1. Correction to proof of Lemma 2/12/13

Lemma 1.1. *Suppose M is a simple (left) module for a ring A with 1. Write $D = \operatorname{End} M$ for the endomorphisms of M as an A-module; this is a division ring, and we regard M as a right vector space over D.*

Suppose E is any m-dimensional D-subspace of M, and $x \notin E$. Then there is an element $r \in A$ such that $rE = 0$, but $rx \neq 0$.

Proof. Induction on m. If $m = 0$, this is obvious; the element $r = 1$ will do. So suppoose $m > 0$, and that we know the result for $m - 1$. Write

$$E = F \oplus Dy,$$

with F an $m-1$-dimensional subspace, and define $J = \operatorname{Ann} F$. We want to show that there is an element $j \in J$ such that

$$jy = 0, \qquad jx \neq 0.$$

Suppose not; that is, suppose that

$$jy = 0 \implies jx = 0 \qquad (j \in J). \tag{1}$$

The inductive hypothesis guarantees that $Jy \neq 0$, so (since M is simple) $Jy = M$. The condition (1) allows us to define an endomorphism α of M by

$$\alpha(jy) = jx \qquad (j \in J).$$

The fact that J is a left ideal implies that α is an A-module map, so $\alpha \in D$. In particular, this means that α respects the action of J, so its definition may be written as

$$j\alpha(y) = jx \qquad (j \in J),$$

or equivalently

$$j(\alpha y - x) = 0 \qquad (j \in J).$$

By the inductive hypothesis, it follows that $\alpha y - x \in F$, and therefore that $x \in E$, a contradiction. □