Definition 0.1. Let $\text{Str}$ be the category of strings. A semantic system on $\text{Str}$ consists of a site $(\mathcal{D}, J)$, together with a function $N: \bigcup(J) \to \text{Str}$. That is, every element $f: D_0 \to D$ of every covering family $j = \{D_\alpha \to D\} \in J(D)$ has an associated name $N(f) \in \text{Str}$.

Let $\mathcal{D}$ be a small site, called the category of domains. Objects of $\mathcal{D}$ may include things like the $\mathbb{R}$ and $\mathbb{Z}$. It may also include things like “Weight” which might be the set of real numbers also, mapping to $\mathbb{R}$ by the identity, but not receiving a map from $\mathbb{R}$. The idea is that a weight measurement is always a real measurement, but not every real measurement is a weight measurement. There may be objects like “weight x volume,” and it may map to objects like “average density.” Basically, $\mathcal{D}$ is the category of ways that things can be measured (as objects) and ways in which these measurements relate (morphisms).

Coverings in $\mathcal{D}$ might be classifications like “over 195 lbs” and “under 200 lbs” which cover.

Definition 0.2. Let $\mathcal{D}$ be a category. Define the category of metrics on $\mathcal{D}$, denoted $\text{Met}(\mathcal{D})$, to be the category whose objects are finite-limit preserving functors $F: \mathcal{D} \to \text{Sets}$, and whose morphisms are natural transformations of functors.

Example 0.3. Suppose that $\mathcal{D}$ is the category with four objects