18.781 Problem Set 1

Due Monday, February 11 in class.

1. There is no really good notation for writing numbers in arbitrary bases, because of the need to make up more and more distinct symbols. For this problem, I'll write a number in base 31 as

$$(15)(6)(25)_{31} = 15 \cdot 31^2 + 6 \cdot 31 + 25 = 14426;$$

here all the numbers on the right, and in parentheses on the left, are base 10. When the base is ten or less, so that each parenthetical term on the left is a single digit, I'll omit the parentheses.

1(a). Write 2019 in base 17.

1(b). What was the most recent year divisible by 17?

1(c). What was the most recent year divisible by 289?

2. Division with remainder works for dividing by *any* positive integer, including 1. Explain why this does not lead to a "base 1" representation of any positive integer as a string of zeros.

3. An integer written in base 10 is divisible by 3 if and only if the sum of the digits is divisible by 3.

3(a). Explain why this is true.

3(b). Give a similar method to decide whether an integer written in base 8 is divisible by 3.

3(c). What's the remainder when $(25)(11)(7)(17)_{31}$ is divided by 30?

4. It's often said that zero is critical to decimal notation, because it keeps track of what power of ten each digit represents. Like many things that everyone knows, this is nonsense. There is a version of the division algorithm in which the remainders are taken not between 0 and $a - 1$ but between 1 and a. Using this algorithm, one finds that every positive integer N has a unique expression

$$N = R_m \cdot 8^m + \cdots + R_1 \cdot 8 + R_0,$$

with $1 \leq R_i \leq 8$. We call this the *super 8 expression*, and write it

$$(R_m)(R_{m-1}) \cdots (R_1)(R_0)^8 = N.$$

For example,

$$368^8 = 370_8, \qquad 588^8 = 610_8.$$

4(a). Find the super 9 expression for $739_{10} = 1010_9$.

4(b). What does this problem have to do with problem 2?