2008 Exam #2 Study Questions for 18.310

1. **Finding Primes:** Describe a reasonably efficient way to find 100 decimal digit primes.
2. **Raising to a power:** Describe an efficient way to raise a number, x, to a high power, y, mod a large number, z.
3. **Groups and Lagrange's theorem:** be prepared to respectively define and state it, prove the latter, and use it.
4. **Euclid's algorithm:** Given 10 digit integers, A and B, how would you implement finding their gcd on a spreadsheet? Expressing that gcd as a linear combination of A and B?
5. **Chinese Remainder Theorem:** State and prove this theorem.
6. **More Chinese Remainder Theorem:** Explain how would you find a number mod PQ that was congruent to A mod P and B mod Q, where P and Q are two relatively prime large numbers.
7. **RSA Algorithm:** Explain how it works.
8. **Carmichael Numbers:** Define Camrichael numbers. Prove that a product of two primes PQ cannot be a Carmichael number.
9. **Five color theorem:** State and prove it.
10. **Kuratowski Theorem:** Given some graphs, which are planar (where the graph is described by edges as vertex pairs or as a diagram)?
11. **FFT:** Describe how one multiplies numbers using the FFT.
12. **More FFT:** Implement it on a spreadsheet using 32^{nd} roots of unity.
13. **Even more FFT:** What is the basic recursion of the FFT?
14. **A different FFT:** What is the Finite Fourier Transform and how can it be inverted?
15. **Sequential Choice:** Explain the optimal strategy for finding the best rank in sequential choice. Prove that it is optimal. How well does it work?
16. **More Sequential Choice:** Use a spreadsheet to compute the exact probability of successfully selecting the best choice in a sequential situation with 20 candidates. Where is the best threshold?
17. **Even more Sequential Choice:** Use a spreadsheet to find the best strategy for minimizing the expected rank in a sequential situation with 20 candidates.
18. **Generating Functions:** Find the formula for the number of ways of tiling a $2\times n$ strip with 2×1 tiles.
19. **More Generating Functions:** Find the formula for the number of ways of tiling a $3\times n$ strip with 2×2 and 1×1 square tiles.
20. **Counting Trees:** Let d(i,T) be the degree of vertex i in tree T. If you sum

$$x_1^{d(1,T)-1}\, x_2^{d(2,T)-1}\, x_3^{d(3,T)-1} \ldots x_n^{d(n,T)-1}$$

over all labeled trees T with n nodes, you obtain

$$(x_1 + x_2 + \ldots + x_n)^{n-2}.$$

Use this formula to calculate the number of trees where the degree of vertex 1 is exactly k.