SIMPLICIAL SETS MODEL NETWORKS.

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- Simplicial sets vs. graphs.
- Simplicial sets are a type of hypergraph (all faces are included).
- Example: word network ([LC]). Wall and Street are vertices on an edge.
- A functor from a simplicial set (representing a network) to the nerve of some category (like the category of vocabularies of various entities).

To be clear, $\mathcal{C}$ is a category of vocabularies if there is a functor $F: \mathcal{C} \to \text{Sets}$ (or perhaps $F: \mathcal{C} \to \text{sSets}$). The objects of $\mathcal{C}$ represent people and the morphisms represent “ways or contexts of communication.” The functor $F$ then says how the vocabularies are translated under these contexts.

Now, if $S$ is a simplicial set representing an arrangement of people in a room, say, then what is a map $S \to N\mathcal{C}$ to the nerve of $\mathcal{C}$? For each vertex in $S$, we get a “person” in $\mathcal{C}$, and for each edge we get a context of communication for the first person to speak to the second. The 2-simplicies are interesting. A person is speaking to two people simultaneously, and we have in mind a way that the second person can translate what is being said to the third person; the third person is then receiving two communications. In the case that $\mathcal{C}$ is a 1-category, we demand that these two communications are equal; in the case of higher categories we make a less stringent demand – there must be a path connecting them.

- Depending on context, you may or may not want ordered simplices. For example in a room of people, each grouping could be considered as unordered. In an English phrase, words are considered ordered.

REFERENCES

[LC] Lin, Chiang. A simplicial complex, a hypergraph, structure in the latent semantic space of document clustering