

### 18.310 Review Questions Exam 1

1. A Weighing: create a scheme for 12 coins one of which is heavy or light all coins must go on scale at least once, B what is max number of coins can be handled in this way without a good coin in  $k$  weighings
2. Sorting: describe heapsort in a few concise sentences. What are its advantages?
3. Median finding algorithm. Suppose  $N$  is gigantic and you pick out 10000 keys at random and find their median what is the probability it is not rank between  $.2N/5$  and  $3N/5$ ? (hint compute probability assuming it has rank exactly  $2N/5$  that half the others lie above it and half below it. Show that as you get further from the middle probabilities go down exponentially, so the answer is only a small factor larger than twice this.
4. Draw a diagram of the comparisons in Batchers algorithm on 16 keys.
5. Huffman and Hu Tucker Given a sequence of frequencies of words., find an optimal Huffman and Hu-Tucker code.
6. give an argument for  $n!$  having form  $(n/e)^n * W(n)$  where  $W(n)$  is  $O(n)$ .
7. What is the Weak Law of Large Numbers? Outline a proof
8. State and outline a proof of Shannons first (clear channel capacity) theorem
9. Describe : Lempel Ziv in your own words as concisely as possible.
10. State and Prove the Fundamental Theorem of Algebra for an arbitrary field.
11. Describe (apply?) a Test for primitivity of a polynomial
12. For a primitive polynomial, of degree 3 show which powers of  $x$  obey which equations. Do the same for a primitive polynomial of degree 4.
13. Shannons second (noisy channel) theorem; describe and outline a proof
14. give a matrix code that encodes four bits and can correct one error show how to find it.
15. given a primitive polynomial  $p$  find  $p^3$
16. find up 3 table for the primitive polynomial given
17. if there are  $k$  errors prove relations between elementary symmetric functions for powers of terms at least  $k$
18. write the error locator polynomial in terms of  $t$ 's for up to 2 errors
19. describe how to test powers to see if there are errors in a 2 error correcting bch code
20. write relevant relations between elementary symmetric functions and power sum symmetric functions for a three error correcting code
21. how many message bits can a primitive polynomial of degree 10 handle. If you construct a 3 error correcting code, how many message bits can there be? If each bit has a probability  $.1/10000$  of an error, what is the probability of there being 4 or more errors in a code on 1023 bits? What (roughly) is the probability that you will make a false decoding