September 26: Dmitry Boyarchenko (University of Chicago): "Characters of unipotent groups over finite fields."

In 1966, J. Thompson conjectured that the dimension of every complex irreducible representation of the group $UL_n(\mathbb{F}_q)$ is a power of q. Here, $\mathbb{F}_q$ is a finite field with q elements and $UL_n(\mathbb{F}_q)$ is the group of unipotent upper-triangular matrices of size n with entries in $\mathbb{F}_q$. A somewhat more general statement was proved by I. M. Isaacs in 1995.

One of the main results to be discussed in my talk explains the "geometry" behind Thompson's conjecture and Isaac's more general theorem. Namely, if G is a unipotent algebraic group over $\mathbb{F}_q$ with the property that every geometric point of G lies in the neutral connected component of its own centralizer, then the dimension of every complex irreducible representation of the finite group $G(\mathbb{F}_q)$ is a power of q.

One of the key ingredients in the proof is the study of L-packets of irreducible representations of $G(\mathbb{F}_q)$, which can be defined for any connected unipotent group G over $\mathbb{F}_q$. Most of my talk will be devoted to a discussion of L-packets and their basic properties. At the end of the talk I will sketch a proof of the theorem stated in the previous paragraph.