Conjugacy classes and group representations

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Joint Mathematics Meeting, January 7, 2016

Outline

The talk in one slide

Want to understand representations of any group G.

I'll give some hints about why this is interesting.

In case you didn't have it beaten into your head by Paul Sally.

And Bert Kostant. And Armand Borel. And Michele Vergne...

Representations of $G \overset{\text{crude}}{\leftrightsquigarrow}$ conjugacy classes in G.

Better: relation is like duality for vector spaces.

dim(representation) $\leftrightsquigarrow$ (size)$^{1/2}$(conjugacy class).

Talk is examples of when things like this are true.

Two cheers for linear algebra

My favorite mathematics is linear algebra.

Complicated enough to describe interesting stuff.

Simple enough to calculate with.

Linear map $T\colon V \to V \rightsquigarrow$ eigenvalues, eigenvectors.

First example: $V =$ fns on $\mathbb{R}$, $S =$ chg vars $x \mapsto -x$.

Eigvals: ± 1. Eigspace for $+1$: even fns (like $\cos(x)$).
Eigspace for -1: odd functions (like $\sin(x)$).

Linear algebra says: to study sign changes in x, write fns using even and odd fns.

Second example: $V =$ functions on $\mathbb{R}$, $T = \frac{d}{dx}$.

Eigenvals: $\lambda \in \mathbb{C}$. Eigspace for λ: multiples of $e^{\lambda x}$.

Linear algebra says: to study $\frac{d}{dx}$, write functions using exponentials $e^{\lambda x}$.

The third cheer for linear algebra

Best part about linear algebra is noncommutativity. . .

Third example: $V =$ fns on $\mathbb{R}$, $S = (x \mapsto -x)$, $T = \frac{d}{dx}$.

S and T don't commute; can't diagonalize both.

Only common eigenvectors are constant fns.

Representation theory idea: look at smallest subspaces preserved by both S and T.

$$W_{\pm\lambda} = \langle \underbrace{\cosh(\lambda x)}_{\text{even}}, \underbrace{\sinh(\lambda x)}_{\text{odd}} \rangle = \langle e^{\lambda x}, e^{-\lambda x} \rangle.$$

Definition of representation

Here's a general setting for not-all-diagonalizable...

I'll talk about groups; same words apply to algebras.

G group; representation of G is

1. (complex) vector space V, and
2. collection of linear maps $\{T_g \colon V \to V \mid g \in G\}$

subject to

$$T_g T_h = T_{gh}, \qquad T_e = \text{identity}.$$

Subrepresentation is subspace $W \subset V$ such that

$$T_g W = W \qquad (\text{all } g \in G).$$

Rep is irreducible if only subreps are $\{0\} \neq V$.

Irreducible subrepresentations are minimal nonzero subspaces of V preserved by all T_g.

Gelfand program. . .

. . . for using repn theory to do other math.

Say group G acts on space X.

Step 1: LINEARIZE. $X \rightsquigarrow V(X)$ vec space of fns on X. Now G acts by linear maps.

Step 2: DIAGONALIZE. Decompose $V(X)$ into minimal G-invariant subspaces.

Step 3: REPRESENTATION THEORY. Study minimal pieces: irreducible reps of G.

Step 4: PRETENDING TO BE SMART. Use understanding of $V(X)$ to answer questions about X.

Hard steps are 2 and 3: how does DIAGONALIZE work, and what do minimal pieces look like?

How many representations are there?

Big step in Gelfand program is describing the set

$$\widehat{G} = \{\text{all irr reps of } G\}.$$

When looking for things, helps to know how many...

Theorem Suppose G is a finite group.

1. $|\widehat{G}| = |\{\text{conj classes in } G\}|$.
2. $\sum_{(V,T)\in\widehat{G}} (\dim V)^2 = |G|$.
3. $\sum_{C\subset G \text{ conj class}} |C| = |G|$.

Theorem suggests two possibilities:

1. Bijection (conj classes in G) $\overset{?}{\leftrightsquigarrow}$ $\widehat{G}$, $C \overset{?}{\leftrightsquigarrow} V_C$.
2. $|C| \overset{?}{=} (\dim V_C)^2$.

Neither is true; but each has elements of truth...

Example: $G = S_n$ = permutations of $\{1, \ldots, n\}$.

Will see that both conj classes in G and $\widehat{G}$ are indexed by partitions of n: expressions $n = p_1 + p_2 \cdots + p_r$, $p_1 \geq p_2 \geq \cdots \geq p_r > 0$.

Partitions, conj classes, repns

S_n = permutations of $\{1, \ldots, n\}$ symmetric group.

$\pi = (p_1, \ldots, p_r)$ decr, $\sum p_i = n$ partition of n.

$S_\pi = S_{p_1} \times S_{p_2} \times \cdots \times S_{p_r} \subset S_n$.

Conj class $C \leftrightsquigarrow$ smallest π so $S_{{}^t\pi} \cap C \neq \emptyset$.

Columns of π = cycle sizes of C.

Irr rep $(V, T) \leftrightsquigarrow$ largest π so $(V, T)|_{S_\pi} \supset$ trivial.

Theorem. These correspondences define bijections

$$(\text{conj classes in } S_n) \leftrightarrow (\text{partitions of } n) \leftrightarrow \widehat{G}$$

S_3			S_4		
$\lvert C_\pi\rvert$	part. π	$(\dim V_\pi)^2$	$\lvert C_\pi\rvert$	part. π	$(\dim V_\pi)^2$
1		1	1		1
3		4	6		9
2		1	3		4
			8		9
			6		1

Lessons learned

Conclusion: For S_n there is a natural bijection

$$\text{(conj classes)} \leftrightarrow \text{(irr repns)}, \qquad C_\pi \leftrightarrow V_\pi.$$

But $|C_\pi|$ not very close to $(\dim V_\pi)^2$.

Maybe interesting question: find bijection relating two formulas $\sum_\pi (\dim V_\pi)^2 = \sum_\pi |C_\pi|$ for $|S_n|$.

$GL_n(\mathbb{F}_q)$: conjugacy classes

Seek (conj classes) $\overset{?}{\longleftrightarrow}$ (irr reps) for other groups.

Try next $GL_n(\mathbb{F}_q)$, invertible $n \times n$ matrices $/\mathbb{F}_q$.

$$\begin{aligned}|GL_n(\mathbb{F}_q)| &= (q^n-1)(q^n-q)\cdots(q^n-q^{n-1})\\ &= (q^{n-1}-1)(q^{n-2}-1)\cdots(q-1)\cdot 1\cdot q\cdots q^{n-1}\\ &= \underbrace{(1+q+\cdots+q^{n-1})(1+q+\cdots+q^{n-2})\cdots(1)}_{q\text{-analog of } n!}\\ &\quad\cdot (q-1)^n \cdot q^{n(n-1)/2}\end{aligned}$$

$GL_n(\mathbb{F}_q)$ is q-analogue of S_n.

partition π of m, Galois orbit $\Lambda = \{\lambda_1, \ldots \lambda_d\} \subset \overline{\mathbb{F}_q}^\times$, $\rightsquigarrow$ conj class $c(\pi, \Lambda) \subset GL_{md}(\mathbb{F}_q)$.

General class in $GL_n(\mathbb{F}_q)$ = partition-valued function π on Galois orbits $\Lambda \subset \overline{\mathbb{F}_q}^\times$ such that

$$\sum_\Lambda |\underbrace{\Lambda}_{\text{eigval}}| \cdot \underbrace{|\pi(\Lambda)|}_{\text{mult}(\Lambda)} = n.$$

($\pi(\Lambda) = \emptyset$ for most Λ.)

$GL_n(\mathbb{F}_q)$: representations

Saw that conj class in $GL_n(\mathbb{F}_q)$ is partition-valued function on Galois orbits on $\bigcup_{d\geq 1} \mathbb{F}_{q^d}^\times$ (19th century linear algebra).

Similarly irr rep of $GL_n(\mathbb{F}_q)$ is partition-valued function on Galois orbits on $\bigcup_{d\geq 1} \widehat{\mathbb{F}_{q^d}^\times}$ Green 1955.

$GL_2(\mathbb{F}_q)$

conj class C	# classes	$\lvert C\rvert$
diag, 2 ev	$(q-1)(q-2)/2$	$q(q+1)$
nondiag, 2 ev	$q(q-1)/2$	$q(q-1)$
$\square\square$	$q-1$	$(q+1)(q-1)$
$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$q-1$	1

repn V	# repns	dim V
princ series	$(q-1)(q-2)/2$	$q+1$
disc series	$q(q-1)/2$	$q-1$
$\square\square$	$q-1$	q
$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$q-1$	1

Conclude (conj classes) $\leftrightarrow$ (irr reps), but not naturally: depends on choice of isom $\mathbb{F}_{q^d}^\times \simeq \widehat{\mathbb{F}_{q^d}^\times}$.

Bijection has $\lvert C_\pi\rvert \approx (\dim V_\pi)^2$.

Back to functions on $\mathbb{R}$

$G_1 = \mathbb{R} =$ translations on $\mathbb{R}$, $(T_t f)(x) = f(x - t)$.

Lie alg $\mathfrak{g}_1 = \mathbb{R}\frac{d}{dx}$; irr rep $\mathbb{C}_\lambda =$ multiples of $e^{-i\lambda x}$.

$G_2 = \mathbb{R} =$ exp mults on $\mathbb{R}$, $(M_\xi f)(x) = e^{-ix\xi} f(x)$.

Lie alg $\mathfrak{g}_2 = \mathbb{R}ix$; irr rep $\mathbb{C}_y =$ delta fns at y.

$Z = \mathbb{R} =$ phase shifts on $\mathbb{R}$, $(P_\theta f)(x) = e^{i\theta} f(x)$.

Lie alg $\mathfrak{z} = i\mathbb{R}$.

Theorem $G =$ group of linear transformations of fns on $\mathbb{R}$ generated by G_1, G_2, and Z.

1. $T_t M_\xi = M_\xi T_t P_{t\xi}$; P_θ commutes with T_t and M_ξ.
2. Every element of G is uniquely a product $T_t M_\xi P_\theta$.
3. $\left[\frac{d}{dx}, ix\right] = i$.
4. $L^2(\mathbb{R}) =$ irr rep of G; unique rep where $P_\theta = e^{i\theta} I$.

Note two versions of canonical comm relations of quantum mechanics; G is Heisenberg group.

Conjugacy classes in Heisenberg group. . .

$\ldots =$ conj classes in Lie alg $\mathfrak{g} = \{t\frac{d}{dx} + i\xi x + i\theta\}$.

$$\mathrm{Ad}(T_{t_1} M_{\xi_1} P_{\theta_1}) \begin{pmatrix} t \\ \xi \\ \theta \end{pmatrix} = \begin{pmatrix} t \\ \xi \\ \theta + t_1\xi - \xi_1 t \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\xi_1 & t_1 & 1 \end{pmatrix} \begin{pmatrix} t \\ \xi \\ \theta \end{pmatrix}.$$

2-diml family of 1-diml conj classes (each fixed $(t, \xi) \neq (0, 0)$), 1-diml family of 0-diml classes $\begin{pmatrix} 0 \\ 0 \\ \theta \end{pmatrix}$.

Repns dual to conj classes: orbits of G on $\mathfrak{g}^*$.

$$\mathrm{Ad}^*(T_{t_1} M_{\xi_1} P_{\theta_1}) \begin{pmatrix} \lambda \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda + \xi_1 z \\ y - t_1 z \\ z \end{pmatrix}.$$

Now have 1-diml fam of 2-diml orbits (each $z \neq 0$), 2-diml fam of 0-diml orbits $\begin{pmatrix} \lambda \\ y \\ 0 \end{pmatrix}$.

Stone-von Neumann Theorem

Theorem (Stone-von Neumann) Irrs of Heisenberg G:

1. for each $z \neq 0$, rep on $L^2(\mathbb{R})_z$:

$$T_t \mapsto \text{transl by } zt, \quad M_\xi \mapsto \text{mult by } e^{-ix\xi}, \quad P_\theta \mapsto e^{iz\theta}.$$

2. for each (λ, y), 1-diml rep on $\mathbb{C}_{\lambda,y}$,

$$T_t \mapsto e^{i\lambda t}, \quad M_\xi \mapsto e^{iy\xi}, \quad P_\theta \mapsto e^{i0\cdot\theta} = 1.$$

Reps corr perfectly to orbits of G on $\mathfrak{g}^*$.

$$L^2(\mathbb{R})_z \leftrightarrow \text{2-diml} \begin{pmatrix} * \\ * \\ z \end{pmatrix}, \quad \mathbb{C}_{\lambda,y} \leftrightarrow \text{0-diml} \begin{pmatrix} \lambda \\ y \\ 0 \end{pmatrix}.$$

"Functional dim" of rep space is half dim of orbit.

Analogue of hope $\dim V_\pi \approx |C_\pi|^{1/2}$ for fin gps.

Philosophy of coadjoint orbits

G Lie group with Lie algebra $\mathfrak{g}$, dual vector space $\mathfrak{g}^*$.

Kirillov-Kostant philosophy of coadjt orbits suggests

$$\{\text{irr reps of } G\} =_{\text{def}} \widehat{G} \overset{?}{\leftrightsquigarrow} \{\text{orbits of } G \text{ on } \mathfrak{g}^*\} \qquad (\star)$$

More precisely. . . restrict right side to "admissible" orbits (integrality cond). Hope to get "most" of $\widehat{G}$: enough for (interesting parts of) Gelfand harmonic analysis.

Hope: orbit $X \leftrightsquigarrow$ rep $V_X =$ fns on Y, $\dim Y = (\dim X)/2$.

Hard part is finding $Y =$ "square root" of space X.

Evidence for orbit philosophy

With the caveat about restricting to admissible orbits...

$$\widehat{G} \overset{?}{\leftrightsquigarrow} \text{orbits of } G \text{ on } \mathfrak{g}^*. \qquad (\star)$$

($\star$) is true for G simply conn nilpotent (Kirillov 1962).

($\star$) is true for G type I solvable (Auslander-Kostant 1971).

($\star$) for algebraic G reduces to reductive G (Duflo 1982).

Case of reductive G is still open.

Actually ($\star$) false for connected nonabelian reductive G.

But there are still theorems close to ($\star$).

$GL_n(\mathbb{R})$

$G = GL_n(\mathbb{R})$ for Lie gps $\leftrightsquigarrow$ $GL_n(\mathbb{F}_q)$ for fin gps.

Lie algebra $\mathfrak{g}$ = all $n \times n$ real matrices $\simeq \mathfrak{g}^*$.

coadjt orbits = conj classes of $n \times n$ real matrices.

Real $n \times n$ matrix has eigenvalues

$$\underbrace{r_1, \ldots, r_p}_{\text{real}}, \underbrace{\{z_1, \overline{z_1}\}, \ldots, \{z_q, \overline{z_q}\}}_{\text{non-real}};$$

say r_i has mult m_i and $\{z_j, \overline{z_j}\}$ mult n_j, with

$$\sum_i m_i + 2\sum_j n_j = n.$$

Conj class $\leftrightsquigarrow$ partitions $|\pi(r_i)| = m_i$, $|\pi(\{z_j, \overline{z_j}\})| = n_j$.

Conj class $\leftrightsquigarrow$ partition-valued fn π on Galois orbits $\Lambda \subset \mathbb{C}$,

$$\sum_\Lambda |\Lambda| \cdot |\pi(\Lambda)| = n.$$

What's a conj class look like?

Fix eigenvalues $(r_i, \{z_j, \overline{z_j}\})$, multiplicities (m_i, n_j).

Ignore partitions for now (or take all to be $1 + 1 + \cdots$).

Matrix in conj class $\leftrightsquigarrow$ decomposition

$$\mathbb{R}^n = (E_1 \oplus \cdots \oplus E_p) \oplus (F_1 \oplus \cdots \oplus F_q), \quad \dim E_i = m_i, \ \dim F_j = 2n_j$$

together with complex structure on each F_j.

Matrix is scalar r_i on E_i, z_j on F_j.

$$\begin{aligned}\text{Conj class} &\simeq \text{manifold of all such decomps of } \mathbb{R}^n \\ &\simeq GL_n(\mathbb{R})/\big[GL_{m_1}(\mathbb{R}) \times \cdots \times GL_{m_p}(\mathbb{R}) \\ &\qquad \times GL_{n_1}(\mathbb{C}) \times \cdots \times GL_{n_q}(\mathbb{C})\big]\end{aligned}$$

Space depends only on ints $(m_i), (n_j)$, not on eigvals.

How do you make a rep from a conj class?

To simplify take real eigenvalues only: r_i with mult m_i.

Conj class $C_{(m_i)} \simeq$ decomps $\mathbb{R}^n = E_1 \oplus \cdots \oplus E_p, \quad \dim E_i = m_i$

$$\simeq GL_n(\mathbb{R})/\left[GL_{m_1}(\mathbb{R}) \times \cdots \times GL_{m_p}(\mathbb{R})\right]$$

Equivariant line bundles on $G/H \leftrightsquigarrow$ characters of H.

Eigenvalues (r_i) define character

$$\chi_{(r_i)} \colon GL_{m_1} \times \cdots \times GL_{m_p} \to \mathbb{C}^\times,$$

$$(g_1, \ldots, g_p) \mapsto |\det g_1|^{ir_1} \cdots |\det g_p|^{ir_p}$$

and so eqvt Herm line bundle $\mathcal{L}_{(r_i)} \to C_{(m_i)}$.

Recall V_C should be fns on a mfld of half dim of C...

$$\mathcal{F}_{(m_i)} = \text{flags } S_1 \subset \cdots \subset S_p = \mathbb{R}^n, \quad \dim S_i/S_{i-1} = m_i.$$

Have eqvt fibration $C_{(m_i)} \to \mathcal{F}_{(m_i)}$,

$$E_1 \oplus \cdots \oplus E_p \mapsto E_1 \subset E_1 \oplus E_2 \subset E_1 \oplus E_2 \oplus E_3 \cdots$$

$\dim \mathcal{F}_{(m_i)} = \dim C_{(m_i)}/2$, and $\mathcal{L}_{(r_i)}$ descends to $\mathcal{F}_{(m_i)}$.

$V_{(m_i),(r_i)} =$ half-density secs of $\mathcal{L}_{(r_i)} \to \mathcal{F}_{(m_i)}$, irr of $GL_n(\mathbb{R})$.

What about complex eigenvalues?

Look at matrices with eigvals $\{z_1, \overline{z_1}\}$, multiplicity n_1; $n = 2n_1$, $z_1 = a_1 + ib_1$. Admissible reqt is $b_1 \in \mathbb{Z}$.

$$\text{Conj class } C_{(n_1)} = \text{class of } a_1 \begin{pmatrix} I_{n_1} & 0 \\ 0 & I_{n_1} \end{pmatrix} + b_1 \begin{pmatrix} 0 & -I_{n_1} \\ I_{n_1} & 0 \end{pmatrix}$$
$$\simeq \text{complex structures on } \mathbb{R}^{2n_1}$$
$$\simeq GL_{2n_1}(\mathbb{R})/GL_{n_1}(\mathbb{C})$$

real manifold of dimension $(2n_1)^2 - 2n_1^2 = 2n_1^2$.

Admissible eigval $\{z_1, \overline{z_1}\}$ defines character

$$\chi_{\{z_1,\overline{z_1}\}} \colon GL_{n_1}(\mathbb{C}) \to \mathbb{C}^\times, \qquad h \mapsto |\det h|^{ia_1} \cdot \left(\frac{\det h}{|\det h|}\right)^{b_1 - n_1}$$

$\rightsquigarrow$ eqvt Herm line bundle $\mathcal{L}_{\{z_1,\overline{z_1}\}} \to C_{(n_1)}$.

Extra $-n_1$ is "half-density" twist.

Want $V_{(n_1),\{z_1,\overline{z_1}\}}$ on fns on a space of half dim of $C_{(n_1)}$.

Subgp $GL_{n_1}(\mathbb{C})$ is maximal: can't fiber $C_{(n_1)}$ over smaller.

Soln: $C_{(n_1)}$ is complex. Replace all fns by hol fns.

Speh representations

$C_{(n_1)} = GL_{2n_1}(\mathbb{R})/GL_{n_1}(\mathbb{C})$ complex mfld, $\dim_{\mathbb{C}} = n_1^2$.

$\chi_{\{a_1 \pm ib_1\}} \colon GL_{n_1}(\mathbb{C}) \to \mathbb{C}^\times, \quad h \mapsto |\det h|^{ia_1} \cdot \left(\frac{\det h}{|\det h|}\right)^{b_1 - n_1}$

$\rightsquigarrow \mathcal{L}_{\{a_1 \pm ib_1\}} \to C_{(n_1)}$ equiv holomorphic line bdle.

Rough idea: $V_{(n_1),\{a_1 \pm ib_1\}} =$ holom secs of $\mathcal{L}_{\{a_1 \pm ib_1\}}$.

Reason: holom fns on $C \approx$ all fns on mfld of half dim C.

Difficulty: $C_{(n_1)}$ has big compact submanifold

$Z_{(n_1)} = O(2n_1)/U(n_1) =$ orthogonal cplx structures,

$\dim_{\mathbb{C}} Z_{(n_1)} = n_1(n_1 - 1)/2$.

Consequence: no holom secs for $b_1 < n_1$.

Solution: replace holom secs by Dolbeault cohom.

$\rightsquigarrow$ irreducible Speh rep $V_{(n_1),\{a_1 \pm ib_1\}}$, $b_1 \leq 0$.

The rest of the story

For $G = GL_n(\mathbb{R})$, $C \subset \mathfrak{g}^*$ coadjt orbit...

...eigvals $\rightsquigarrow$ line bundle $\mathcal{L}$ on C.

Case of real eigvalues: get G rep by

1. [eigspaces $\rightsquigarrow$ flags] $\rightsquigarrow$ fibration $C \to F$
2. representation $=$ secs of $\mathcal{L}$ constant on fibers.

Case of complex eigvalues: get G rep by

1. [cplx structure on eigspaces] $\rightsquigarrow$ complex structure on C
2. representation $=$ (sheaf cohom of) holom secs of $\mathcal{L}$.

Combining ideas, get reps if C diagonalizable over $\mathbb{C}$.

Same ideas apply to any reductive Lie group G.

Need to replace (flag $\rightsquigarrow$ parabolic)

But that's just jargon, and we're all GREAT at jargon.

Get rep $V_C \leftrightsquigarrow$ any coadjt orbit $C \subset \mathfrak{g}^*$ semisimple.

What about conj classes of nilpotent matrices?

Conjugacy classes of nilpotent matrices

$C_\pi \subset \mathfrak{gl}_n(\mathbb{R})^*$ conj class of nilp matrices

$\leftrightsquigarrow \pi = (p_1, \ldots, p_r)$ partition of n (Jordan blocks).

Define ${}^t\pi = (q_1, \ldots, q_s)$ transpose partitition.

$$q_j = \#\{i \mid \pi_i \geq j\} = \dim\left(\ker X^j / \ker X^{j-1}\right) \qquad (X \in C_\pi)$$

Recall $\mathcal{F}_{{}^t\pi} =$ flags $(S_1 \subset \cdots \subset S_s = \mathbb{R}^n)$, $\quad \dim S_j/S_{j-1} = q_j$.

$\rightsquigarrow$ fibration $C_\pi \to \mathcal{F}_{{}^t\pi}$, $X \mapsto \left(\ker X \subset \ker X^2 \subset \cdots \subset \ker X^s = \mathbb{R}^n\right)$.

So can define $V_{C_\pi} =$ half-densities on $\mathcal{F}_{{}^t\pi}$.

This is like a case we already did...

$\leftrightsquigarrow C_{(r_j),(q_j)} =$ matrices with s eigvals (r_j), mults (q_j).

$C_{(r_j),(q_j)} \to \mathcal{F}_{{}^t\pi}$, $X \mapsto (\ker(X - r_1) \subset \cdots \subset \ker(X - r_1) \cdots (X - r_s))$.

Nilpotent class C_π is limit of semisimple conj classes $C_{(r_j),(q_j)}$.

Other Lie groups

Do we understand irreducible reps for Lie group G?

Recall Duflo more or less reduced to case G reductive.

For G reductive, attaching rep to $C \subset \mathfrak{g}^*$ reduces to C nilp.

Good news: $C =$ limit of semisimple $\rightsquigarrow$ know what to do.

Bad news: $G \neq GL_n\mathbb{R} \implies$ nilp $C \neq \lim(\text{semisimple})$.

Good news: there's still more math to do!

Perhaps there should be some sort of organization to support that?