1. 18.757 Homework 5

Due Thursday, May 2.

1. Use the Weyl dimension formula to list all the (complex continuous) irreducible representations of the compact connected Lie group of type G_2, of dimension at most 200.

2. Suppose G is a compact connected Lie group with maximal torus T, and $\xi \in X^*(T)$. Write $\pi(\xi)$ for the unique irreducible representation of G of extremal weight ξ. Show that the character $\Theta(\xi)$ of $\pi(\xi)$ is real-valued if and only if ξ is conjugate to $-\xi$ under the Weyl group $W(G, T)$.

3. List the dominant weights of the 64-dimensional irreducible representation of G_2.