18.354J/3541
Nonlinear dynamics: continuum systems

Spring 2020

Lectures:	MW 2:30-4.00 in 253-7826	
Instructor:	Jörn Dunkel	
Contact:	`dunkel@mit.edu`	253-7826 (office phone)
Office Hours:	Thursdays 2:30-3:30 (2-381)	
Course website:	`math.mit.edu/~dunkel/Teach/18.354/`	

This course introduces the basic ideas for understanding the dynamics of continuum systems, by studying specific examples from a range of different fields. Our goal will be to explain the general principles, and also to illustrate them via important physical effects. A parallel goal of this course is to give you an introduction to mathematical modeling.

The equations of motion that we derive for continuum systems are typically nonlinear partial differential equations, for which it is very difficult to obtain analytical solutions. We shall therefore briefly foray into dimensional analysis to see how it is possible to obtain qualitative information about a system without having to solve the full equations of motion.

The first part of the course will study diffusion, to demonstrate how continuum descriptions arise from averaging microscopic degrees of freedom. We will discuss how solution techniques for the diffusion equation generalize to Fokker-Planck equations and quantum dynamical systems.

We shall then study the 'calculus of variations', which is a minimization approach to finding solutions of continuous systems. We are familiar with these techniques for discrete systems and now adapt the ideas to help us with continuous systems. First, we examine the classical brachiostrome problem posed by Bernoulli, and then consider an array of problems in many different physical systems (e.g., shapes of soap films, bending of elastic beams, etc.)

In the second half of the course we will examine hydrodynamic problems. We will discuss derivations of Navier-Stokes-type equations and study particular solutions. In the last part, we will give an outlook on the mathematical description of solitons, active biological systems and topological defects.

GRADING

- 60%: Problem sets
- 10%: Project proposal
- 30%: Final project presentation + report

TEXTBOOKS

Although there are no textbooks which follow the precise spirit of this course, there are literally hundreds of textbooks where the topics we will cover are discussed. For most lectures, typed notes can be downloaded from the course webpage. Useful books for further reading include:

- M. Cross, *Pattern Formation and Dynamics in Non-equilibrium Systems*, Cambridge University Press (2009).
- A. Goriely, *The Mathematics and Mechanics of Biological Growth*, Springer (2017).
- B. Audoly & Y. Pomeau, *Elasticity & Geometry*, Oxford University Press (2010).
- D. J. Acheson, *Elementary Fluid Dynamics*, Oxford University Press (1990).

HOMEWORK - PROBLEM SETS

Homework will be assigned roughly every two-three weeks. We aim to have 4 problem sets in total. Each homework set may contain analytical and computational problems, and even the odd experiment. Assignments must be handed in at the start of class on the due date.

First unexcused late homework score will be multiplied by 3/4. No subsequent unexcused late homework is accepted.

You are welcome to discuss solution strategies and even solutions, but please write up the solution on your own. Be sure to support your answer by explaining important steps or listing any relevant theorems. Be as clear and concise as possible. I strongly encourage the computational problems to be written in MATHEMATICA, MATLAB or JULIA.

FINAL PROJECT

The ideas we will be discussing have applications to many fields, many of which we will not cover. To give you a chance to explore an area of interest to you, the course will require a final project, in which you explore in depth something of interest to you and within the course's scope. Final projects will be presented in class during the final two classes.

IMPORTANT DATES

- Wed Feb 26 - Problem Set 1 DUE
- Wed Mar 11 - Problem Set 2 DUE
- Wed Mar 18 - Proposal (1 page) for final project DUE
- Wed Apr 1 - Problem Set 3 DUE
- Wed Apr 22 - Problem Set 4 DUE
- May 3, 5 & 10 - Final project presentations
- May 10 - Final project report DUE

Note: The exact due dates for the P-sets may be subject to change