1. 18.757 Homework 3

Due Tuesday, February 26.

1. Recall from the first problem set the space

$$H^k(\mathbb{C}^n)$$

of degree k harmonic polynomials in n variables, carrying a representation of the orthogonal group $O(n)$. Prove that the restriction of this representation to $O(n-1)$ decomposes as a direct sum

$$H^k(\mathbb{C}^n)|_{O(n-1)} \simeq \bigoplus_{j=0}^{k} H^j(\mathbb{C}^{n-1}).$$

(Possible hint: try to prove some statement about *all* polynomials first.)

2. Prove that $H^k(\mathbb{C}^n)$ is an irreducible representation of $O(n)$. (Hint: the Frobenius reciprocity result proved in class can be helpful.)

3. Give an example of a finite group G, a field k, and an irreducible representation V_π over k, with the property that $D_\pi = k$, and the composite of the maps

$$C(G,k)(\pi) \hookrightarrow C(G,k) \twoheadrightarrow \mathrm{End}_k(V_\pi) \xrightarrow{\sim} V_\pi \otimes_k V_\pi^* \xrightarrow{\sim} C(G,k)(\pi)$$

is zero. Here the second map is the operator-valued Fourier transform

$$f \mapsto \sum_{x \in G} f(x)\pi(x),$$

and the last is the inverse Fourier transform discussed in class.