We begin by recalling the definition of a site.

**Definition 0.0.1.** A site is a pair \((\mathcal{C}, J)\), where \(\mathcal{C}\) is a category and \(J\) is a function that assigns to each \(C \in \text{Ob}(\mathcal{C})\), a set \(J(C)\) in which each element \(T \in J(C)\) is itself a set \(T = \{t_\alpha : c_\alpha \to C\}\) of morphisms in \(\mathcal{C}\) with target \(C\); these data are required to satisfy axioms which we present after defining some terms. We call the sets \(T \in J(C)\) trait classifications on \(C\) and individual morphisms \(t \in T\) trait classes on \(C\). We require the following:

1. For each trait class \(t : c \to C\) on \(C\) and each map \(D \to C\) in \(\mathcal{C}\), the fiber product \(c \times_C D\) exists in \(\mathcal{C}\).
2. For any map \(f : D \to C\) in \(\mathcal{C}\) and any trait classification \(T \in J(C)\), the pullback \(f^*T = \{g^*(t) | t \in T\}\) is a trait classification on \(D\).
3. If \(T = \{t_\alpha : c_\alpha \to C\}\) is a trait classification on \(C\) and for each \(t_\alpha \in T\) the set \(U_\alpha = \{u_{\alpha,\beta} : b_{\alpha,\beta} \to c_\alpha\}\) is a trait classification on \(c_\alpha\), then the family of composites
   \[
   \bigcup_{t_\alpha \in T} \{b_{\alpha,\beta} \to C | u_{\alpha,\beta} \in U_\alpha\}
   \]
   is a trait classification on \(C\).
4. If \(f : B \to C\) is an isomorphism in \(\mathcal{C}\) then the singleton set \(\{f\}\) is a trait classification on \(C\).

**Definition 0.0.2.** Let \((\mathcal{C}, J)\) denote a site, \(X\) a topological space, and \(\text{Shv}(X)\) the category of sheaves of sets on \(X\). A functor \(F : \mathcal{C} \to \text{Shv}(X)\) is called topological if it preserves the finite limits which exist in \(\mathcal{C}\) and if, for every trait classification \(T \in J\), the induced map of sheaves
\[
\left( \prod_{\{t_\alpha : c_\alpha \to C\} \in T} F(c_\alpha) \right) \to F(C)
\]
is surjective.

Given a pair of topological functors \(F, G : \mathcal{C} \to \text{Shv}(X)\), we will say that a natural transformation \(a : F \to G\) is topological if, for every trait class \(c \to C\), the induced diagram of sheaves
\[
\begin{array}{ccc}
F(c) & \longrightarrow & G(c) \\
\downarrow & & \downarrow \\
F(C) & \longrightarrow & F(C)
\end{array}
\]
is a pullback square in \(\text{Shv}(X)\).
Example 0.0.3. If $X$ is a point, then $\text{Shv}(X) = \text{Sets}$ is the category of sets. A functor $F: C \to \text{Sets}$ is topological if it preserves finite limits and takes trait classifications to surjections.

I think that if $X$ is sober, then a functor $F: C \to \text{Shv}(X)$ is local if, for each point $x \in X$, the induced functor $F_x: C \to \text{Sets}$ on stalks (given by composition of $F$ with $\colim_{x \in U} -: \text{Shv}(X) \to \text{Sets}$) is topological.

Definition 0.0.4. Let $F: C \to \text{Sets}$ be a topological functor, and let $D = \{(C_1, T_1), \ldots, (C_n, T_n)\}$ denote a finite set of pairs $(C_i, T_i)$ in which $C_i \in \text{Ob}(C)$ is an object and $T_i \in J(C_i)$ is a trait classification on $C$. The D-display of $F$ is