October 14: David Vogan (MIT), “Representations of Hecke algebras and Hermitian forms.”

Suppose (W, S) is a Weyl group. The Hecke algebra of W is an algebra $\mathcal{H}_{\mathbb{Z}}$ over $\mathbb{Z}[q]$ generated by elements $T_s (s \in W)$ subject to the braid relations and to

$$(T_s + 1)(T_s - q) = 0.$$

Kazhdan-Lusztig character theory involves representations of this Hecke algebra (free over $\mathbb{Z}[q]$).

In character theory, the element $m \in \mathbb{Z}$ typically represents m copies of some irreducible representation. This comes ultimately from thinking of $1 \in \mathbb{Z}$ as representing a one-dimensional vector space. In order to study not just characters but also Hermitian forms, it is useful to enlarge $\mathbb{Z}$ to the ring

$$\mathbb{W} = \mathbb{Z}[\omega]/(\omega^2 = 1) = \mathbb{Z} + \omega\mathbb{Z},$$

in which ω represents a one-dimensional space with a negative-definite Hermitian form. In this setting, the Hecke algebra may naturally be replaced by an algebra $\mathcal{H}_{\mathbb{W}}$ over $\mathbb{W}[q]$ generated by elements T_s subject to the braid relations and to

$$(T_s + 1)(T_s - q\omega) = 0.$$

I'll explain how to use Kazhdan-Lusztig theory for Hermitian forms to define representations of $\mathcal{H}_{\mathbb{W}}$.