

March 30, 2016: David Vogan (MIT), *Langlands parameters and finite-dimensional representations II*.

This is a continuation of the talk February 3, but is meant to be accessible without having seen that.

Again the goal is to find something like parametrizations of representations of compact groups in terms of arithmetic and Langlands dual groups. I'll state three theorems and a conjecture. If F is a local field, write $W_{F,c}$ for the (unique) maximal compact subgroup of the Weil group of F , and $WD_{F,c}$ for the corresponding Weil-Deligne group.

- (1) If K is compact connected Lie and ${}^\vee G$ a complex dual group, then \widehat{K} is in one-to-one correspondence with homomorphisms from $S^1 = W_{\mathbb{C},c}$ to ${}^\vee G$ (modulo ${}^\vee G$ conjugacy). This is Cartan-Weyl highest weight theory.
- (2) Suppose G is connected reductive complex algebraic, endowed with an inner class of real forms σ , with L -group ${}^L G$. Write $K(\sigma)$ for a maximal compact subgroup of $G(\mathbb{R}, \sigma)$. Then the union over σ of the sets $\widehat{K(\sigma)}$ is in one-to-one correspondence with ${}^\vee G$ -equivariant local systems on the space of L -homomorphisms from $W_{\mathbb{R},c}$ to ${}^L G$.

This is a version of my thesis, formulated in this way about ten years ago. Even if $K(\sigma)$ is connected, this parametrization of the representations of $K(\sigma)$ may be quite different from the one in (1).

- (3) Suppose F is non-archimedean local and G is connected reductive \overline{F} -algebraic, endowed with an inner class of rational forms σ , with L -group ${}^L G$. Write $\{K_j(\sigma)\}$ for the collection of maximal compact subgroups of $G(\sigma)$. Then (conjecturally) the union over σ of the set of lowest depth representations of $K_j(\sigma)$ (appearing in various irreducibles for $G(\sigma)$) is in one-to-one correspondence with ${}^\vee G$ -equivariant local systems on the space of L -homomorphisms from $WD_{F,c}$ to ${}^L G$.

If the center of G is not connected, then the theorem in (2) and the conjecture in (3) need to be stated with “strong rational forms” to be correct.

I'll explain (this is the third theorem) how the conjecture in (3) includes (for the case of depth zero representations) Lusztig's classification of the irreducible representations of finite Chevalley groups.