Seven Sketches in Compositionality:  
Real-World Applications of Category Theory

Topic 1. Cascade effects — Posets and adjunctions

I. Posets, and transitivity
II. Monotone functions, variance, and structure preserving maps
III. Galois connections and adjunctions
IV. Closures, kernels, monads, comonads
V. Contagion

Topic 2. Data transformations — Categories, functors, and universal constructions

I. Databases: schemas, instances, transformations
II. Definition of category and functor, the category Set
III. Graphs and autonomous dynamical systems

Topic 3. Resource theory — Monoidal posets and categorification

I. Resources requirements and process plans
II. Definition of monoidal posets
III. Categorification: “from whether to which”

Topic 4. Collaborative design — Enriched categories and profunctors

I. Problem statement: doing one part in a larger project
II. Monotone design problems
III. Boolean-enriched categories, functors, and profunctors
IV. The compact-closed monoidal category
V. Framed bicategory structure

Topic 5. Signal flow graphs — Props and graphical proof systems

I. Props
II. Generators and relations
III. The prop of linear relations
IV. Theorem: Sound and complete
V. Signal flow graphs and stream semantics
VI. Direction of signal flow as a derived concept

**Topic 6.** Electric circuits — *Wiring diagrams and functorial semantics*

I. String diagrams for categories and monoidal categories
II. Wiring diagrams (operads)
III. The hypergraph category of labelled graphs
IV. Functors that preserve extra structure
V. The hypergraph functor from circuits of resistors to linear relations

**Topic 7.** Logic of behavior — *Sheaves, toposes, and internal languages*

I. Problem statement: logic for composing machines
II. The topos of sheaves on the interval domain.
III. Type theories and semantics
IV. How a safety proof might look