

18.100B Problem Set 5

Due in class Monday, March 16. You may discuss the problems with other students, but you should write solutions entirely on your own.

1. Text, page 78, number 7. (Hint: the Schwartz inequality Theorem 1.35 can help.)

2. Text, page 81, number 16.

3. Suppose that X and Y are metric spaces, $f: X \rightarrow Y$ is any function, and $p \in X$. Show (using Definition 4.5) that f is continuous at p if and only if the following condition is satisfied: for every sequence $\{p_n\}$ in X that converges to p , we have

$$\lim_{n \rightarrow \infty} f(p_n) = f(p)$$

in Y .

4. (If you've seen one convergent sequence, you've seen them all.) Let X be the metric space consisting of the points $\{1/n | n = 1, 2, 3, \dots\}$ and 0 in \mathbb{R} , with the distance function coming from \mathbb{R} . Let Y be any metric space, $\{p_n\}$ a sequence in Y , and p_0 any point of Y . Define a function $f: X \rightarrow Y$ by

$$f(1/n) = p_n, \quad f(0) = p_0.$$

Show that f is continuous if and only if the sequence $\{p_n\}$ converges to p_0 .