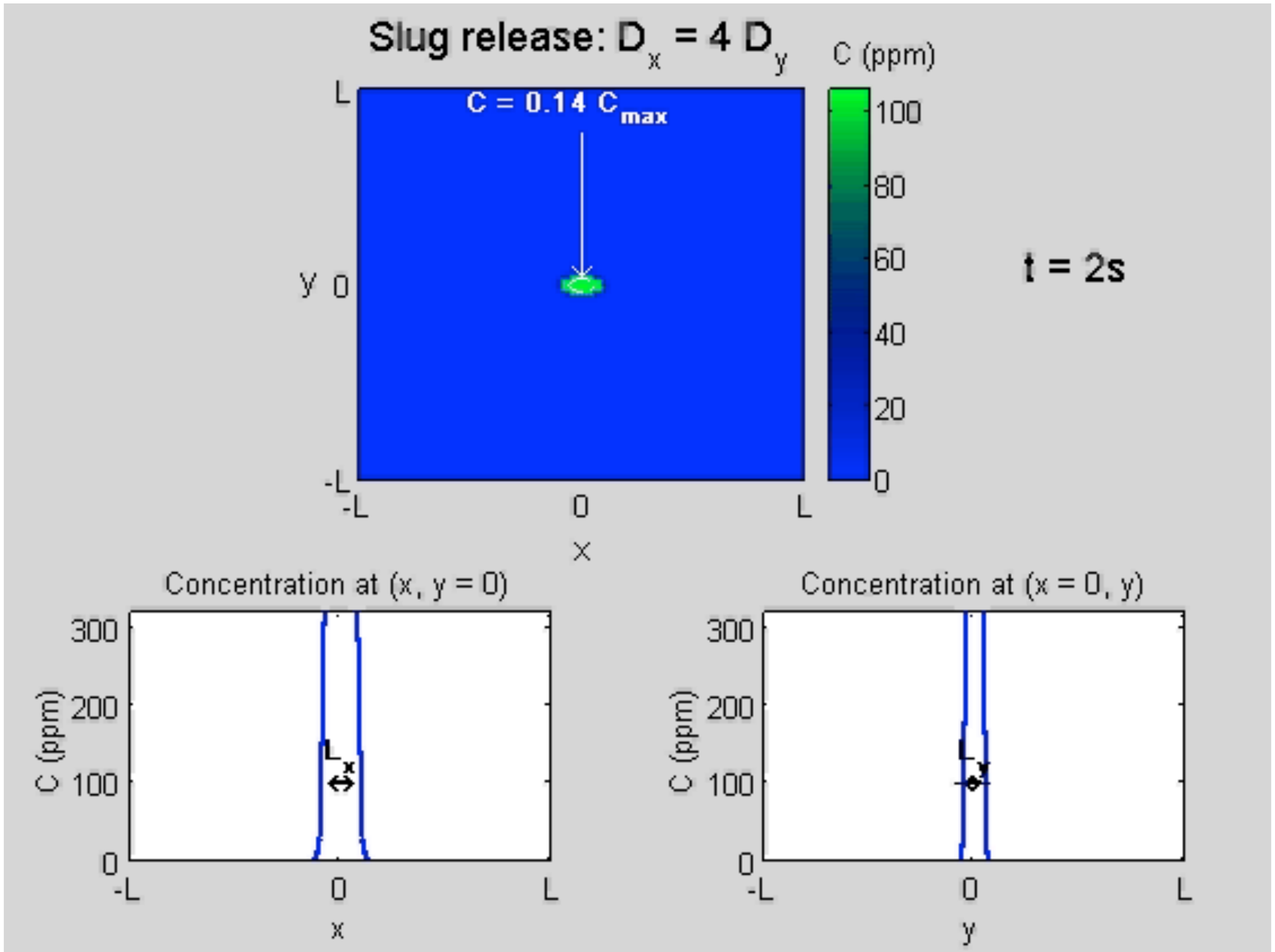
2D diffusion equation



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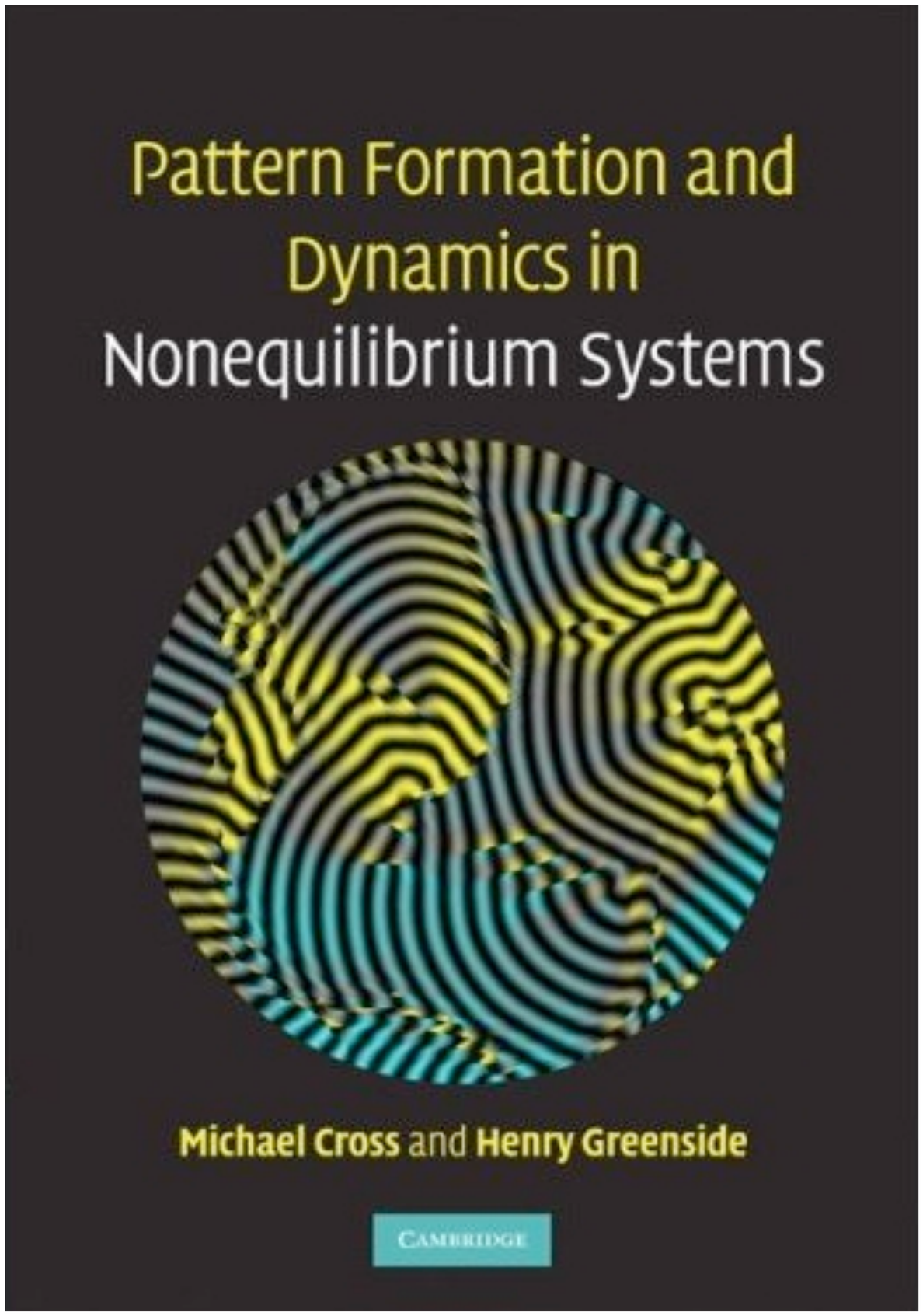

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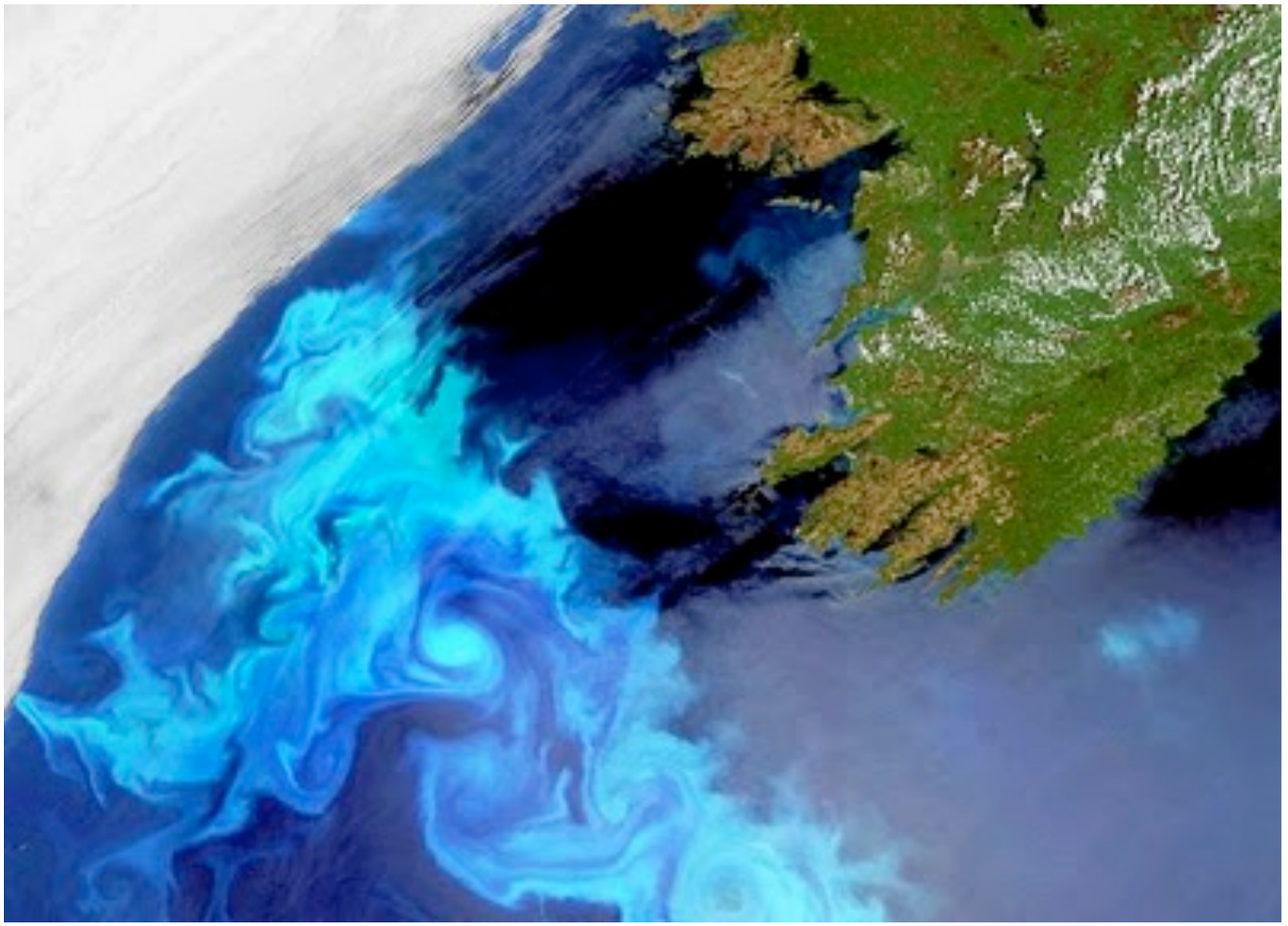
Plankton



ESA cost of Ireland

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Animals

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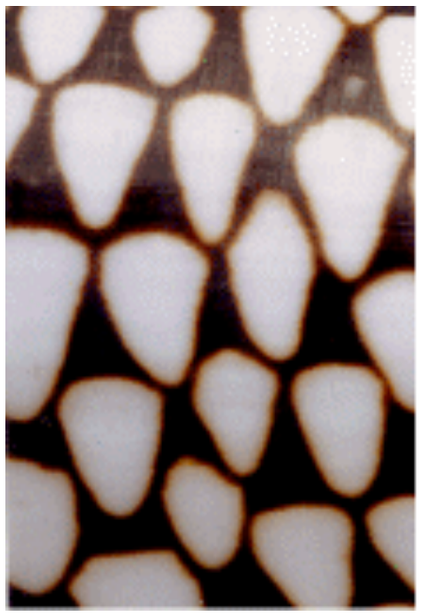
Shells





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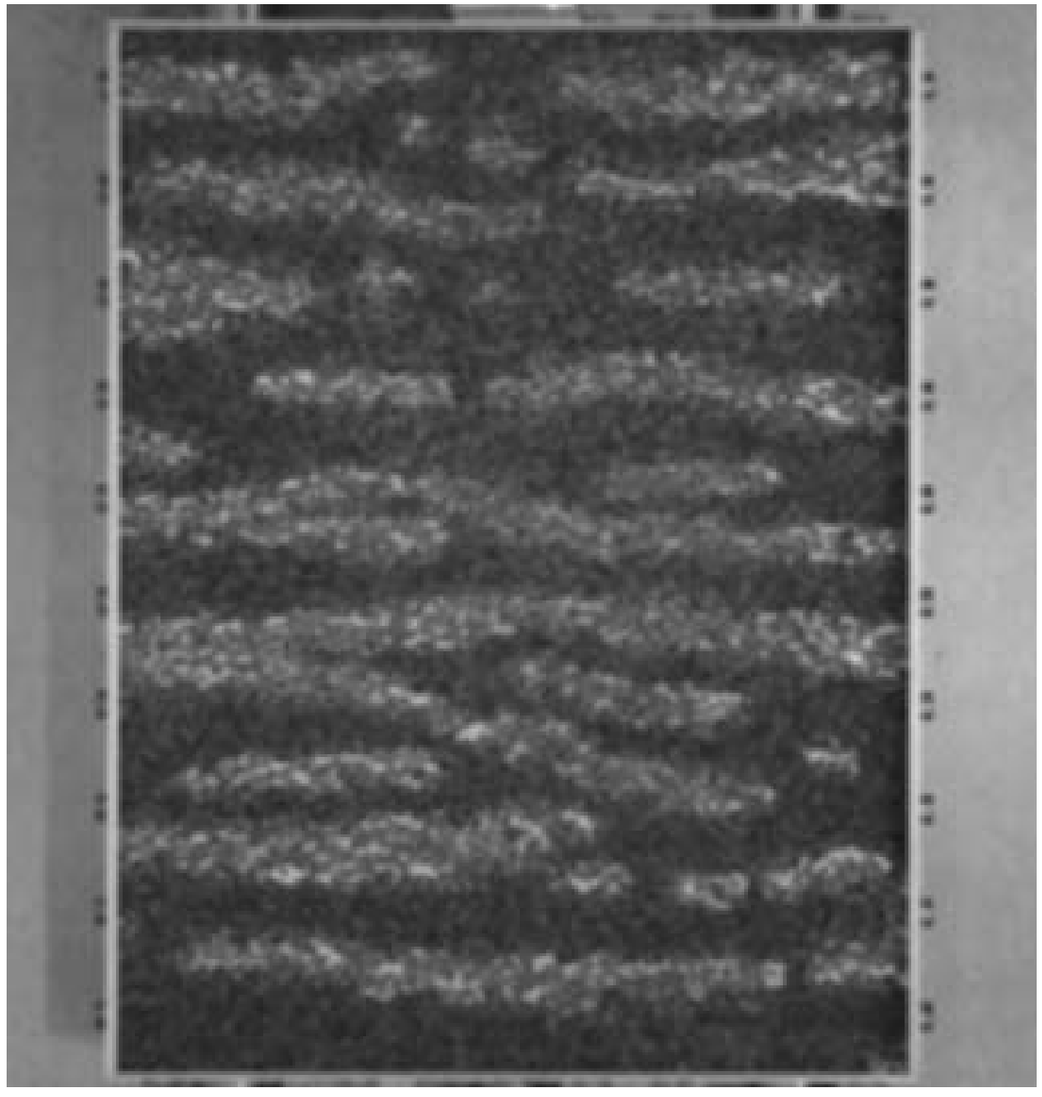
Compare: vibrated granular media



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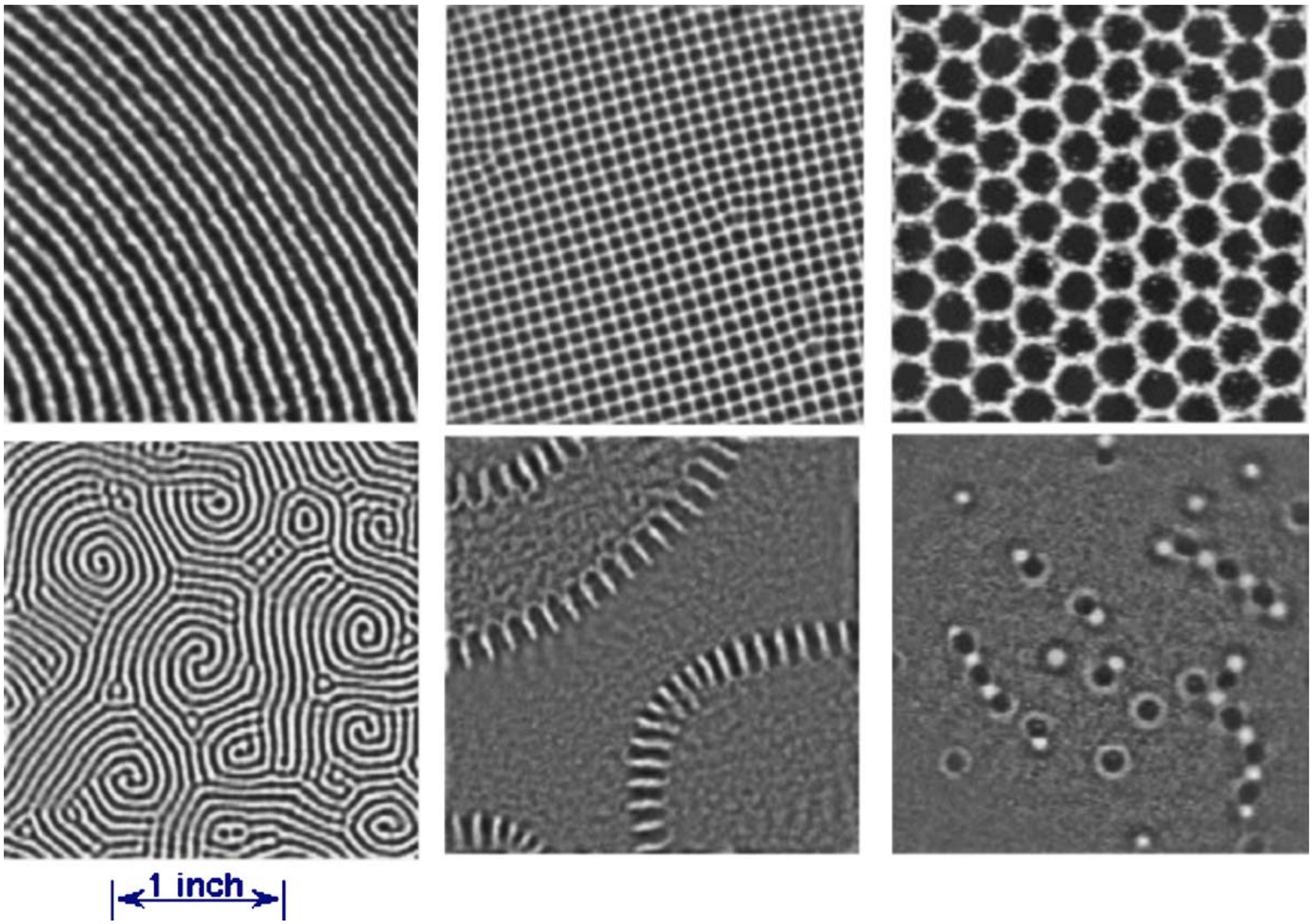

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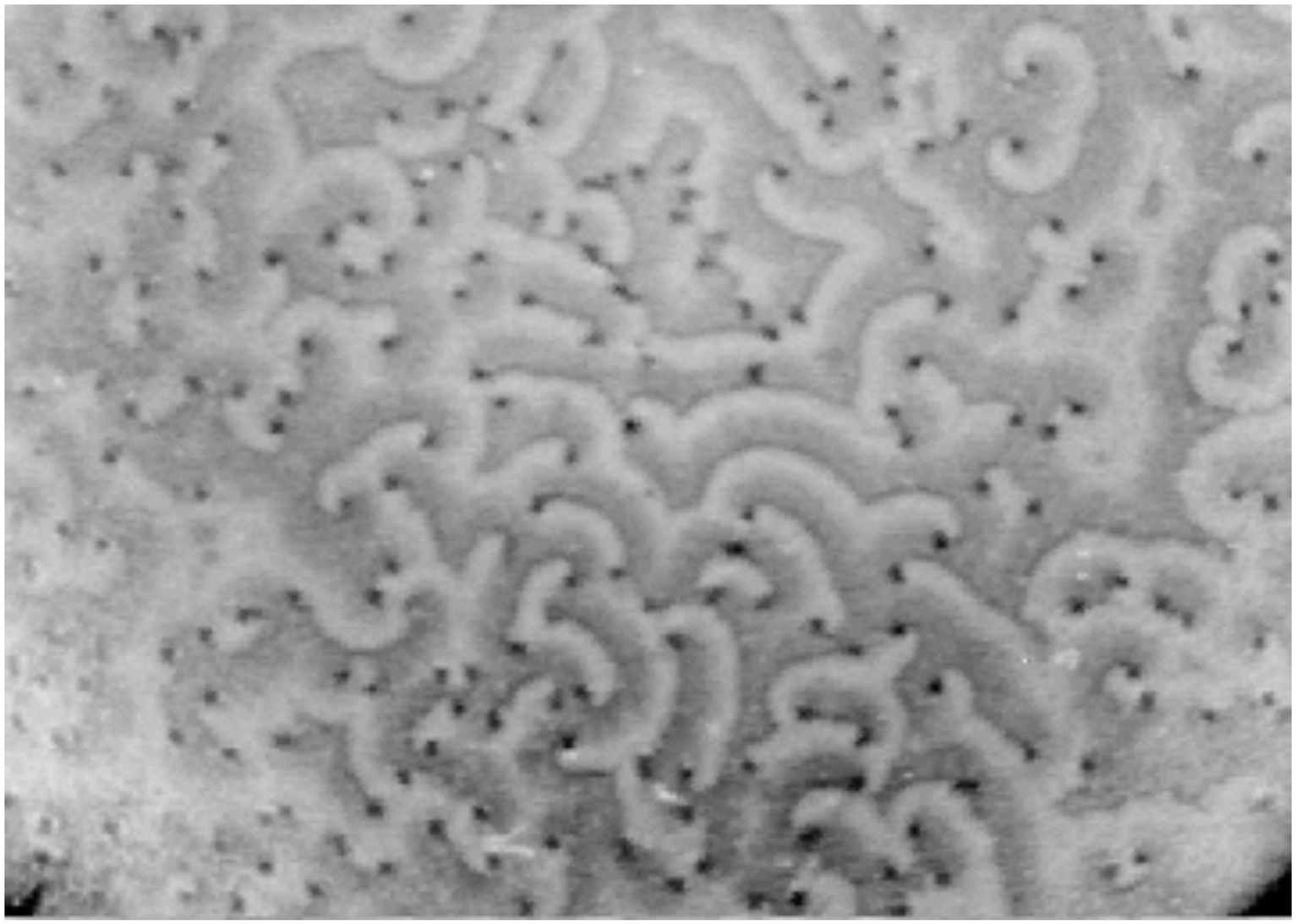
Slime mold



aggregation of a starving slime mold (credit: Florian Siegert)

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Belousov-Zhabotinsky reaction

http://www.youtube.com/watch?feature=player_detailpage&v=bH6bRt4XJcw

Mix of potassium bromate, cerium(IV) sulfate, malonic acid and citric acid in dilute sulfuric acid

the ratio of concentration of the cerium(IV) and cerium(III) ions oscillated

This is due to the cerium(IV) ions being reduced by malonic acid to cerium(III) ions, which are then oxidized back to cerium(IV) ions by bromate(V) ions

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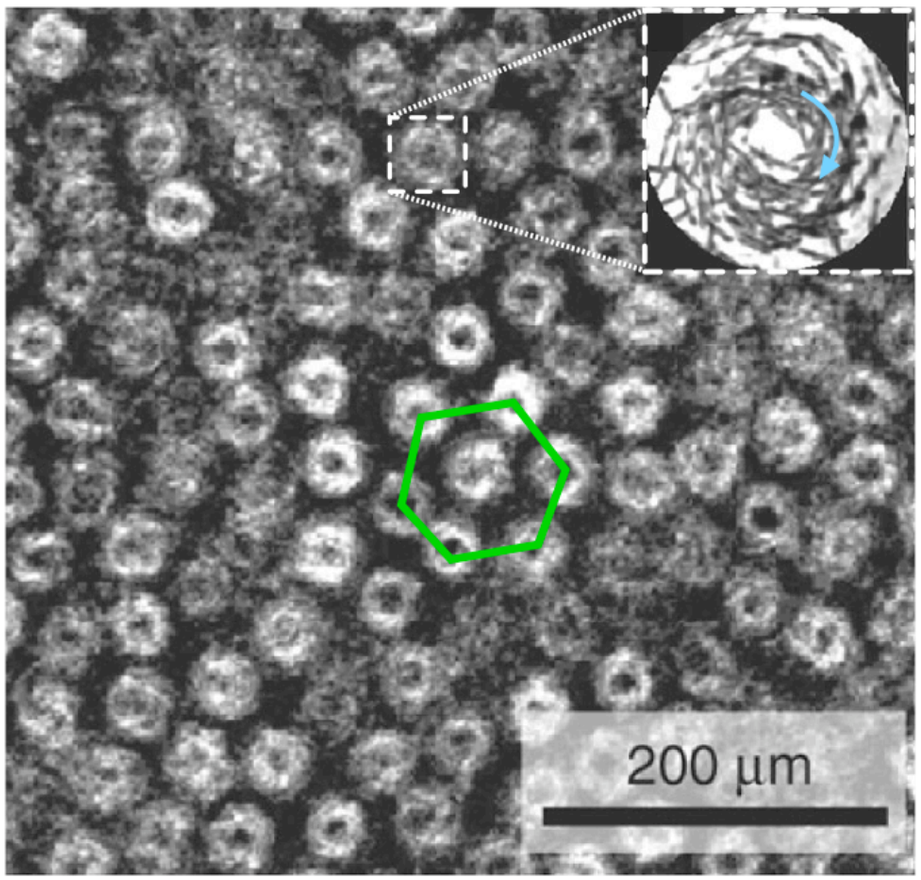
2d Swift-Hohenberg model

arxiv: 1208.4464

Sea urchin sperm cells
near surface
(high concentration)

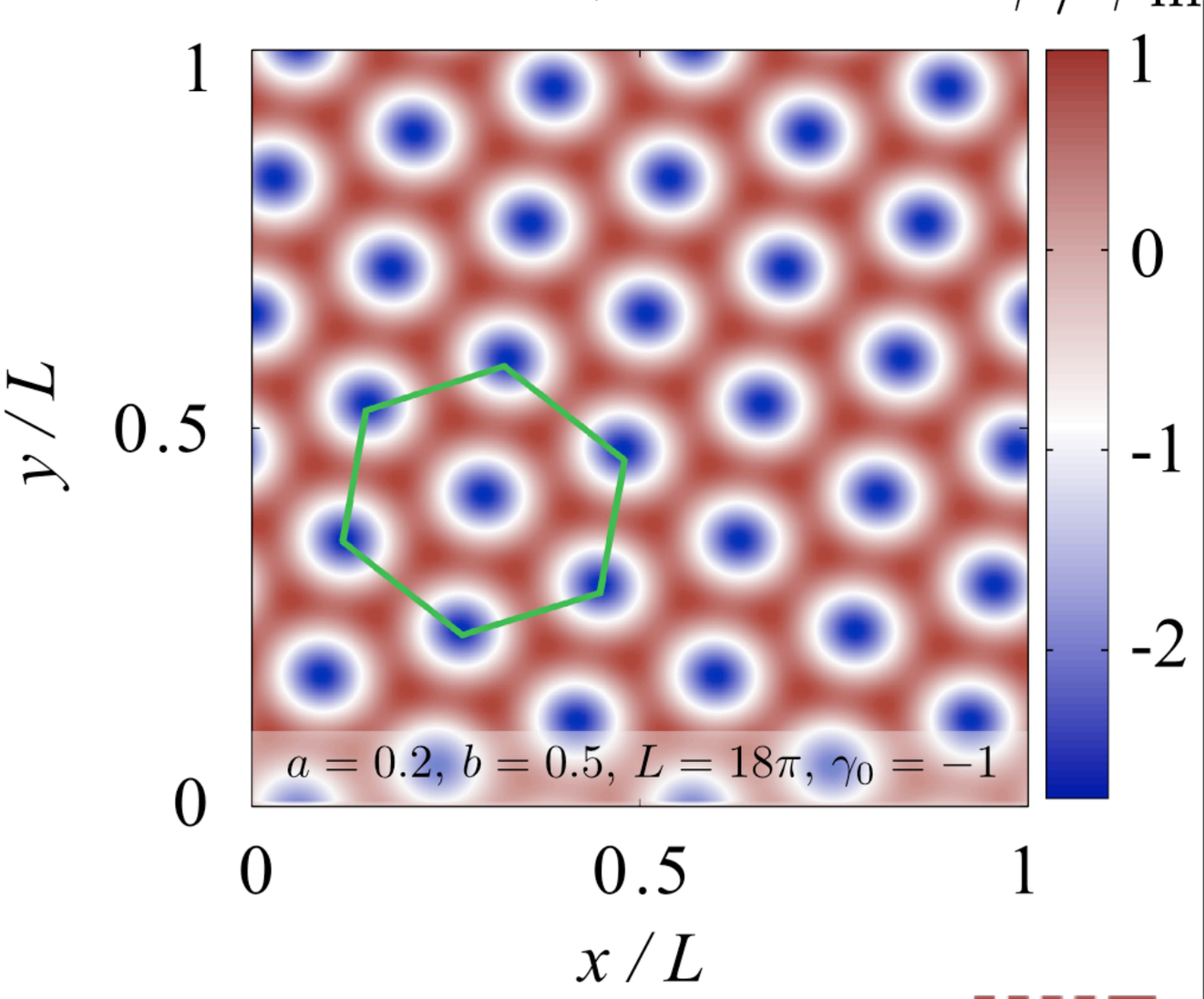


Riedel et al (2007) Science

broken
reflection-symmetry

$$b \neq 0$$

$$\psi \not\mapsto -\psi$$



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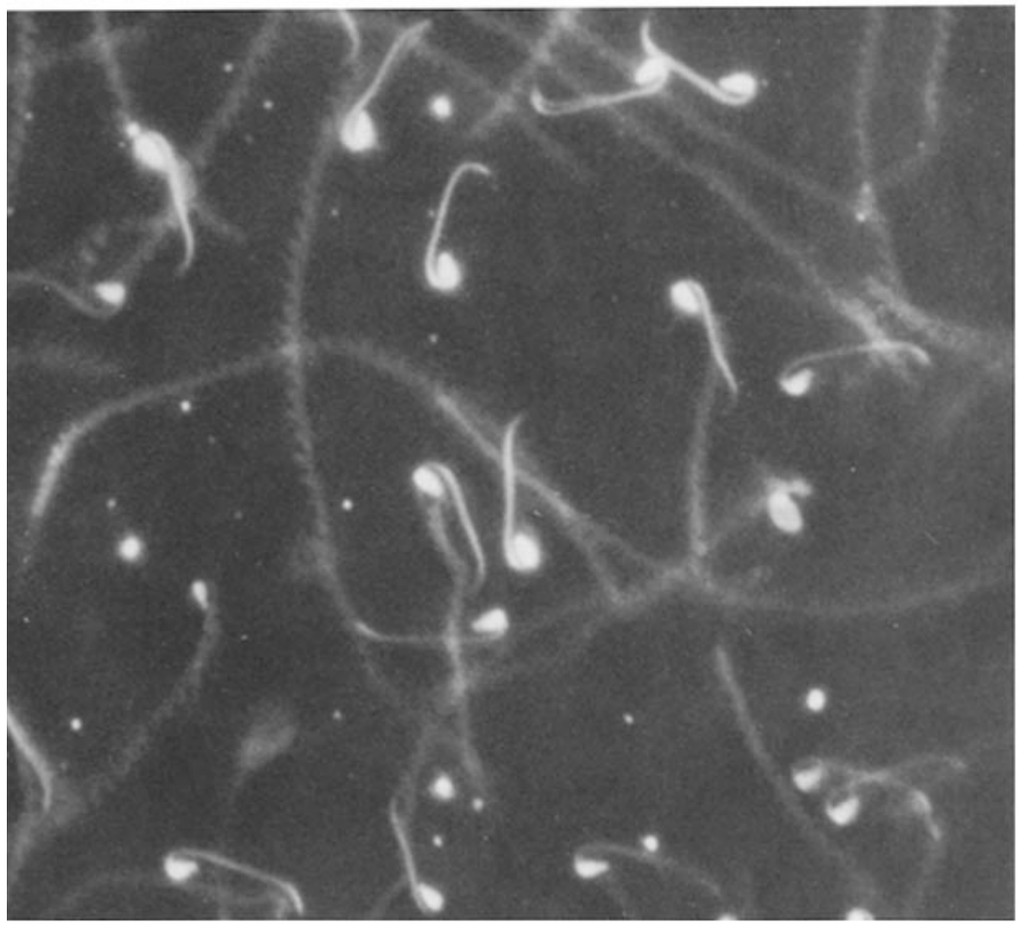
Symmetry breaking near boundaries

Broken reflection-symmetry at surfaces

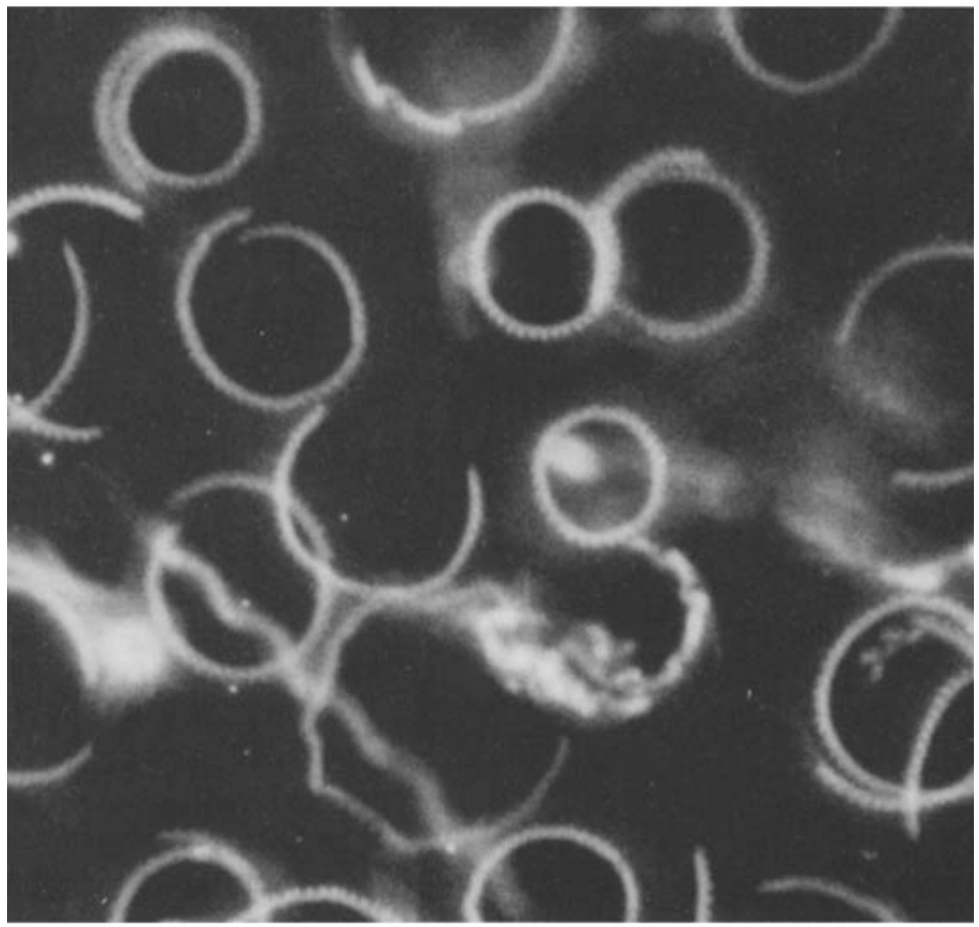
Gibbons (1980) JCB

Sea urchin sperm

in bulk (dilute)



near surface (dilute)



similar for bacteria (E. coli): Di Luzio et al (2005) Nature

2d Swift-Hohenberg model

arxiv: 1208.4464

$$\partial_t \psi = -U'(\psi) + \gamma_0 \nabla^2 \psi - \gamma_2 (\nabla^2)^2 \psi$$

reflection-symmetry

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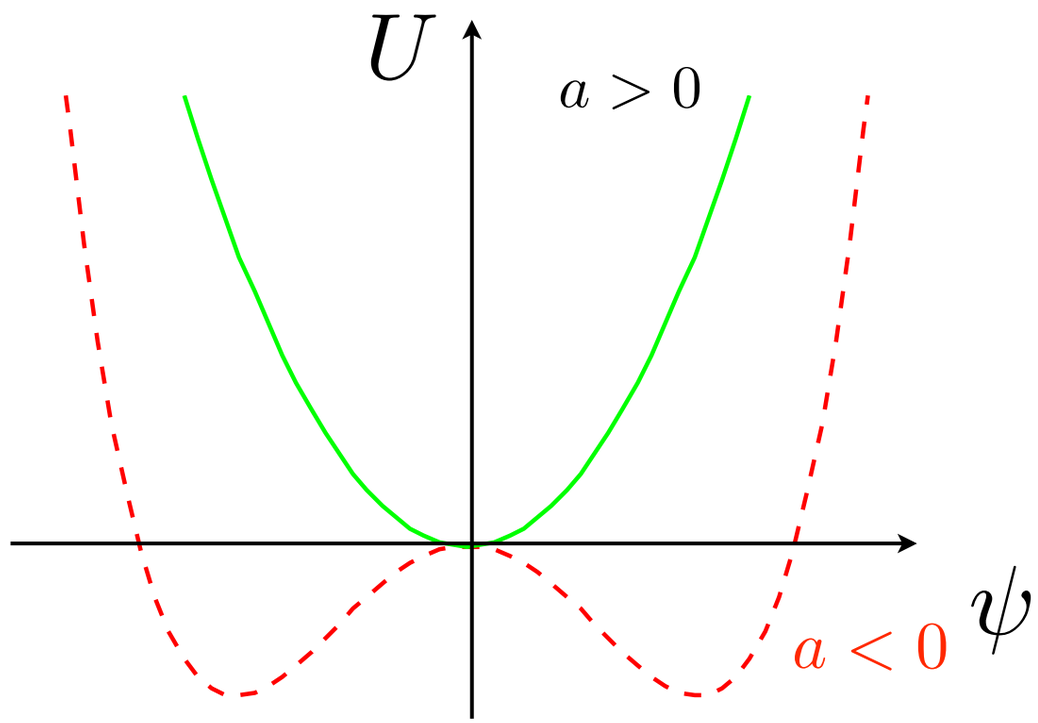
$$\psi \mapsto -\psi$$

$$U(\psi) = \frac{a}{2}\psi^2 + \cancel{\frac{b}{3}\psi^3} + \frac{c}{4}\psi^4$$



$$\psi(t, \boldsymbol{x}) = \nabla \times \boldsymbol{v}$$

2d Swift-Hohenberg model

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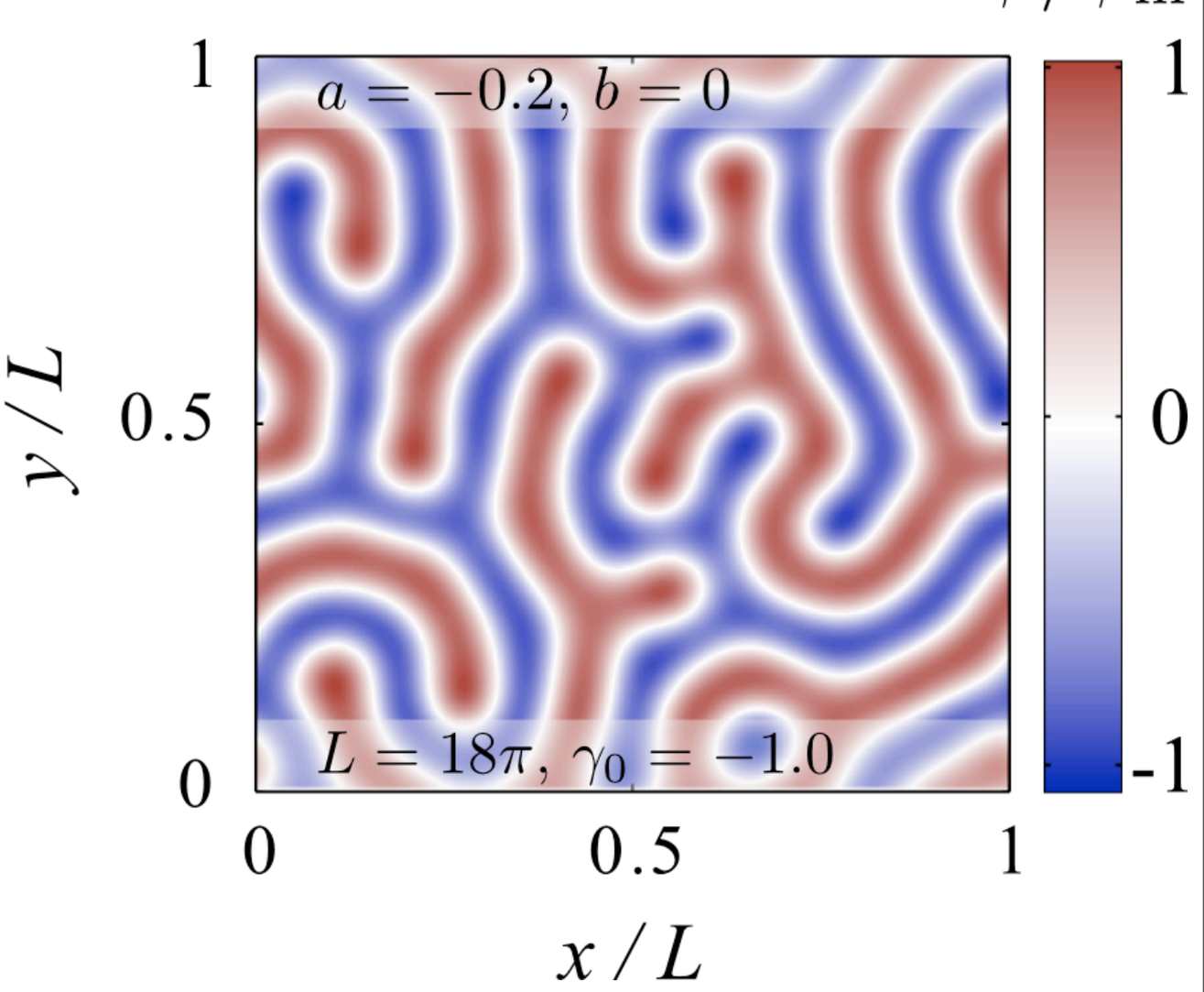

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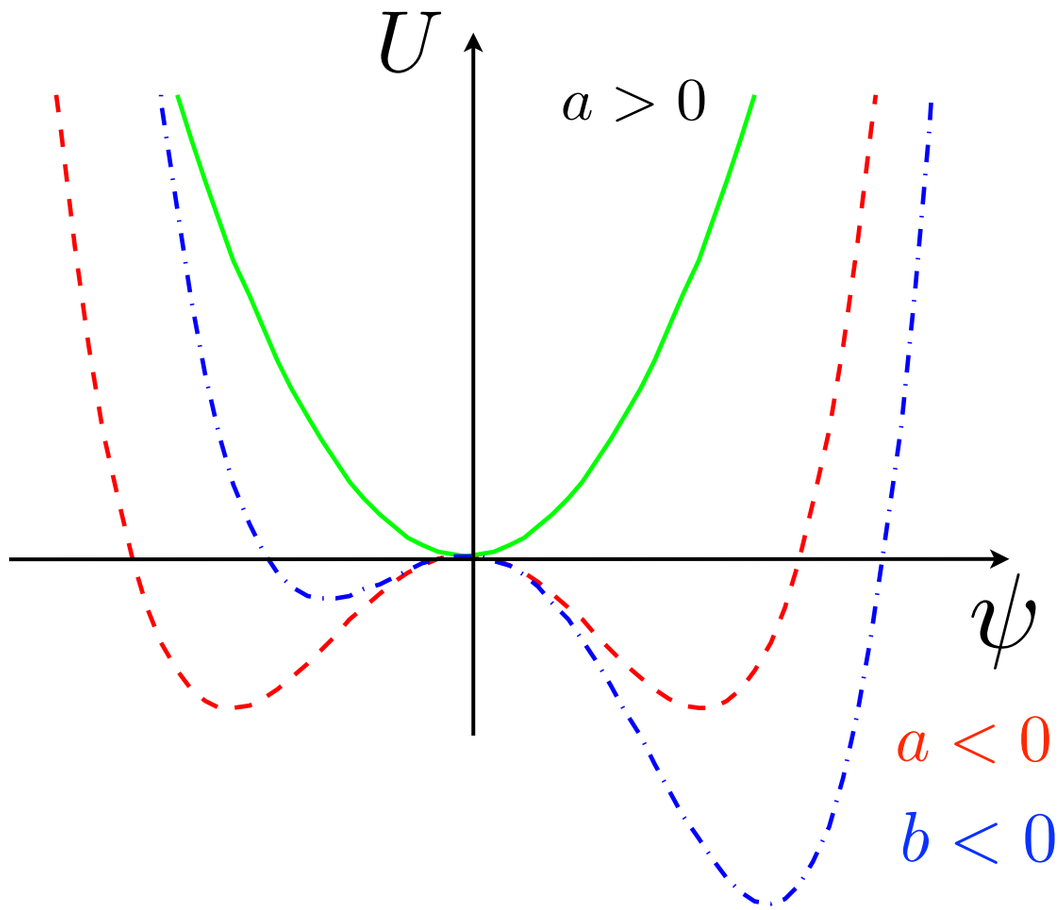

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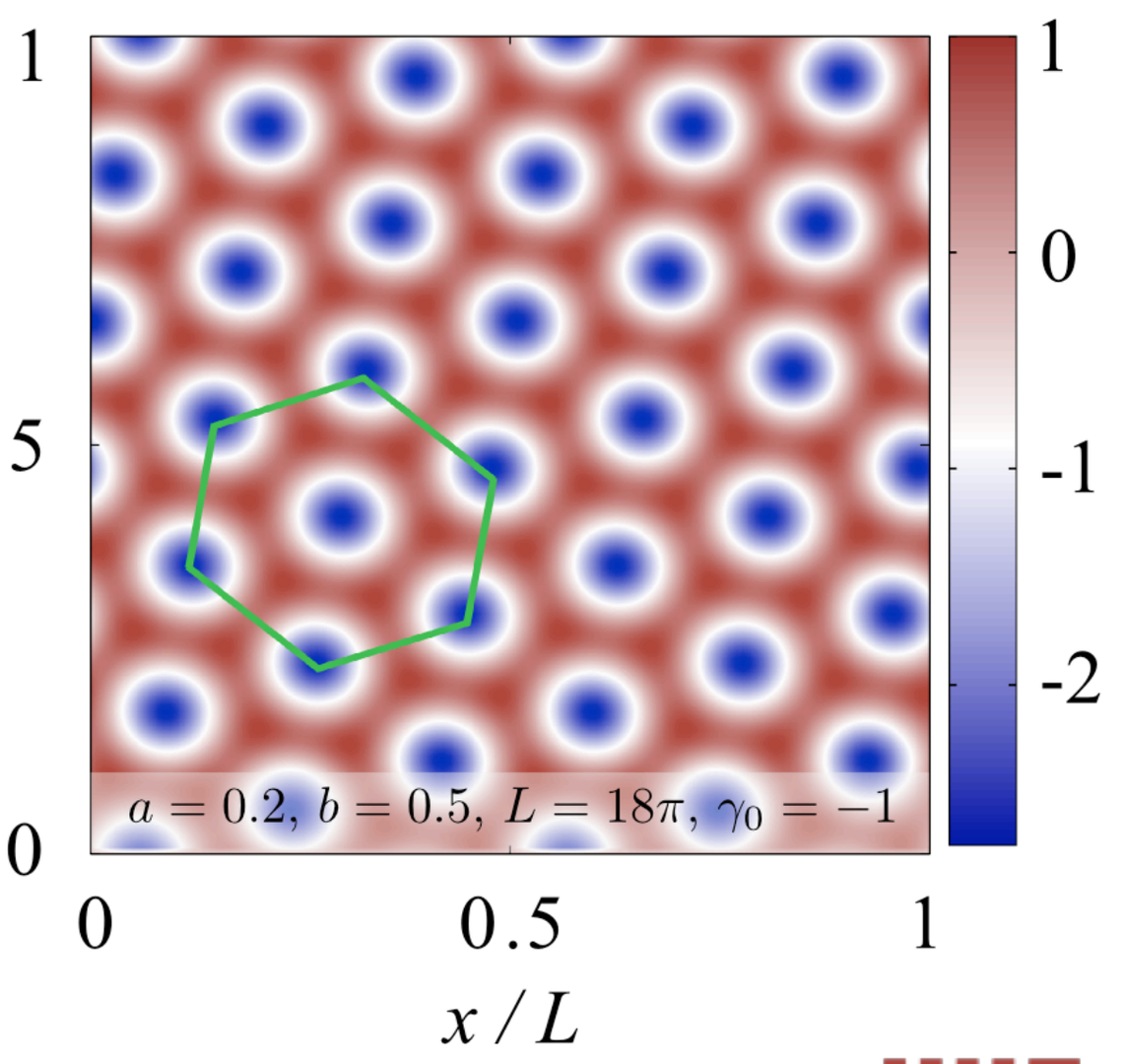

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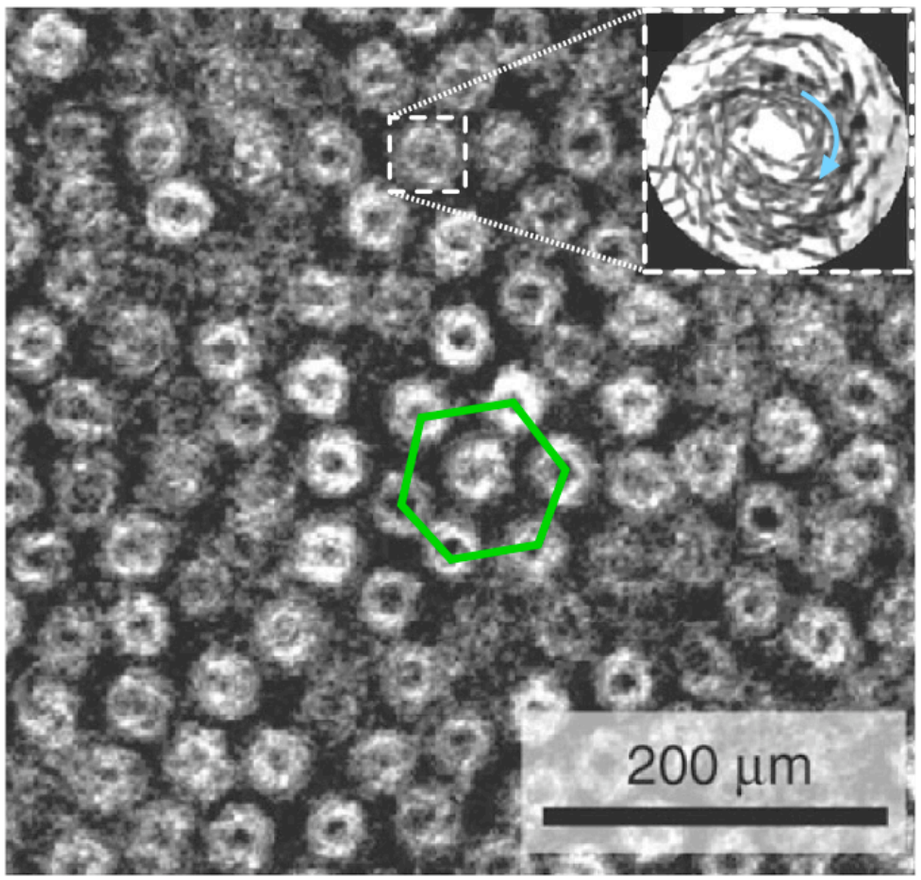

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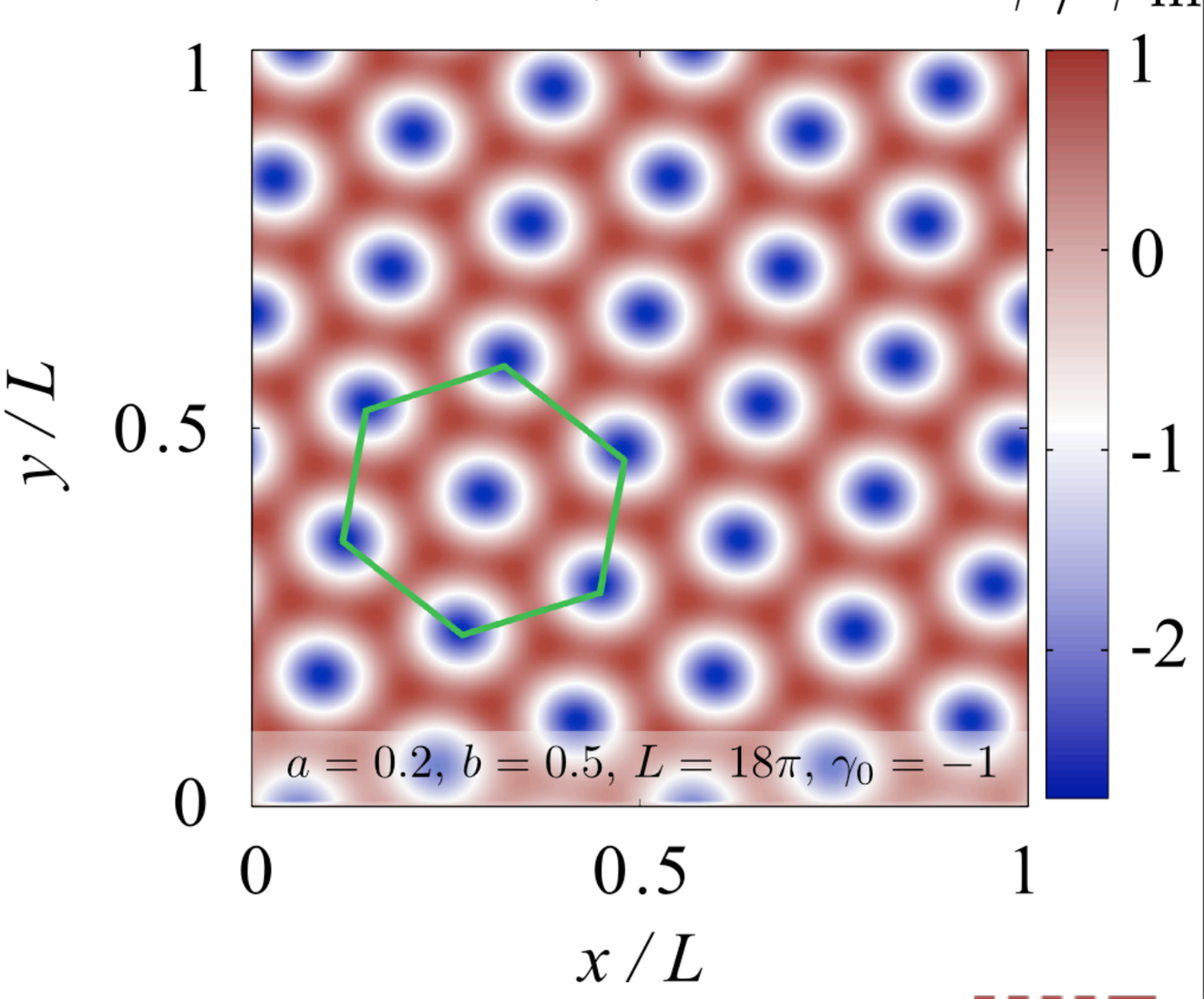
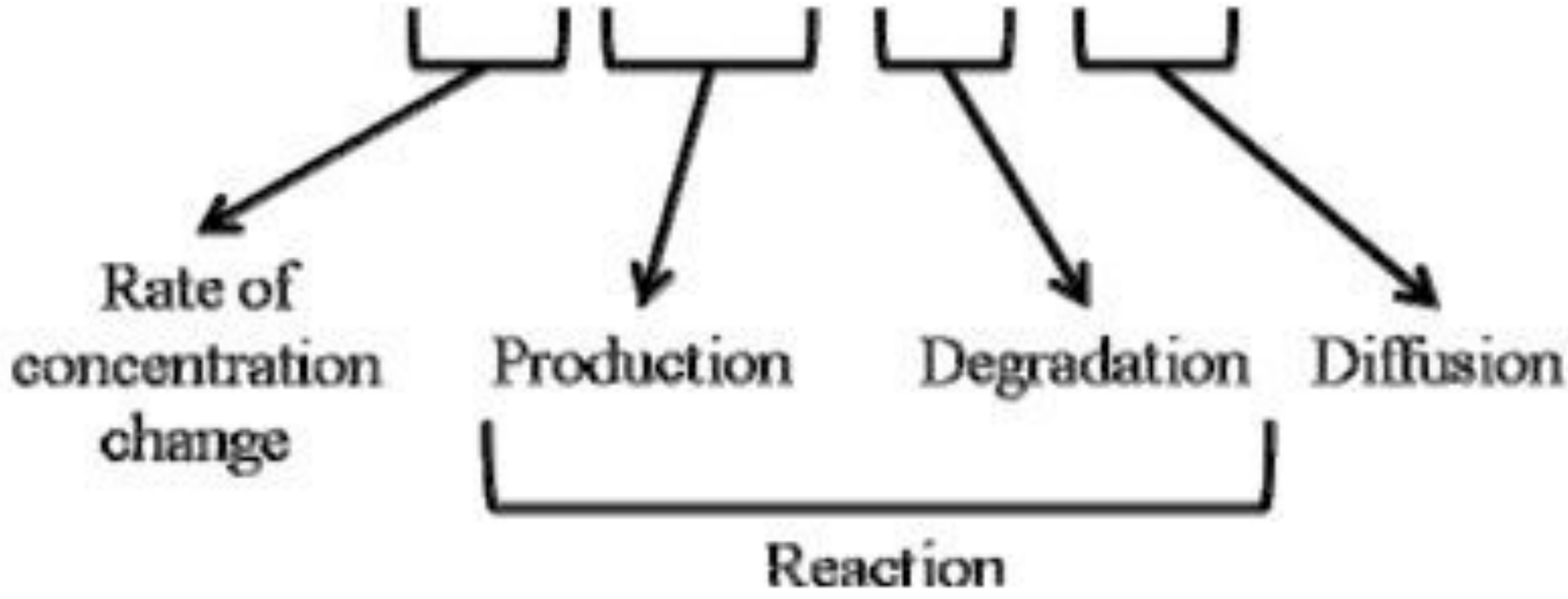


Turing model

$$\frac{\partial u}{\partial t} = F(u,v) - d_u v + D_u \Delta u$$

$$\frac{\partial v}{\partial t} = G(u,v) - d_v v + D_v \Delta v$$



A. M. Turing. **The chemical basis of morphogenesis.** *Phil. Trans. Royal Soc. London. B* **327**, 37–72 (1952)

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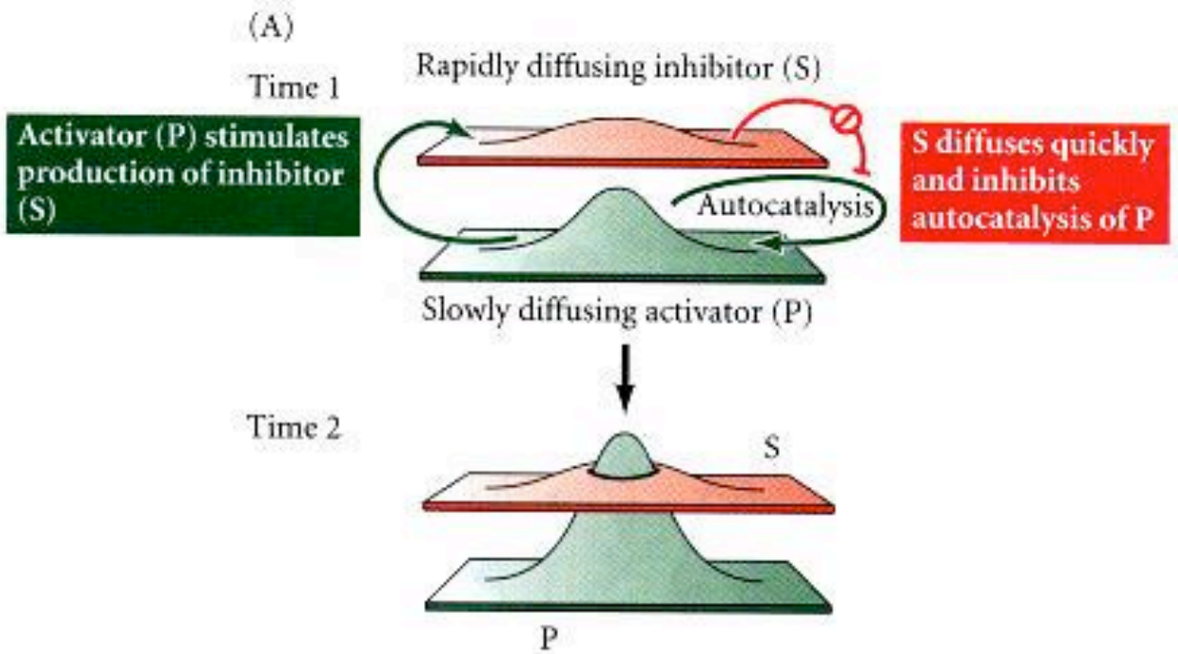
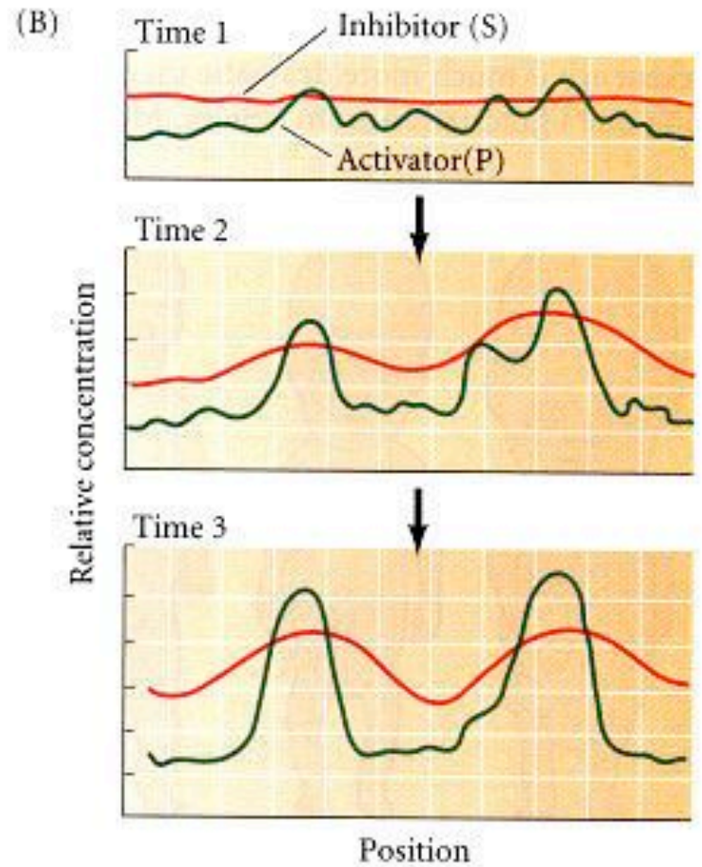
Turing model





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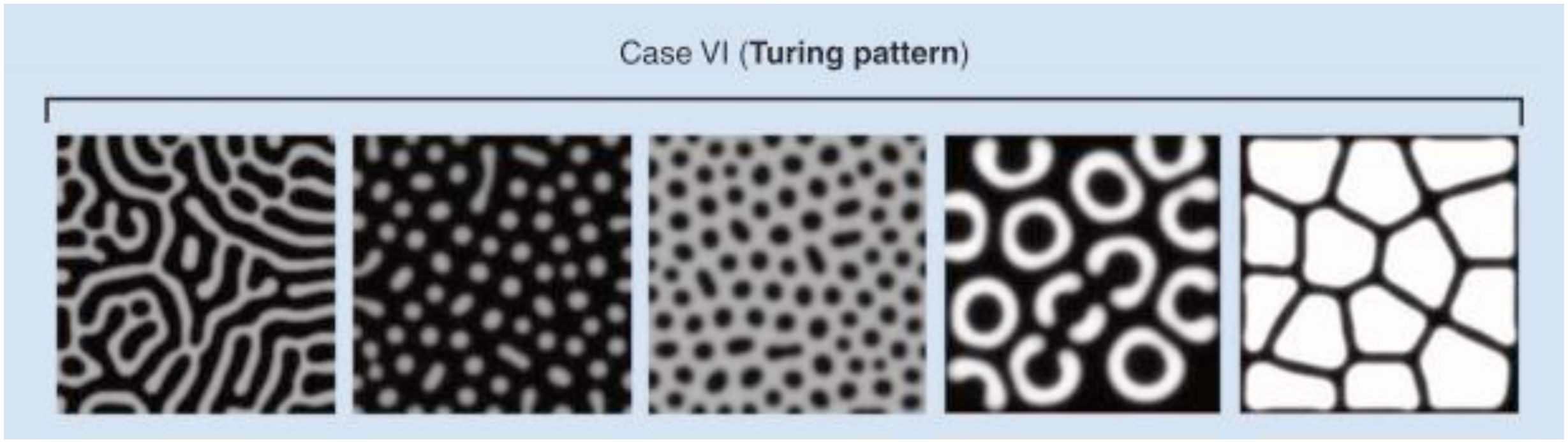
Turing pattern



Kondo S, & Miura T (2010). Reaction-diffusion model as a framework for understanding biological pattern formation. *Science, 329* (5999), 1616-20

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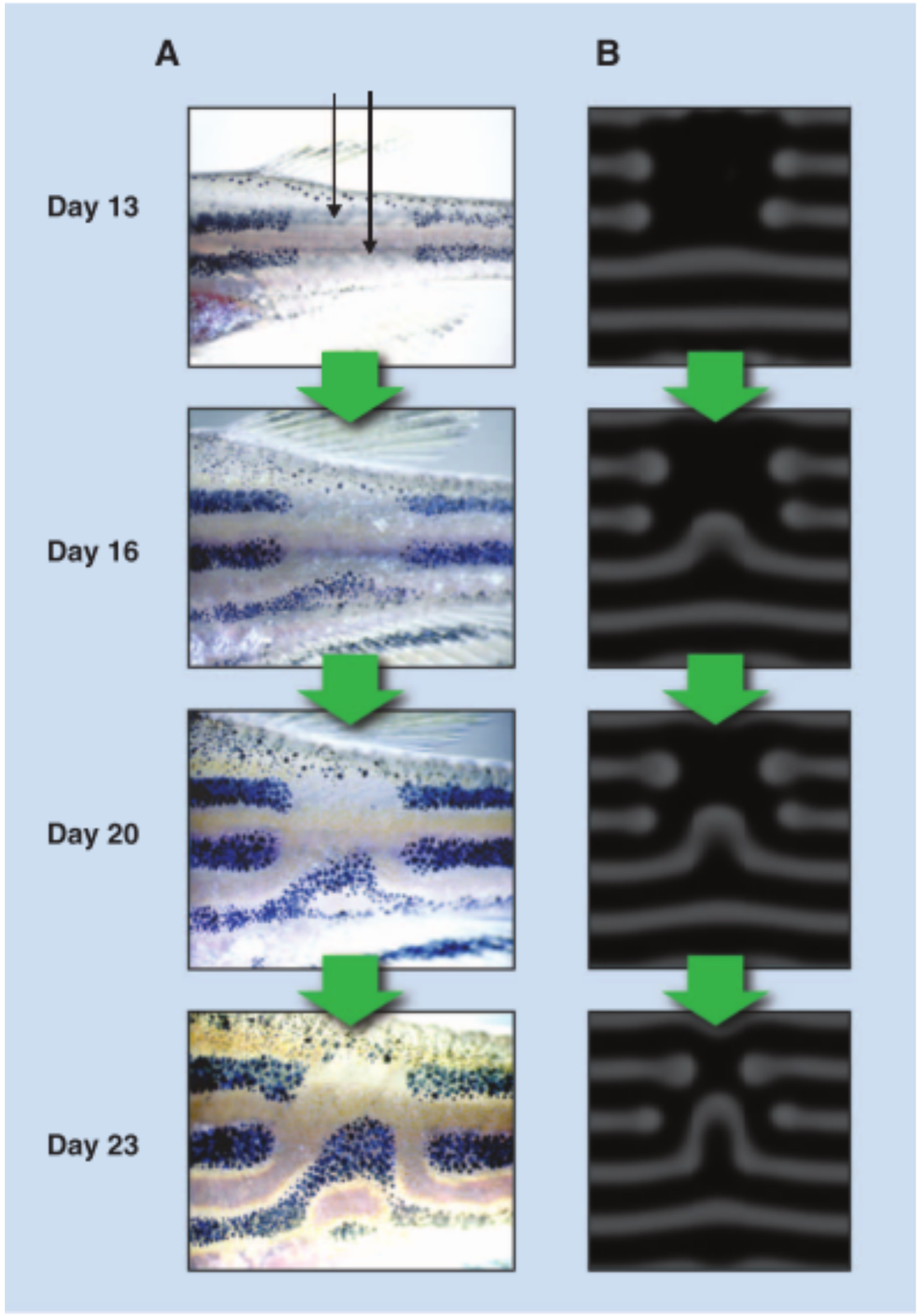
The matching of zebrafish stripe formation and a Turing model



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