

18.100B Problem Set 3

Due in class Monday, February 23. You may discuss the problems with other students, but you should write solutions entirely on your own.

1. Text, page 44, number 12.

2. Text, page 44, number 17.

A metric space X is said to be *totally bounded* if for every positive real number r , X is contained in a finite union of neighborhoods of radius r . That is, (for every $r > 0$) there are finitely many points p_1, \dots, p_n in X so that

$$X = N_r(p_1) \cup \dots \cup N_r(p_n).$$

3.

- a) Suppose that X is totally bounded and that $Y \subset X$. Show that Y is totally bounded.
- b) Show that the unit interval $[0, 1]$ (with its usual metric) is totally bounded.

4. This problem concerns a metric on the set L of infinite words in the letters 0 and 1, introduced in the second problem set. If

$$x = a_1 a_2 \dots, \quad y = b_1 b_2 \dots \quad (a_i, b_j \in \{0, 1\})$$

are two elements of L , define

$$d(x, y) = \begin{cases} 0, & \text{if } x = y \\ 2^{-(j-1)}, & \text{if } a_1 = b_1, \dots, a_{j-1} = b_{j-1}, \text{ but } a_j \neq b_j \end{cases}$$

(This is called the *2-adic distance from x to y* .) For example,

$$d(10011\dots, 00010\dots) = 1,$$

and

$$d(10011\dots, 10010\dots) = 1/16.$$

- a) Show that d is a metric on L .
- b) Show that L is totally bounded.