

18.310 Assignment 1: Due Wednesday, September 10, 2008

Weekly assignments due at 4PM in the Undergraduate Math Office (UMO).

1) Verify that one can have a good non-adaptive 3 weighing scheme with **no good coin** for each of 9 through 13 coins; by producing a three-row matrix having each number of columns, obeying the conditions that each row sums to 0, and no 2 columns are the same or negative of one another; for each of these values of n . For 12 for fewer coins be able to determine if the bad coin is heavy or light.

Hint: this is very easy if you use the construction given in class for $n=13$
(You can do it any other way you want if you can find one.)

2) Construct a 4 weighing optimum scheme with the maximum number of coins, including one good coin (which you know), using the procedure described in class.. This means you construct a matrix which is 4 by something. It is best done on a spreadsheet. It is easy on a spreadsheet. Verify that this scheme is optimal. Explain why one good coin is needed for this optimal scheme. Use your scheme to build a 4 weighing automatic bad coin finder on a spreadsheet. This finder consist of your matrix and spaces to indicate the results of the weighing. Given those results you want to determine which of the coins is bad and whether it is heavy or light, if you can determine that.

(I find it useful to assign a ternary number to each column, and use the use the "if", and "sum" functions on the spreadsheet to accomplish this.)

3) Suppose the balance only had two outcomes: balance or unbalance, so that you do not know which side is heavier. Find a lower bound on the number of weighings for n coins. Find an explicit general solution that meets this bound.

4) Suppose we are given k weighings, no good coin, and we want to find the one bad coin and determine whether it is light or heavy. Find a good scheme for $(3^k)/2-x$ coins, for $x = 5/2, 7/2, 9/2$. That is generalize the results of problem 1.

5) Suppose there were seven weighings using the scheme and notations described in class, and the results were 0,1,0,0,1,-1,0
Was the bad coin heavy or light? Justify your answer, and give a general solution that works for any input and any number of weighings.

6) Suppose there are two bad coins. Find a pigeonhole principle lower bound on the number of weighings need to find them given there are n coins to begin with, and no good coins.