Experimental Evidence for Maximal Surfaces in a 3 Dimensional Minkowski Space.

R. M. Kiehn
Emeritus Professor of Physics
Physics Department, University of Houston,
Houston, TX 77004 USA.
http://www.cartan.pair.com

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Abstract: Conventional physical dogma, justified by the local success of Newtonian dynamics for particles, assigns a Euclidean metric with signature (plus, plus, plus) to the three spatial dimensions. Minimal surfaces are of zero mean curvature and negative Gauss curvature in a Euclidean space, which supports affine evolutionary processes. However, experimental evidence now indicates that the non-affine dynamics of a fluid admits a better description in terms of a 3 dimensional space with a Minkowski metric of signature (plus, plus, minus). Three dimensional spaces with a Minkowski metric admit *maximal* surfaces of zero mean curvature, with conical or isolated singularities, and with positive Gauss curvature, in contrast to Euclidean 3D metrics. Such properties are also associated with the Hopf map, which generates two surfaces of zero mean curvature and positive Gauss curvature in a 4D Euclidean space. Falaco Solitons, easily created as topological defects in a swimming pool, are experimental artifacts of maximal surfaces (of zero mean curvature, but positive Gauss curvature) in a 3D Minkowski space. The topological defects in the otherwise flat surface of fluid density discontinuity appear as a pair of zero mean curvature surfaces, with a conical singularity at each end. The two conical singularies of the Falaco Soliton pair appear to be connected with a 1D string under tension. The singular conical points are associated with rotation (not translation) about a rotational axis or "a fixed point", and are not mapped globally by affine transitive transformations. The experimental evidence indicates that the relativistic idea whereby the presence of matter determines the physical metric (such as in the theory of gravity) must be augmented by the realization that the state of matter also has metrical influence. In particular, the metric of 3D space with matter, and its resultant dynamics, need not be Euclidean.

Keywords: Falaco Solitons, Minimal and Maximal surfaces, Cosmic Strings, Global Stability

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1 Maximal Surfaces

Maximal surfaces are generated by immersive maps from a two dimension space into a 3 dimensional space with a Lorentz metric [2]. The maximal surface is defined in terms of a space like immersion with positive Gauss curvature and with zero mean curvature. Such surfaces are related to minimal surfaces in a space with a Euclidean metric, but minimal surfaces in Euclidean space have negative Gauss curvature. Maximal Surfaces can admit isolated, or "conical", singularities, where Minimal surfaces can not. Maximal surfaces can mimic catenoidal and helical surfaces of Euclidean theory, but may exhibit singular subsets of points.

Consider a space with a Minkowski - Lorentz metric of the form

$$(ds)^2 = (dx)^2 + (dy)^2 - (dz)^2. \tag{1}$$

The immersion

$$R(s,\sigma) = (1/a)[(\sinh(a\sigma + b)\cos(s), (\sinh(a\sigma + b)\sin(s), (a\sigma + b)] \tag{2}$$

generates a space-like maximal surface in a space with a Minkowski metric. The surface is of zero mean curvature, but the metric vanishes at the conical singular point, and the Gauss curvature becomes infinite. The surface is similar to the minimal surface Catenoid in Euclidean geometry, but here, unlike the Euclidean catenoid, the Minkowski catenoid has a singular point.

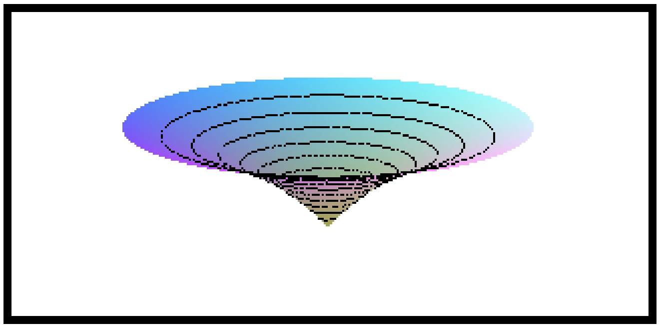


Fig. 1. A Maximal (Catenoidal) Surface with Conical Singularity in a 3D Minkowski space.
Other examples of zero mean curvature surfaces in both Euclidean and Minkowski spaces can be found at http://www22.pair.com/csdc/download/maxlor.pdf

The zero mean curvature surfaces, with a singular point, can be formed experimentally in a fluid. The experimental evidence is presented below. The idea that 3-dimensional space may or may not be Euclidean challenges a dogmatic precept of modern physics, where it is rarely perceived that physical 3D space can be anything but Euclidean. However, as discussed in the following section, the occurence of long lived rotational structures in the free surface of a water, which have been described as Falaco Solitons, exhibit the features of maximal surfaces in a Lorentz - Minkowski space. The Falaco Solitons are topological defect structures easily replicated in an experimental sense. Optical measurements indicate that the surface defect structures have a zero mean curvature. In addition, the surface defect structures have an apparent conical singularity which is an artifact of the signature of a maximal space-like surface in Minkowski space.

2 Falaco Solitons

Falaco Solitons came to the attention of the writer in 1986, while visiting an old MIT roomate in Rio de Janeiro. The Falaco Solitons are topologically universal phenomena created experimentally by a macroscopic rotational dynamics in a continuous media with a discontinuity surface, such as that found in a swimming pool. The topologically coherent structure of Falaco Solitons appears to replicate certain features found at all physical scales, from spiral arm galaxies and cosmic strings to microscopic hadrons. The easy to replicate experiment indicates the creation of "stationary" thermodynamic states (or solitons) far from equilibrium, which are locally unstable in a Euclidean sense, but are experimentally globally stabilized.

The original analysis was conducted without utilization of 3D space having Minkowski properties. Several exact solutions to the Navier-Stokes equations, *in a rotating frame of reference but with a Euclidean metric assumption*, have been used to demonstrate bifurcations to structures that are close to Falaco Solitons with an "open throat". However, the Navier-Stokes solutions found so far, do not permit the formation of the conical singularity that is observed experimentally. These results suggest that the open throat Falaco Soliton solutions can appear as cosmological realizations of Wheeler's wormholes in Euclidean 3 space, but are not directly related to the realization of closed throat solutions related to a Minkowski 3 space. The difference leads to the definition of Euclidean Falaco Solitons (Wormholes) and Minkowski Falaco Solitons.

The Minkowski Falaco Solitons could represent spin pairing mechanisms in the microscopic Fermi surface. Expermentally in the swimming pool, the Minkowski Falaco Solitons do exhibit the confinement problem of sub microscopic quarks on the end of a string. If you try to cut the confinement string, the quarks (zero mean curvature surfaces) disappear. To the eye the Minkowski Falaco Solitons appear to be macroscopic realizations of sub-submicroscopic strings connecting branes, or, at the very large macroscopic level, as cosmological strings.

The existence of the Falaco Solitons indicates that the presence of matter in the *fluid* state can impose a Minkowski-Lorentz metric structure on a 3 dimensional space.

3 The Falaco Soliton - A Topological Defect in a swimming pool.

3.1 Preface

During March of 1986, while visiting an old friend in Rio de Janeiro, Brazil, the present author became aware of a significant topological event involving solitons that can be replicated experimentally by almost everyone with access to a swimming pool. Study the photo which was taken by David Radabaugh, in the late afternoon, Houston, TX 1986.

Fig 2. Three Falaco Soliton pairs

The extraordinary photo is an image of 3 pairs of what are now called Falaco Solitons, a few minutes after their creation. The Falaco Soliton consists of a pair of globally stabilized rotational indentations in the free water-air surface of the swimming pool. The pair of conta-rotating dimples are connected by an (unseen in the photograph) interconnecting thread from the vertex of one dimple to the vertex of the other dimple of the rotational pair. These Solitons are apparently long-lived non-equilibrium states of matter far from thermodynamic equilibirum. They will persist for many minutes in a still pool of water. Their stability is presumed to be globally established by a connecting thread (under tension) that connects the two conical singularities. If the singular thread is abruptly severed (experimentally), the endcaps disappear in a rapid non-diffusive manner.

The black discs are formed on the bottom of the pool by Snell refraction of a rotationally induced dimpled surface of zero mean curvature. Careful examination of the contrast in the photo will indicate the region of the dimpled surface as deformed artifacts to the left of each black spot at a distance about equal to the separation distance of the top right pair and elevated above the horizon by about 25 degrees. The photo was taken in late afternoon. The fact that the projections are circular and not ellipses indicates that the dimpled surface is a surface of zero mean curvature. (Photo by David Radabaugh, Schlumberger, Houston, 1986.) A better photo, also taken by D. Radabaugh in 2004 in a swimming pool in Mazan, France, demonstrates the dimpled surface and the Snell refraction.

Fig 3. Surface Indentations of a Falaco Soilton
The photo is in effect a single frame of a digital movie that may be downloaded from
http://www22.pair.com/csdc/download/blackspots.avi.

The fluid motion is a local (non-rigid body) rotation motion about the interconnecting thread. In the photos note that the actual indentations of the free surface are of a few millimeters at most. The lighting and contrast optics enables the dimpled surface structures to be seen (although highly distorted) above and to the left of the black spots on the bottom of the pool. The experimental details of creating these objects are described below. From a mathematical point of view, the Falaco Soliton is a connected pair of two dimensional topological defects connected by a one dimensional topological defect or thread. The Falaco soliton is easily observed in terms of the black spots associated with the surface indentations. The black circular discs on the bottom of the pool are created by Snell refraction of sunlight on the dimpled surfaces of zero mean curvature. Also the vestiges of mushroom spirals in the surface structures around each pair can be seen. Such surface spiral arms can be visually enhanced by spreading chalk dust on the free surface of the pool.

The surface defects of the Falaco Soliton are observed dramatically due the formation of circular black discs on the bottom of the swimming pool. The very dark black discs are emphasized in contrast by a bright ring or halo of focused light surrounding the black disc. All of these visual effects can be explained by means of the unique optics of Snell refraction from a surface of zero mean curvature. (This explanation was reached on the day, and about 30 minutes after, the present author became aware of the Falaco effect, while standing under a brilliant Brazilian sun and in the white marble swimming pool of his friend in Rio de Janeiro. An anecdotal history of the discovery is described below.) During the initial few seconds of decay to the metastable soliton state, each large black disk is decorated with spiral arm caustics, remindful of spiral arm galaxies. The spiral arm caustics contract around the large black disk during the stabilization process, and ultimately disappear when the steady soliton state is achieved. It should be noted that if chalk dust is sprinkled on the surface of the pool during the formative stages of the Falaco soliton, then the topological signature of the familiar Mushroom Spiral pattern is exposed. The dimpled surface created appears to be (almost) a minimal surface of zero mean curvature. This conclusion is justified by the fact that the Snell projection to the floor of the pool is almost conformal, preserving the circular appearance of the black disc, independent from the angle of solar incidence. (Notice that the black spots on the bottom of the pool in the photo are circular and not distorted ellipses, even though the solar elevation is less than 30 degrees.) The conformal projection property is a property of normal projection from minimal surfaces [45].

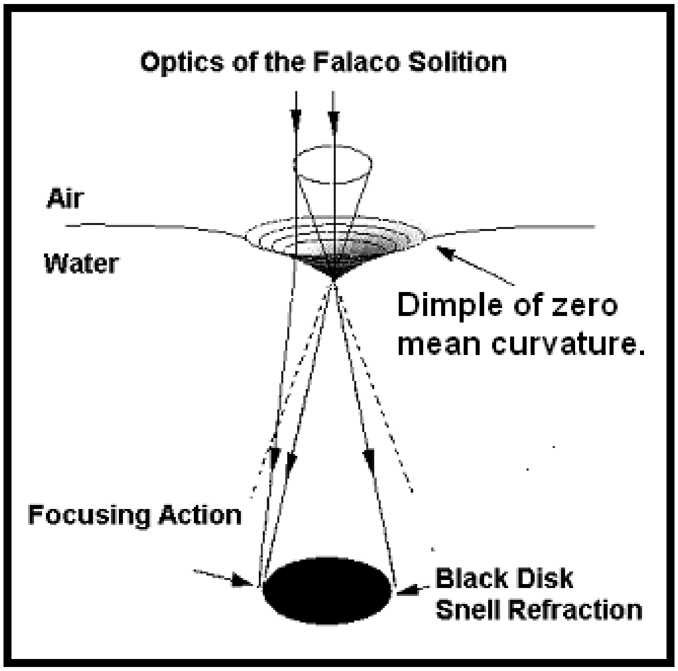


Fig 4. Optics of the Falaco Soliton

The Figure presented above was originally constructed in 1987-1989, before the concept of a Maximal Surface in Minkowski 3 Space was appreciated. However, note the similarity of the visually observed dimples presented in 1987 and the computed Minkowski maximal surface constructed in Figure 1.

A feature of the Falaco Soliton [11] that is not immediately obvious is that it consists of a pair of two dimensional topological defects, in a surface of fluid discontinuity, which are *connected* by means of a topological singular thread. Dye injection near an axis of rotation during the formative stages of the Falaco Soliton indicates that there is a unseen thread, or 1-dimensional string singularity, in the form of a circular arc that connects the two 2-dimensional surface singularities or dimples. Transverse Torsional waves of dye streaks can be observed to propagate, back and forth, from one dimple vertex to the other dimple vertex, guided by the "string" singularity. The effect is remindful of the whistler propagation of electrons along the guiding center of the earth's magnetic field lines.

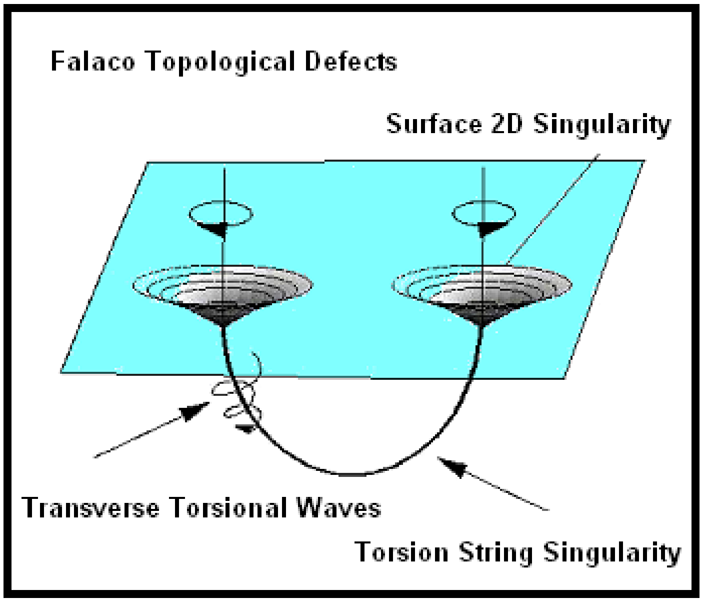


Fig 5. Falaco Topological Defects

It is conjectured that the tension in the singular connecting thread provides the force that maintains the global stability of the pair of locally unstable, dimpled surface structures. The equilibrium mode for the free surface implies that the surface should be flat, of zero Gauss curvature, without dimpled distortions. If the thread is severed, the endcap singularities disappear almost immediately, and not diffusively.

However, as a soliton, the topological system retains its coherence for remarkably long time - more than 15 minutes in a still pool. The long lifetime of the Falaco Soliton is due to this *global stabilization* of the connecting string singularity, even though the surface of negative Gauss curvature is locally unstable. The long life of the soliton state in the presence of a viscous media indicates that the flow vector field describing the dynamics is probably harmonic. This result is in agreement with the assumption that the fluid can be represented by a Navier-Stokes equation with a dissipation that is represented by the product of a shear viscosity and the vector Laplacian of the velocity field. If the velocity field is harmonic, the vector Laplacian vanishes, and the shear dissipation goes to zero no matter what is the magnitude of the shear viscosity term. Hence a palatable argument is offered for the existence of the long lifetime. More over it is known that minimal surfaces of zero mean curvature are generated by harmonic vector fields, hence the surface endcaps of zero mean curvature give further credence to the idea of a harmonic velocity field.

The bottom line is that it is possible to produce, hydrodynamically, in a viscous fluid with a surface of discontinuity, a long lived coherent structure that consists of a set of macroscopic topological defects. The Falaco Solitons are representative of non-equilibrium long lived structures, or "stationary states", far from equilibrium. These observation were first reported at the 1987 Dynamics Days conference in Austin, Texas [11], [12], and subsequently in many other places, mostly in the hydrodynamic literature [19], [20], [30], [27], [32], as well as several APS meetings.

These, long-lived topologically coherent objects, dubbed the Falaco Solitons (for reasons explained below), have several features equivalent to those reported for models of the sub-microscopic hadron. String theorists take note, for the structure consists of a pair of topological 2-dimensional locally unstable rotational defects in a surface of discontinuity, globally connected and stabilized globally in the fluid by a 1 dimensional topological defect or string with tension. (In Euclidean space, the surface defects are of negative Gauss curvature, and are, therefor, locally unstable.) As mentioned above, the experimental equilibrium state is a flat surface of zero Gauss curvature. However, it is conjectured that the local instability is overcome globally by a string whose tension globally stabilizes the locally unstable endcaps. These observational conjectures were explained partially in terms of a bifurcation process and solutions to the Navier-Stokes equations in a rotating frame of reference [49]. Now a better description would be in terms of a fluid with a Minkowski metric.

The reader must remember that the Falaco Soliton is a topological object that can and will appear at all scales, from the microscopic to the macroscopic, from the sub-submicroscopic world of strings connection branes to the cosmological level of spiral arm galaxies connected by threads. At the microscopic level, the method offers a view of forming spin pairs that is different from Cooper pairs and could offer insight into Superconductivity. At the level of Cosmology, the concept of Falaco Solitons could lead to explanations of the formation of flat spiral arm galaxies. At the submicroscopic level, the Falaco Solitons mimic quarks on a string. At the macroscopic level, the topological features of the Falaco Solitons can be found in solutions to the Navier-Stokes equations in a rotating frame of reference. Under deformation of the discontinuity surface to a flattened ball, the visual correspondence to hurricane structures between the earth surface and the tropopause is remarkable. In short, the concept of Falaco Solitons is a universal phenomena.

3.2 The Experiment

The Falaco Soliton phenomena is easily reproduced by placing a large circular disc, such as dinner plate, vertically into the swimming pool until the plate is half submerged and it oblate axis resides in the water-air free surface. Then move the plate slowly in the direction of its oblate axis. At the end of the stroke, smoothly extract the plate (with reasonable speed) from the water, imparting kinetic energy and distributed angular momentum to the fluid. Initially, the dynamical motion of the edges of the plate will create a pair of vortex structures in the free surface of the water (a density discontinuity which can also be mimicked by salt concentrations). If these vortex structures were Rankine vortices of opposite rotation, they would cause the initially flat surface of discontinuity to form a pair of parabolic concave indentations of positive Gauss curvature, indicative of the "rigid body" rotation of a pair of contra-rotating vortex cores of uniform vorticity. However, in a few seconds the vortex surface depressions will decay into a pair of convex dimples of zero mean curvature (in a Euclidean setting the structures would be described as having a negative Gauss curvature - as long as the singularity did not appear). Associated with the evolution in the early stages of formation is a visible set of spiral arm caustics, As the convex dimples form, the surface effects can be observed in bright sunlight via their Snell projections as large black spots on the bottom of the pool. In a few tries you will become an expert experimentalist, for the drifting spots are easily created and, surprisingly, will persist for many minutes in a still pool. The dimpled depressions are typically of the order of a few millimeters in depth, but the zone of circulation typically extends over a disc of some 10 to 30 centimeters or more, depending on the plate diameter. This configuration, or coherent topological defect structure, has been defined as the Falaco Soliton. For purposes of illustration , the vertical depression has been greatly exaggerated in Figures 3 and 4.

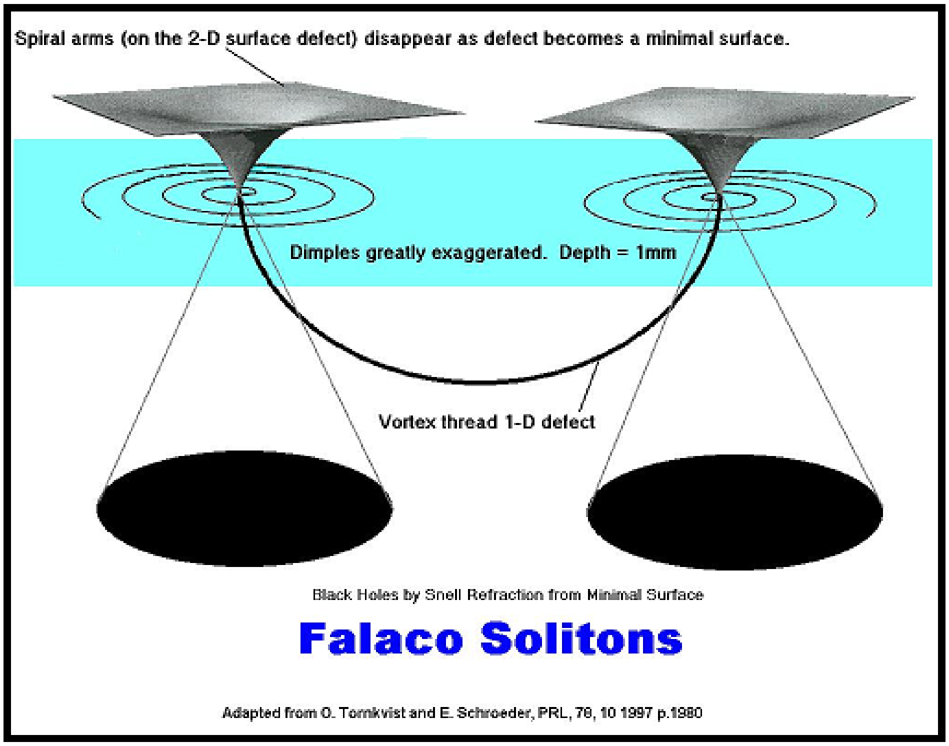
If a thin broom handle or a rod is placed vertically in the pool, and the Falaco soliton pair is directed in its translation motion to intercept the rod symmetrically, as the soliton pair comes within range of the scattering center, or rod, (the range is approximately the separation distance of the two rotation centers) the large black spots at first shimmer and then disappear. Then a short time later, after the soliton has passed beyond the interaction range of the scattering center, the large black spots coherently reappear, mimicking the numerical simulations of soliton coherent scattering. For hydrodynamics, this observation firmly cements the idea that these objects are truly coherent "Soliton" structures. This is the only (known to this author) macroscopic visual experiment that demonstrates these coherence features of soliton scattering in numerical studies.

If the string connecting the two endcaps is sharply "severed", the confined, two dimensional endcap singularities do not diffuse away, but instead disappear almost explosively. It is this observation that leads to the statement that the Falaco soliton is the macroscopic topological equivalent of the illusive hadron in elementary particle theory. The two 2-dimensional surface defects (the quarks) are bound together by a string of confinement, and cannot be isolated. The dynamics of such a coherent structure is extraordinary, for it is a system that is globally stabilized by the presence of the connecting 1-dimensional string.

4 Falaco Solitons as Landau Ginsburg structures in micro, macroscopic and cosmological systems

The Falaco experiments lead to the idea that such topological defects are available at all scales. The Falaco Solitons consist of spiral "vortex defect" structures (analogous to CGL theory) on a two dimensional minimal surface, one at each end of a 1-dimensional "vortex line" or thread (analogous to GPG theory). Remarkably the topological defect surface structure is locally unstable, as the surface is of negative Gauss curvature. Yet the pair of locally unstable 2-D surfaces is *globally* stabilized by the 1-D line defect attached to the "vertex" points of the minimal surfaces.

For some specific physical systems it can be demonstrated that period (circulation) integrals of the 1-form of Action potentials, A, lead to the concept of "vortex defect lines". The idea is extendable to "twisted vortex defect lines" in three dimensions. The "twisted vortex defects" become the spiral vortices of a Complex Ginsburg Landau (CGL) theory , while the "untwisted vortex lines" become the defects of Ginzburg-Pitaevskii-Gross (GPG) theory [46].



In the macroscopic domain, the experiments visually indicate "almost flat" spiral arm structures during the formative stages of the Falaco solitons. In the cosmological domain, it is suggested that these universal topological defects represent the ubiquitous "almost flat" spiral arm galaxies. Based on the experimental creation of Falaco Solitons in a swimming pool, it has been conjectured that M31 and the Milky Way galaxies could be connected by a topological defect thread [11]. Only recently has photographic evidence appeared suggesting that galaxies may be connected by strings.

Interacting Galaxies NGC 1409 and NGC 1410 **HST • WFPC2**
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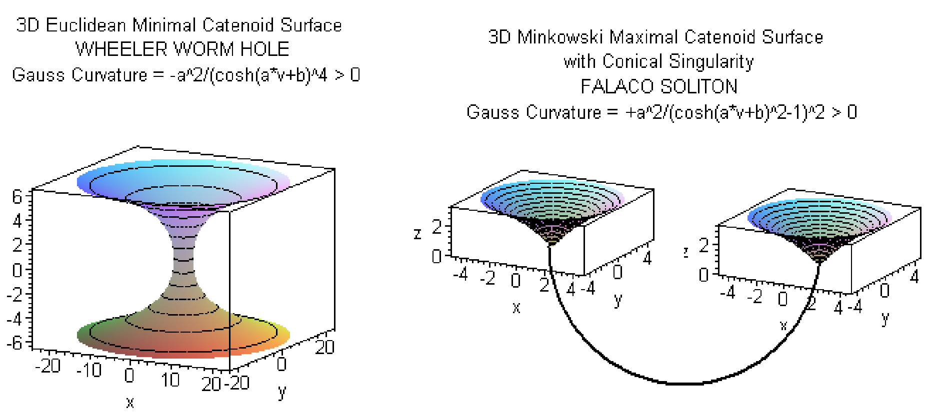
At the other extreme, the rotational minimal surfaces of zero mean curvature which form the two endcaps of the Falaco soliton, like quarks, apparently are confined by the string. If the string (whose "tension" induces global stability of the unstable endcaps) is severed, the endcaps (like unconfined quarks in the elementary particle domain) disappear (in a non-diffusive manner). In the microscopic electromagnetic domain, the Falaco soliton structure offers an alternate, topological, pairing mechanism on a Fermi surface, that could serve as an alternate to the Cooper pairing in superconductors.

It is extraordinary, but the Falaco Solitons appear to be macroscopic realizations of the Wheeler wormhole. This structure was presented early on by Wheeler (1955), but was considered to be unattainable in a practical sense. To quote Zeldovich p. 126 [54]

> "The throat or "wormhole" (in a Kruskal metric) as Wheeler calls it, connects regions of the same physical space which are extremely remote from each other. (Zeldovich then gives a sketch that topologically is equivalent to the Falaco Soliton in a Euclidean metric). Such a topology implies the existence of 'truly

geometrodynamic objects' which are unknown to physics. Wheeler suggests that such objects have a bearing on the nature of elementary particles and anti particles and the relationships between them. However, this idea has not yet borne fruit; and there are no macroscopic "geometrodynamic objects" in nature that we know of. Thus we shall not consider such a possibility."

This quotation dates back to the period 1967-1971. Now the experimental evidence justifies (again) Wheeler's intuition. However, the concept of a wormhole as a catenoidal surface of zero mean curvature in a Euclidean space space is transformed into a Falaco Soliton as a catenoidal surface of zero mean curvature in a Minkowski 3 space. The catenoidal surface of zero mean curvature in a 3D Euclidean space is a Wheeler Wormhole, while the catenoidal surface of zero mean curvature and its conical singular point in a 3D Minkowski space is a part of the fluid Falaco Soliton.



The Wheeler Wormhole has an open throat.
The Falaco Soliton has conical singularities with a connecting string.

5 A Cosmological Conjecture

The objective of this section is to summarize certain topological aspects and defects of thermodynamic physical systems and their possible continuous topological evolution, creation, and destruction on a cosmological scale. The creation and evolution of stars and galaxies can be interpreted in terms of the creation of topological defects and evolutionary phase changes in a very dilute turbulent, non-equilibrium, thermodynamic system of maximal Pfaff topological dimension 4 [48] [49]. The cosmology so constructed is opposite in viewpoint to those efforts which hope to describe the universe in terms of properties inherent in the quantum world of Bose-Einstein condensates, super conductors, and superfluids [47]. Both approaches utilize the ideas of topological defects, but thermodynamically the approaches are opposite in the sense that the quantum method involves, essentially, equilibrium systems, while the approach presented herein is based upon non-equilibrium systems. Based upon the single assumption that the universe is a non-equilibrium thermodynamic system of Pfaff topological dimension 4 leads to a cosmology where the universe, at present, can be approximated in terms of the non-equilibrium states of a very dilute van der Waals gas near its critical point. The stars and the galaxies are the topological defects and coherent - but not equilibrium - structures of Pfaff topological dimension 3 in this non-equilibrium system of Pfaff topological dimension 4. The topological theory of the ubiquitous van der Waals gas leads to the concepts of negative pressure, string tension, and a Higgs potential as natural consequences of a topological point of view applied to thermodynamics. Perhaps of more importance is the fact that these concepts do not depend explicitly upon the geometric constraints of metric or connection, and yield a different perspective on the concept of gravity.

The original motivation for this conjecture is based on the classical theory of correlations of fluctuations presented in the Landau-Lifshitz volume on statistical mechanics [3]. However, the methods used herein are not statistical, not quantum mechanical, and instead are based on Cartan's methods of exterior differential forms and their application to the topology of thermodynamic systems and their continuous topological evolution [21]. Landau and Lifshitz emphasized that real thermodynamic substances, near the thermodynamic critical point, exhibit extraordinary large fluctuations of density and entropy. In fact, these authors demonstrate that for an almost perfect gas near the critical point, the correlations of the fluctuations can be interpreted as a $1/\mathrm{r}$ potential giving a $1/\mathrm{r}^2$ force law of attraction. Hence, as a cosmological model, the almost perfect gas - such as a very dilute van der Waals gas - near the critical point yields a reason for both the granularity of the night sky and for the $1/\mathrm{r}^2$ force law ascribed to gravitational forces between for massive aggregates.

A topological (and non statistical) thermodynamic approach can be used to demonstrate how a four dimensional variety can support a turbulent, non-equilibrium, physical system with universal properties that are homeomorphic (deformable) to a van der Waals gas [41]. The method leads to the necessary conditions required for the existence, creation or destruction of topological defect structures in such a non-equilibrium system. For those physical systems that admit description in terms of an exterior differential 1-form of Action potentials of maximal rank, a Jacobian matrix can be generated in terms of the partial derivatives of the coefficient functions that define the 1-form of Action. When expressed in terms of intrinsic variables, known as the similarity invariants, the Cayley-Hamilton 4 dimensional characteristic polynomial of the Jacobian matrix generates a universal phase equation. Certain topological defect structures can be put into correspondence with constraints placed upon those (curvature) similarity invariants generated by the Cayley-Hamilton 4 dimensional characteristic polynomial. These constraints define equivalence classes of topological properties.

The characteristic polynomial, or Phase function, can be viewed as representing a family of implicit hypersurfaces. The hypersurface has an envelope which, when constrained to a minimal hypersurface, is related to a swallowtail bifurcation set. The swallowtail defect structure is homeomorphic to the Gibbs surface of a van der Waals gas. Another possible defect structure corresponds to the implicit hypersurface surface defined by a zero determinant condition imposed upon the Jacobian matrix. On 4 dimensional variety (space-time) , this non-degenerate hypersurface constraint leads to a cubic polynomial that always can be put into correspondence with a set of non-equilibrium thermodynamic states whose kernel is a van der Waals gas. Hence this universal topological method for creating a low density turbulent non-equilibrium media leads to the setting examined statistically by Landau and Lifshitz in terms of classical fluctuations about the critical point.

The conjecture presented herein is that non-equilibrium topological defects in a non-equilibrium 4 dimensional medium represent the stars and galaxies, which are gravitationally attracted singularities (correlations of fluctuations of density fluctuations) of a real gas near its critical point. Note that the Cartan methods do not impose (*a priori.)* a constraint of a metric, connection, or gauge, but do utilize the topological properties associated with constraints placed on the similarity invariants of the universal phase function.

Based upon the single assumption that the universe is a non-equilibrium thermodynamic system of Pfaff topological dimension 4 leads to a cosmology where the universe, at present, can be approximated in terms of the non-equilibrium states of a very dilute van der Waals gas near its critical point. The stars and the galaxies are the topological defects and coherent (but not equilibrium) self-organizing structures of Pfaff topological dimension 3 formed by irreversible topological evolution in this non-equilibrium system of Pfaff topological dimension 4.

The turbulent non-equilibrium thermodynamic cosmology of a real gas near its critical point yields an explanation for:

1. The granularity of the night sky as exhibited by stars and galaxies.
2. The Newtonian law of gravitational attraction proportional to $1/\mathrm{r}^2$.
3. The expansion of the universe (4th order curvature effects).
4. The possibility of domains of negative pressure (explaining what has recently been called dark energy) due to a classical Higgs mechanism for aggregates below the critical temperature (3rd order curvature effects)
5. The possibility of domains where gravitational effects (2nd order Gauss curvature effects) appear to be related to entropy and temperature properties of the thermodynamic system.
6. The possibility of cohesion properties (explaining what has recently been called dark matter) due to string or surface tension (1st order Mean curvature effects)
7. Black Holes (generated by Petrov Type D solutions in gravitational theory [1]) are to be related to Minimal Surface solutions to the Universal thermodynamic 4th order Phase function.

The bulk of these ideas were established befor it was realized that the Falaco Solitons were artifacts of zero mean curvature in Minkowski 3D space. The only modification to the thermodynamic (metric independent) theory is to demonstrate that the not only the presence of matter, but also the state of matter, can induce a physical metric on the topological domain.

6 Summary

As the Falaco phenomena appears to be the result of a topological defect, it follows that as a topological property of hydrodynamic evolution, it could appear in any density discontinuity, at any scale. This rotational pairing mechanism, as a topological phenomenon, is independent from size and shape, and could occur at both the microscopic and the cosmic scales. In fact, as mentioned above, during the formative stages of the Falaco Soliton pair, the decaying vortex structures exhibit spiral arms easily visible as caustics emanating from the boundary of each vortex core. The observation is so striking that it leads to the conjecture: Can the nucleus of M31 be connected to the nucleus of our Milky way galaxy by a tubular cosmic thread? Can material be ejected from one galaxy to another along this comic thread? Can barred spirals be Spiral Arm galaxies at an early stage of formation - the bar being and exhibition of material circulating about the stabilizing thread? At smaller scales, the concept also permits the development of another mechanism for producing spin-pairing of electrons in the discontinuity of the Fermi surface, or in two dimensional charge distributions. Could this spin pairing mechanism, depending on transverse wave, not longitudinal wave, coupling be another mechanism for explaining superconductivity? As the defect is inherently 3-dimensional, it must be associated with a 3-form of Topological Torsion, A^dA, introduced by the author in 1976 [4] [19] [20] [27], but now more commonly called a Chern Simons term when associated with geometrical connections. These ideas were exploited in an attempt to explain high TC superconductivity [22]. To this author the importance of the Falaco Solitons is that they offer the first clean experimental evidence of topological defects taking place in a dynamical system. Moreover, the experiments are fascinating, easily replicated by anyone with access to a swimming pool, and stimulate thinking in almost everyone that observes them, no matter what his field of expertise. They certainly are among the most easily produced solitons.

7 Some History

Just after WW II, one of my first contacts at MIT was a Brazilian young man named JOSE' HAROLDO RIBEIRO FALCÃO. He was in metallurgy and I was in physics. We became close friends and roommates during the period 1946-1950. He spent much of his time chasing the girls and playing soccer for MIT. Now MIT is not known for its athletic achievements, and when one weekend Haraldo scored two goals - giving MIT one of its few wins (ever) - the sports section of one of the Boston papers, misspelled his name with the headline ~

"FALACO SCORES TWO GOALS - MIT WINS"

Frankly I do not remember the exact headline from more than 55 years ago, but one thing is sure: Haraldo was known as FALACO ever since.

Haraldo moved back to Brazil and our ways parted. I became interested in many things, the most pertinent to this story included topological defects and topological evolution in physical systems. In 1986 I thought it would be great fun to go to Rio to see my old college friend, and then go to Machu Pichu to watch Haley's comet go by. My son was an AA pilot, so as parents we got a free Airline Ticket ticket to Brazil. Haraldo had married into a very wealthy family and had constructed a superb house, that his wife had designed, hanging onto a cliff-side above Sao Coronado beach south of Rio. Haraldo had a white marble swimming pool next to the house fed by a pristine stream of clear water.

The morning after my wife and I arrived in Rio (Haraldo's chauffeur met us at the airport in a big limo) I got up, after sleeping a bit late, and went to the pool for a morning dip. Haraldo and his family had gone to work, and no one was about. I sat in the pool, wondering about the fortunes of life, and how Haraldo - who I had help tutor to get through MIT - was now so very wealthy, and here I was - just a poor university professor. I climbed out of the pool, and was met by two servants who had been waiting in the wings. One handed me a towel and a terry cloth robe, and the other poured coffee and set out some breakfast fruit, croissants, etc.

I put a lot of sugar into the strong Brazilian coffee, as Haraldo had taught me to do long ago, and was stirring the coffee when I turned toward the pool (about 5-10 minutes after climbing out of the pool). In the otherwise brilliant sunshine, two black disks (about 15 cm in diameter) with bright halo rings were slowing translating along the pool floor. The optics caught my attention. Is there something about the southern hemisphere that is different? Does the water go down the drain with a different rotation? What was the cause of these Black Discs?

I went over to the pool, jumped in to investigate what was going on, and Voila!!!, the black discs disappeared. I thought: Here was my first encounter of the third kind and I blew it.

I climbed out of the pool, again, and then noticed that a pair of what I initially thought to be Rankine vortices was formed as my hips left the water, and that these rotational surfaces (which would be surface depressions of positive Gauss curvature if they were Rankine vortex structures) decayed within a few seconds into a pair of rotational surfaces of negative Gauss curvature. Each of the ultimate rotational surfaces were as if someone had depressed slightly a rubber sheet with a pencil point forming a dimple. As the zero mean curvature surfaces stabilized, the optical black disks were formed on the bottom of the pool. The extraordinary thing was that the surface deformations, and the black spots, lasted for some 15 minutes !!!. They were obviously rotational solitons.

The rest is history, and is described on my website and several published articles in some detail. The first formal presentation was at the 1987 Austin Dynamic Days get together, where my presentation and photos cause quite a stir. The Black Discs were quickly determined to be just an artifact of Snell's law of refraction of the solar rays interacting with the dimpled surfaces of negative Gauss curvature. What was not at first apparent was that there is a circular "string" – a 1D topological defect – that connects the two 2D topological defects of zero mean curvature curvature. The string extends from one dimple to the other, and is evident if you add a few drops of dye to the water near the rotation axis of one of the "dimples". Moreover, experimentation indicated that the long term soliton stability was due to the global effect of the "string" connecting the two dimpled rotational surfaces. If the arc is sharply severed, the dimples do not "ooze" away, as you would expect from a diffusive process; instead they disappear quite abruptly. It startled me to realize that the Falaco Solitons have the confinement properties (and problems) of two quarks on the end of a string.

I called the objects FALACO SOLITONS, for they came to my attention in Haraldo's pool in Rio. Haraldo will get his place in history. I knew that finally I had found a visual, easily reproduced, experiment that could be used to show people the importance and utility of Topological Defects in the physical sciences, and could be used to promote my ideas of Continuous Topological Evolution.

The observations were highly motivating. The experimental observation of the Falaco Solitons greatly stimulated me to continue research in applied topology, involving topological defects, and the topological evolution of such defects which can be associated with phase changes and thermodynamically irreversible and turbulent phenomena. When colleagues in the physical and engineering sciences would ask "What is a topological defect?" it was possible for me to point to something that they could replicate and understand visually at a macroscopic level.

The topological ideas have led [48] ultimately to

1. A non-statistical method of describing processes that are thermodynamically irreversible.
2. Applications of Topological Spin and Topological Torsion in classical and quantum field theories.
3. Another way of forming Fermion pairs
4. A suggestion that spiral galaxies may be stabilized by a connecting "thread", and an explanation of the fact that stars in the far reaches of galactic spiral arms do not obey the Kepler formula.
5. A number of patentable ideas in fluids, electromagnetism, and chemistry.

More detail (with downloadable pdf files of almost all publications) may be found on the web site:

http://www.cartan.pair.com
RMK
69 Saint Donat
84380 Mazan, France

The original observation was first described at a Dynamics Days conference in Austin, TX, [11] and has been reported, as parts of other research, in various hydrodynamic publications, but it is apparent that these concepts have not penetrated into other areas of research. As the phenomena is a topological issue, and can happen at all scales, the Falaco Soliton should be a natural artifact of both the sub-atomic and the cosmological worlds. The reason d'etre for this short article is to bring the idea to the attention of other researchers who might find the concept interesting and stimulating to their own research

8 Acknowledgements

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10 About the Author

Professor R. M. Kiehn, B.Sc. 1950, Ph.D. 1953, Physics, Course VIII, MIT, started his career working (during the summers) at MIT, and then at the Argonne National Laboratory on the Navy's nuclear powered submarine project. Argonne was near his parents home in the then small suburban community known as Elmhurst, Illinois. At Argonne, Dr. Kiehn was given the opportunity to do nuclear experiments using Fermi's original reactor, CP1. The experience stimulated an interest in the development of nuclear energy. After receiving the Ph. D. degree as the Gulf Oil Fellow at MIT, Dr. Kiehn went to work at Los Alamos, with the goal of designing and building a plutonium powered fast breeder reactor, a reactor that would produce more fissionable fuel then it consumed. He was instrumental in the design and operation of LAMPRE, the Los Alamos Molten Plutonium Reactor Experiment. He also became involved with diagnostic experiments on nuclear explosions, both in Nevada on shot towers above ground, and in the Pacific from a flying laboratory built into a KC-135 jet tanker. He is one of the diminishing number of people still alive who have witnessed atmospheric nuclear explosions.

Dr. Kiehn has written patents that range from AC ionization chambers, plutonium breeder reactor power plants, to dual polarized ring lasers and down-hole oil exploration instruments. He is active, at present, in creating new devices and processes, from the nanometer world to the macroscopic world, which utilize the features of Non Equilibrium Systems and Irreversible Processes, from the perspective of Continuous Topological Evolution.

Dr. Kiehn left Los Alamos in 1963 to become a professor of physics at the University of Houston. He lived about 100 miles from Houston on his Pecan Orchard - Charolais Cattle ranch on the banks of the San Marcos river near San Antonio. As a pilot, he would commute to Houston, and his classroom responsibilities, in his Cessna 172 aircraft. He was known as the "flying professor".

He is now retired, as an "emeritus" professor of physics, and lives in a small villa at the base of Mount Ventoux in the Provence region of southeastern France. He maintains at active scientific website at

http://www.cartan.pair.com